New Approach for Non-Periodic Short-Term Forecasting: GARCH(p,q) Smoothing Hybrid Grey-CLMS with BPNN Weighting

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Abstract

A new approach, Garch smoothing hybrid GREY-CLMS prediction with BPNN weighting, is introduced herein for the applications of the non-periodic short-term forecasting. The Grey-CLMS hybrid prediction with BPNN weighting for the applications of the non-periodic short-term forecasting in order to overcome the crucial problem, overshooting and undershooting predicted outputs, by the way of compensation between grey and cumulative 3 points least squared linear predictions to yield the pretty satisfactory results. However, some predicted values have been shown not precisely enough as the observations are really far away from the both grey and cumulative 3 points least squared linear prediction outputs. Therefore, this paper proposes a new approach, GARCH(p,q) smoothing hybrid GREY-CLMS prediction with BPNN weighting, in which ARMAX/GARCH composite model has been incorporated into the hybrid GREY-CLMS prediction, and keep employing back-propagation neural net to train/simulate their weights so as to highly improve the prediction accuracy due to the smoothness enhanced. The proposed method is tested successfully in the empirical examples on the topics of international stock price indexes.

Keyword：GARCH smoothing hybrid GREY-CLMS prediction with BPNN-weighting, GM(1,1|α) prediction model, Cumulative 3 points least mean squared linear model.

1. INTRODUCTION

1.1 Background

The forecast system, that provides the services concerning the predictive information for supporting the decision-making to the public or private sector in society, in fact involves highly complicated and uncertain situation or structure [1]. Modeling a forecasting system is widely discussed and studied for years, especially the topic about the trend analysis on time series or index series. Basically, the forecast is executed through means of extrapolating a value at the next time instant according to the forecast model equation. However, some of models, e.g. Holt-Winters smoothing, Regression method, and Box-Jenkins, usually require a lot of observed data for fitting their models to build better approach [2][3] so that these models probably do not be suitable for short-term forecast because of only fewer data available. Secondly, some of models, e.g., Holt-Winters smoothing, Regression method, and Time series method, are applicable for the problems where data characterizes the cyclic variation or normal distribution [3]. Contrarily, GM(1,1|α) model introduced in [4] just needs fewer data for modeling without any training processes that means a simple and fast method. Thus, GM(1,1|α) model is often utilized in the short-term forecast for years.

1.2 Motivations

Although GM(1,1|α) model takes the advantage of simple and fast to predict the future output, the prediction accuracy is also still arguable in many papers [5] since the overshooting problem causes a big residual error around the turning points where the extreme peak or valley output occurs. However, cumulative 3 points least squared linear model introduced in this study results in the damped outputs around the turning points. Therefore, a hybrid GREY-CLMS prediction with BPNN weighting model approach overcomes the crucial problems happened mentioned above by the way of compensation between grey and cumulative 3 points least squared linear prediction outputs [6] to yield the pretty satisfactory results in the forecasts. Nevertheless, some predicted values have been shown not precisely enough as the observations are really far away from the both grey and cumulative 3 points least squared linear prediction outputs as shown from Fig. 3 to 5. This situation can be explained that the data sequences have the volatility clustering phenomenon and the generalization of hybrid GREY-CLMS prediction can not track this trip too long, i.e. smoothing not enough. Accordingly, this paper proposes a new approach, Garch smoothing hybrid GREY-CLMS prediction with BPNN weighting, in which ARMAX/GARCH [7][8]composite model has been incorporated into the hybrid GREY-CLMS prediction, and keep employing back-propagation neural net to train/simulate their weights so as to adaptively improve the prediction accuracy. The verification of several experiments in the stock price indexes forecasting has done successfully.

2. NON-PERIODIC SHORT-TERM PREDICTION MODELS

The major prediction models applied in this study will be described in the following subsections for non-periodic short-term forecast. The first subsection explains ARMAX/GARCH composite model. The second one introduces the gery prediction model GM(1,1|α), and the third shows the cumulative least mean squared polynomial model.

