# Asymmetric Duopoly under Different Market Structures

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# Abstract

When goods are substitutes (complements), we find a clear price (output) ranking across five duopoly markets, namely Cournot, Bertrand, Cournot-Stackelberg, Bertrand-Stackelberg, and joint profit maximization. We explain these rankings in terms of levels of conjectural variation.

Key words: market comparison; price/output ranking; conjectural variation

JEL classification: L11; L13; D43

## 1. Introduction

It is well known that the outcome of a duopoly market varies when competition takes different forms: Bertrand, Cournot, Cournot-Stackelberg (C-S), Bertrand-Stackelberg (B-S), and joint profit maximization (JPM). Comparative studies of these market structures, such as Cournot versus Bertrand, have obtained interesting results, but a complete comparison for the five market structures has not been conducted. For instance, Hathaway and Rickard (1979) and Cheng (1985) showed that at least one firm's output (price) must be higher (lower) under Bertrand duopoly than under Cournot. In an asymmetric linear duopoly, Singh and Vives (1984) obtained more definite results: both firms' outputs (prices) are higher (lower) in Bertrand equilibrium than in Cournot. Vives (1985) and Okuguchi (1987) also found that Cournot prices are always higher than Bertrand, given substitute goods. The output and price comparisons involving sequential Stackelberg games have only been carried out under symmetry assumptions. Anderson and Engers (1992) showed that, with symmetric firms, the C-S equilibrium price is lower and the output is

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higher than in Cournot equilibrium. With symmetric firms but non-linear demand, Dastidar (2004) found that the B-S market is generally, but not always, more competitive than the C-S market.

Symmetry assumptions impose serious limitations on the generality of the theory, since firms are rarely identical in the real world. In fact, even most obvious conclusions in symmetric cases may fail to hold in asymmetric ones. For example, the JPM market structure is commonly considered to be the least competitive one, often associated with the lowest outputs and the highest prices. But this is not necessarily the case in asymmetric duopolies, even if goods are substitutes. Hence, it remains unresolved whether there exists a clear ranking for the equilibrium prices or quantities in these five market structures.

The first aim of the present paper is to fill this gap. With linear asymmetric demand and cost functions, we obtain a clear-cut price (output) ranking for the five market structures when goods are substitutes (complements). The second aim is to explain these rankings. The five market structures differ in three aspects: (i) sequential versus simultaneous moves (ii) price versus quantity competition, and (iii) cooperative versus non-cooperative equilibrium. Given this diversity, it seems difficult to find a single framework to analyze the rankings. Interestingly, we find that the price and output rankings are related to an old economic concept, conjectural variation (CV). Indeed, they coincide with the rankings of the levels of CV corresponding to the equilibrium outcomes of the five markets. The reason for obtaining the same rankings is that each ranking is an indication of market competitiveness, which in the case of CV is the consequence of the toughness of firm behavior. In other words, toughness of firm behavior leads to higher market competitiveness.

Since its introduction by Bowley (1924), the acceptance of CV has not been without debate. It is widely acknowledged that CV is not usually consistent with unbounded rationality. However, the vast literatures on behavioral economics and experimental economics convincingly demonstrate that human behavior often deviates far from unbounded rationality, even in very simple laboratory circumstances. Thus, CV is still valuable for analyzing the behavior of firms. Indeed, empirical evidence of CV behavior have been found in Iwata (1974), Brander and Zhang (1990), Haskel and Martin (1994), and Erickson (1997). In the words of Schmalensee (1988), CV can be "best interpreted as reduced form parameters that summarize the intensity of rivalry that emerges from what may be complex patterns of behavior" (p. 650). Cabral (1995) is of the same opinion. Moreover, the consistency of CV has been revived by researchers using frameworks of bounded rationality and evolutionary processes. Among others, this includes McMillan (1984), Dixon and Somma (2003), Dockner (1992), Friedman and Mezzetti (2002), Figuières et al. (2004b), Jean-Marie and Tidball (2006), and Müller and Normann (2005, 2007). The adoption of the CV approach in applied theory papers can also be found in Green (1999) and Saracho (2005). Giocoli (2005) and Figuières et al. (2004a) provide good surveys on the recent CV literature.

Up to now, the CV literature has mainly focused on symmetric cases. For

instance, it is known that in symmetric Cournot oligopoly, CV can generate the entire range of outcomes between Bertrand and JPM (see Fama and Laffer, 1972; Kamien, 1975; Anderson, 1977). In a symmetric Bertrand duopoly, Pfaffermayr (1999) argues that CV can be interpreted as collusive behavior under optimal punishment strategies, covering the full range of possible outcomes from Bertrand equilibrium to JPM. However, it is unclear whether these findings also apply to asymmetric situations. Furthermore, there seems to be lack of a thorough investigation about the link between the competitiveness of the five markets and the level of CV in asymmetric duopoly.

The next section presents the asymmetric duopoly model and solves the equilibrium prices and outputs for the five market structures. In Section 3, the price and output rankings in the five market structures are obtained. Section 4 relates these rankings to the levels of CV associated with each market structure. Section 5 concludes.

#### 2. Model

In this section we look at a linear asymmetric duopoly model. We assume the representative consumer has a quadratic utility function  $u = x_0 + a_1x_1 + a_2x_2 - 0.5(b_1x_1^2 + b_2x_2^2 + 2rx_1x_2)$ , where  $x_1$  and  $x_2$  are the duopoly outputs,  $x_0$  is a numeraire good, and  $a_1$ ,  $a_2$ ,  $b_1$ , and  $b_2 > 0$  are parameters. We assume  $b_1b_2 > r^2$ , so the utility function is strictly concave in  $x_1$  and  $x_2$ . Given the two goods' prices,  $p_1$  and  $p_2$ , the consumer maximizes her utility subject to a budget constraint  $x_0 + p_1x_1 + p_2x_2 \le m$ , where *m* is the consumer's income. When *m* is sufficiently high, the utility maximization yields the following inverse and direct asymmetric demand functions for good *i* (*i*, *j* = 1, 2 and  $i \ne j$ ):

$$p_i = a_i - b_i x_i - r x_j, \tag{1}$$

$$x_{i} = \frac{b_{j}(a_{i} - p_{i}) - r(a_{j} - p_{j})}{b_{1}b_{2} - r^{2}}.$$
(2)

Each firm *i* has a constant marginal