2.1 ARMAX/GARCH composite model

A GARCH prediction model allows a flexible model description of conditional mean, using a general ARMAX form, and conditional variance, employing a GARCH(p,q) process [9]. This ARMAX/GARCH composite model can performance simulation, forecasting, and parameter estimation of univariate time series in the presence of conditional heteroscedasticity, especially in financial time series applications like asset return problem. ARMAX models encompass autoregressive (AR), moving average (MA), and regression (X) models, in any combinations as described below.

$$y(t) = C + \sum_{i=1}^{r} R_i y(t-i) + \varepsilon(t) + \sum_{j=1}^{m} M_j \varepsilon(t-j) + \sum_{k=1}^{N_x} \beta_k X(t,k) \qquad (1)$$

where X is an explanatory regression matrix in which each column is a time series and X(t,k) denotes the t th row and k th column.

The GARCH(p,q) models the conditional variance as a standard GARCH process with Gaussian innovations and its mathematical expression is shown as follows.

$$\sigma^2(t) = K + \sum_{i=1}^{p} G_i \sigma^2(t-i) + \sum_{j=1}^{q} A_j \varepsilon^2(t-j) \qquad (2)$$

where $\sigma^2(t)$ represents the conditional variance and $\varepsilon^2(t-j)$ stands for the j-lag residual from ARMAX modeling with Gaussian distribution.

2.2 Grey prediction model GM(1,1|α)

A grey prediction model GM(1,1|α) is introduced in the grey system theory [4]. This model needs a series of data to be equipped with a smoothing exponential trend, and applies this trend for predicting the unseen output. Transforming the given data by 1-AGO process aims to generate a set of data with approximately exponential curve, and thus these data facilitate to build the appropriate prediction model. The following steps briefly describe GM(1,1|α) modeling.

Step 1: starting with accumulated generating operation once (1-AGO)

$$x^{(1)}(k) = \sum_{j=1}^{k} x^{(0)}(j), \quad k = 1,2,...,n \qquad (3)$$

$x^{(0)}(k)$: the original sampled data that is a nonnegative sequence

Step 2: finding developing coefficient a and control coefficient b by using grey difference equation

$$x^{(0)}(k) + az^{(1)}(k) = b, \; k = 2,3,...,n \qquad (4)$$

$$z^{(1)}(k) = \alpha x^{(1)}(k) + (1-\alpha)x^{(1)}(k-1), \; 0 \le \alpha \le 1 \qquad (5)$$

$z^{(1)}(k)$: the background value

$$a = \frac{\sum_{k=2}^{n} x^{(0)}(k) \sum_{k=2}^{n} z^{(1)}(k) - (n-1)\sum_{k=2}^{n} x^{(0)}(k) z^{(1)}(k)}{(n-1)\sum_{k=2}^{n} z^{(1)}(k)^2 - (\sum_{k=2}^{n} z^{(1)}(k))^2} = \frac{\Delta_a}{\Delta} \qquad (6)$$

$$b = \frac{\sum_{k=2}^{n} x^{(0)}(k) \sum_{k=2}^{n} z^{(1)}(k)^2 - \sum_{k=2}^{n} z^{(1)}(k) \sum_{k=2}^{n} x^{(0)}(k) z^{(1)}(k)}{(n-1)\sum_{k=2}^{n} z^{(1)}(k)^2 - (\sum_{k=2}^{n} z^{(1)}(k))^2} = \frac{\Delta_b}{\Delta} \qquad (7)$$

Step 3: solving the predicted value $\hat{x}^{(1)}(k)$ through the grey differential equation and performing inverse of accumulated generating operation once (1-IAGO) to obtain $\hat{x}^{(0)}(k)$

$$\frac{d\hat{x}^{(1)}(k)}{dk} + a\hat{x}^{(1)}(k) = b \qquad (8)$$

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) = (x^{(0)}(1) - \frac{b}{a})(e^{-a(k-1)} - e^{-a(k-2)}), \; k = 2,3,... \qquad (9)$$

2.3 Cumulative least mean squared polynomial model

The cumulative least mean squared polynomial model is constructed as follows:

Step 1: accumulated generating operation once (1-AGO)

$$y^{(1)}(k)=\sum_{j=1}^{k}y^{(0)}(j),\quad k=1,2,...,n \tag{10}$$

$y^{(0)}(k)$: the original sampled data that is a nonnegative sequence.

Step 2: Building least mean squared polynomial model generally in the following way:

$$\hat{y}^{(1)}(i)=c_0+c_1x^{(0)}(i)+c_2x^{(0)^2}(i)+\cdots\cdots+c_kx^{(0)^k}(i) \tag{11}$$

In order to solve the coefficients $c_0,c_1,\cdots\cdots,c_k$ in Eq. (11), the least square method is employed so as to minimize the sum of square of the residual error in Eq. (12) expressed below:

$$\text{min.}\quad q=\sum_{i=1}^{n}\varepsilon_i^2=\sum_{i=1}^{n}(y^{(1)}(i)-\hat{y}^{(1)}(i))^2 \tag{12}$$

$$s.t.\quad \hat{y}^{(1)}(i)=c_0+c_1x^{(0)}(i)+c_2x^{(0)^2}(i)+\cdots\cdots+c_kx^{(0)^k}(i)$$

The least square method is again used to solve the best approximation solution for x to the equation of

$$X_aC=Y'. \tag{13}$$

According to the definition of the following vectors:

$$X_a=\begin{bmatrix}X_a(1)^T\\X_a(2)^T\\ \cdot\\ \cdot\\ \cdot\\X_a(n)^T\end{bmatrix}$$: Matrix X_a

$X_a(i)=[1,x^{(0)}(i),x^{(0)^2}(i),...,x^{(0)^k}(i)]^T$: Augment vector $X_a(i)$

$Y'=[y^{(1)}(1),y^{(1)}(2),...,y^{(1)}(k)]^T$: Vector Y'

$C=[c_0,c_1,c_2,...,c_k]^T$: Vector C

Step 3: Derived the normal equation to find pseudo inverse matrix

Solving for Eq. (13) typically turns out to be a normal equation [10],

$$X_a^TX_aC=X_a^TY' \tag{14}$$

, in which matrix C is a coefficient vector for $c_0,c_1,\cdots\cdots,c_k$ in Eq. (12) and Y' is observed values given by in Eq. (13).

Step 4: Solving the appropriate coefficients and Predicting the next output

The solution to C in the normal equation is equal to X_a^+Y' where X_a^+ is a pseudo inverse of matrix X_a defined as $(X_a^TX_a)^{-1}X_a^T$.

$$C=(X_a^TX_a)^{-1}X_a^TY'=X_a^+Y' \tag{15}$$

$$\hat{y}^{(1)}(n+1)=X_a(n+1)C. \tag{16}$$

$$\hat{y}^{(0)}(n+1)=\hat{y}^{(1)}(n+1)-\hat{y}^{(1)}(n). \tag{17}$$

3. GARCH(p,q) SMOOTHING HYBRID GREY-CLMS PREDICTION WITH BPNN-WEIGHTING

The main objective of this paper is to propose a GARCH smoothing hybrid GREY-CLMS prediction for solving the problem that always occurred in the traditional models mentioned above. The reason why we apply GARCH smoothing hybrid GREY-CLMS into the applications of non-periodic short-term forecasts is to avoid the consequence of overshooting or undershooting effects in the predicted results.

3.1 Motivation for hybrid models prediction

According to the analysis discussed in [11], decreasing the number of sampling points as possible as we can, and lessening the effect of the magnitude of original data can lower the residual error of GM(1,1|α) model. Thus, using a few sampled points for GM(1,1|α) prediction would achieve the better prediction accuracy. This imply that this kind of GM(1,1|α) model is thus applicable of being a non-periodic short-term forecast. Furthermore, how to alleviate the effect of the magnitude of the original given data so as to reduce the residual error of GM(1,1|α) model is considered as a crucial issue. Based on the phenomena discovered in [12], the prediction of GM(1,1|α) model is always to reveal an overshooting output around the turning points since the extreme magnitude (too high or too low) happens there as shown in Fig. 3, 4, 5, and 6.

Accordingly, the predicted value from the grey prediction model will turn out to be an overestimated result at the position of turning points. However, a cumulative least mean squared linear forecast using the most recent few sampled points is introduced herein to compensate the problem in GM(1,1|α) so as for lessening the residual error resulted from the overshooting effect. Notice that the advantage of first accumulated generating operation (1-AGO) in effect is to convert the randomly distributed data to a set of data that is characterized to be approximately exponential distribution [13]. That is, seeking a regular property in data set instead of the irregular one is a great deal of facilitating model construction. Likewise, the GM(1,1|α) modeling also takes 1-AGO transformation, and hence the accumulated non-negative input data can form a kind of approximately exponential curve for facilitating the forecast work [4]. As we know, the cumulative least mean squared linear prediction causes the predicted results having the underestimated outputs around the turning points region, as shown in Fig. 1. This situation is conversely to the situation happened to GM(1,1|α) forecasts. Therefore, we intuitively can apply this reverse characteristic to offset the overshooting results such that in fact alleviating the effect of the residual error in GM(1,1|α) forecasts can be achieved [6]. A cumulative 3 points least mean squared linear prediction combining with GM(1,1|α) prediction is therefore exploited for a hybrid prediction $\hat{z}^{(0)}(k)$ as follows.

$$\hat{z}^{(0)}(k) = w_1\hat{x}^{(0)}(k) + w_2\hat{y}^{(0)}(k)\,, \qquad w_1 + w_2 = 1 \tag{18}$$

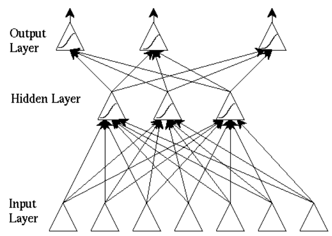
In Eq. (18), $\hat{x}^{(0)}(k)$ and $\hat{y}^{(0)}(k)$ stand for the predicted value from a grey model and the predicted value from a cumulative 3 points least mean squared linear model, respectively; moreover, the w_1 and w_2 represent the weight of $\hat{x}^{(0)}(k)$ and $\hat{y}^{(0)}(k)$, respectively. Accordingly, this proposed hybrid prediction is thus denoted as GREY-CLMS in abbreviation where CLMS means the cumulative least mean squared. In order to obtain the values of the weights w_1 and w_2, we have to build a learning machine to determine the appropriate weights for $\hat{x}^{(0)}(k)$ and $\hat{y}^{(0)}(k)$. To do so, the intelligent computation can be considered as a learning machine approach to train and simulate the weight values in w_1 and w_2. However, the random walk signal always shows a sign of volatility clustering in many social works, e.g. stock price index or asset returns of financial time series. This is because the residuals exerting from the modeling of ARMAX or ARIMAX possibly implies the time-varying conditional variance and its volatility cluster behavior in data sequences [9]. Nevertheless, some predicted values have been shown not precisely enough as the observations is really far away from the both grey and cumulative 3 points least squared linear prediction outputs as shown from Fig. 3 to 5. This situation can be explained that the data sequences have the volatility clustering phenomenon and the generalization of hybrid GREY-CLMS prediction can not track this trip too long, i.e. smoothing not enough. Therefore, we have to add something control factor to the hybrid GREY-CLMS prediction so as for enhancing the capability about the smoothing effect or generalization in the forecasts. A GARCH(p,q) model has been incorporated into the hybrid GREY-CLMS prediction so that this GARCH model can actualize the function of smoothness among the variant models. In this way, the output of GARCH smoothing hybrid GREY-CLMS prediction with intelligent weighting can be done as follows.

$$\hat{z}^{(0)}(k) = w_1\hat{h}^{(0)}(k) + w_2\hat{x}^{(0)}(k) + w_3\hat{y}^{(0)}(k)\,, \qquad w_1 + w_2 + w_3 = 1 \tag{19}$$

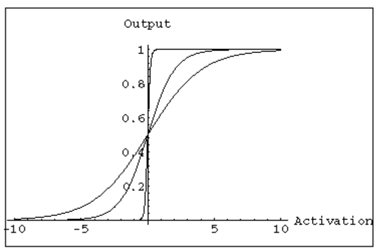
In Eq. (19), $\hat{h}^{(0)}(k)$ represents the predicted value from a GARCH(p,q) model as indicated by $y(t)$ in Eq. (1), and both $\hat{x}^{(0)}(k)$ and $\hat{y}^{(0)}(k)$ stand for the same denotation in Eq. (18). Moreover, the w_1, w_2, and w_3 represent the weight of $\hat{h}^{(0)}(k)$, $\hat{x}^{(0)}(k)$, and $\hat{y}^{(0)}(k)$, respectively.

3.2 BPNN-Based Weighting Grey-CLMS Outputs

A well-known intelligent computing machine, back-propagation neural net (BPNN) [14], is utilized in this GARCH smoothing hybrid GREY-CLMS prediction as tuning the weights for the hybrid forecast output $\hat{z}^{(0)}(k)$ in Eq. (19). Even though the BPNN is suitable for the long-term learning applications and needs many historical data for its adaptive training to model an unknown system [15], the weighting function used in this GARCH smoothing hybrid GREY-CLMS short-term prediction can actually perform well as just providing a few observed data into network for training. This is because the objective of the applied BPNN is merely to tune the appropriate weights for the outputs of GARCH smoothing hybrid GREY-CLMS, not for simulating the whole mapping feature or representation between input patterns and output targets in the specific topics, e.g., pattern recognition or functional approximation. By examining mean-square-error of the forecast outputs after several experiments as shown in the following section, the simulation results allow to be acceptable from the point of view in forecast accuracy. Therefore, we apply the BPNN, as shown in Fig.1, as weighting machine to yield the weights for the weighted-sum between the outputs of GARCH smoothing hybrid GREY-CLMS.



(a)



(b)

Fig. 1. (a) A typical BPNN architecture, and (b) Tangent sigmoid function as activations for neurons.

The block diagram of this GARCH smoothing hybrid GREY-CLMS non-periodic short-term prediction is depicted in Fig. 2. This diagram has clearly illustrated the procedure of training phase and simulation phase, and explained in detail the operation of GARCH smoothing hybrid GREY-CLMS forecast introduced in this study. Based on this system, the applications of either stochastic type forecast (e.g. options price indexes) or random walk type forecast (e.g. stocks or futures price indexes) both of them can be accomplished successfully.

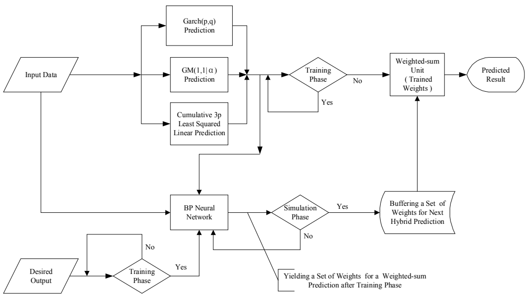


Fig. 2. The block diagram represents the procedure of GARCH smoothing hybrid GREY-CLMS prediction with BPNN weighting for short-term non-periodic forecasting system.

4. EXPERIMENTAL RESULTS AND DISCUSSIONS

This section provides an experiment for the verifications of the proposed hybrid stochastic type prediction, that is, international stock price indexes forecast. Computer simulations have illustrated below followed by the discussions for verifying the proposed advance hybrid prediction algorithm performed successfully.

The international stock price indexes [16] have been cited herein for testing the proposed hybrid stochastic type prediction algorithm to fit in with the real world dynamic marketing analysis. This forecast analysis has sampled a set of data for a period of 15 months from December 2000 to February 2002. These data have referred to the stock price indexes for four stock exchange markets — U.S.A, Taiwan, Japan, and South Korea.

The hybrid stochastic type prediction is applied to these four international stock price indexes for forecasting the next possibility index at the next time instant as shown in Fig. 3, 4, 5, and 6. A performance criterion derived by Mean-Square-Error, will be used to compare the forecast performance for every simulation, and several competitor models applied for the same forecast topic, as listed in Table 1, have also discussed in this study. For operation in forecast, a set of 4 most recent actual data is sampled to construct different prediction models for evaluating the single-step-ahead predicted result that is the output of the hybrid prediction. As the desired value at this time moment is obtained, discarding the first datum in the current data set and adding the newest observed data into the data set placing the fourth position to form a new data set that will be used to build the prediction models for next single-step-ahead forecast. This process repeats until the final point is predicted. In Fig. 3, 4, 5, and 6 the predicted sequence 1 represents as the output of GM(1,1|α) prediction, the predicted sequence 2 stands for the output of cumulative 3 points least mean squared linear prediction, the predicted sequence 3 denotes the output of the hybrid GREY-CLMS prediction, and the predicted sequence 4 indicates the output of proposed GARCH smoothing hybrid GREY-CLMS prediction. In Table 1, the comparison of MSE between the competitor models has been made, and the method we proposed reaches the best accuracy in the forecasts. Furthermore, the goodness of model fit for GARCH smoothing hybrid GREY-CLMS prediction with BPNN weighting is also test by Q-test [1], and null hypothesis cannot be rejected after verification of p-value greater than level of significance (5%).

5. CONCLUSIONS

In order to get rid of overshooting and improve the prediction accuracy, this study presents the GARCH smoothing hybrid GREY-CLMS prediction with BPNN weighting for the non-periodic short-term forecasting. Reducing the residual error in the predicted results to increase the prediction accuracy is an important idea because it deeply affects the performance of forecasts. The proposed approach fulfils this idea and obtains the satisfactory results in the non-periodic short-term forecasts. The following statements summarize the achievement of this proposed hybrid prediction.

(a) A compensation mechanism is introduced by employing GARCH smoothing hybrid GREY-CLMS prediction that GARCH(p,q) can yield the smoothness for affecting an underestimated output from cumulative 3 points least mean squared linear prediction to offset the overshooting predicted result from the GM(1,1|α) prediction to effectively solve the problem of a huge singleton residual error around the turning point region, and also can significantly highly smooth the outputs among three models to dramatically increase the overall forecast accuracy.

(b) Exploiting back-propagation neural net to tune the appropriate weights among outputs of GARCH(p,q) prediction, GM(1,1|α) prediction, and cumulative 3 points least mean squared linear prediction so as for incorporating the intelligent learning ability into non-periodic short-term forecast applications.

(c) The GARCH smoothing hybrid GREY-CLMS prediction with BPNN weighting we proposed herein can be applied suitably into stochastic or random walk type forecasts. As examining in Table 1, the fact is that the application employing GARCH smoothing hybrid GREY-CLMS prediction is better than that using hybrid GREY-CLMS prediction thus due to the more generalization or smoothing effect enhanced by GARCH.

6. ACKNOWLEDGEMENTS

This work is fully supported by the National Science Council, Taiwan, R.O.C., under grant number **NSC91-2626-E-230-002**.

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Table 1. The mean squared error (MSE) in International Stock Price Indexes Forecast Dated from Dec. 2000 to Feb. 2002 up to 15 months. (unit=10^5)

Methods	NY D.J. Indus. Index	Taiwan TAIEX Index	Japan Nikkei Index	Korea Composite Index	Average of MSE
GM	5.4701	4.6396	6.5163	0.0499	4.1690
C3LSP	4.0171	4.7252	8.5029	0.0585	4.3259
4LSP	5.2073	5.6145	7.9089	0.0522	4.6957
HW	7.3462	9.2697	12.6903	0.0801	7.3466
BJ	13.4619	8.9066	6.2920	0.1024	7.1907
RBFNN	2.87088	3.7655	6.2989	0.0653	3.2501
GRNN	2.5912	3.6922	6.0469	0.0589	3.0973
GARCH	4.4743	5.2811	5.7104	0.0396	3.8764
GCBPW	2.5783	3.2496	5.0311	0.0324	2.7229
GGCBPW	2.5075	3.2560	4.7847	0.0316	2.6450

Note: Method abbreviation
1. GM- GM(1,1|α) Model
2. C3LSP- Cumulative 3 Points Least mean squared Linear Model
3. 4LSP- 4 Points Least mean squared Linear Model
4. HW-Holt-Winters Smoothing Model
5. BJ-Box-Jenkins Forecasting Model
6. RBFNN- Radial Basis Function Neural Net
7. GRNN- General Regression Neural Net
8. GARCH- Generalized Auto-Regressive Conditional Heteroskedasticity
9. GCBPW- Grey-CLMS BPNN-Weighting
10. GGCBPW- GARCH Smoothing Hybrid Grey-CLMS with BPNN-Weighting

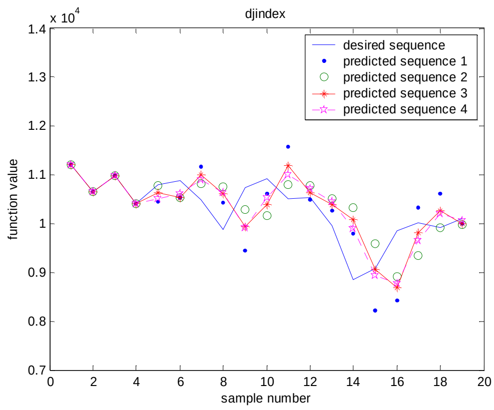

Fig. 3. The stochastic type prediction for the forecast of New York D.J. Indus. Index dated from December 2000 to February 2002 for 15 months.

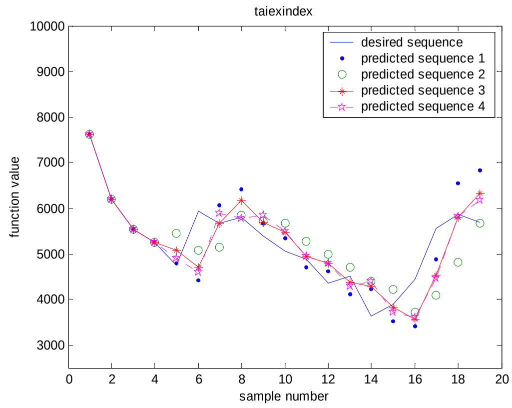

Fig. 4. The stochastic type prediction for the forecast of Taiwan TAIEX Index dated from December 2000 to February 2002 for 15 months.

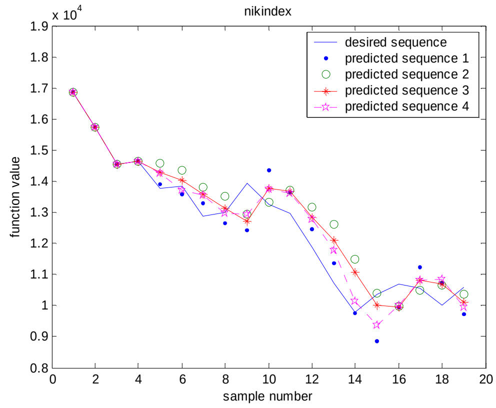

Fig. 5. The stochastic type prediction for the forecast of Japan Nikkei Index dated from December 2000 to February 2002 for 15 months.

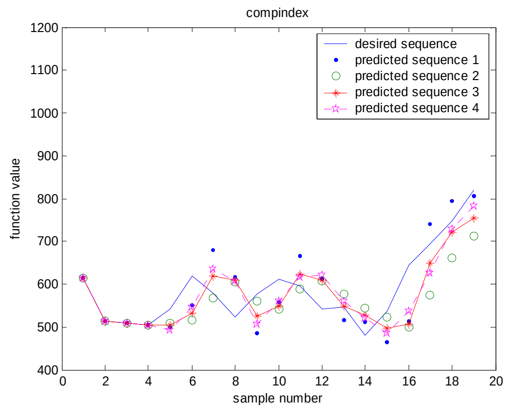

Fig. 6. The stochastic type prediction for the forecast of Korea Composite Index dated from December 2000 to February 2002 for 15 months.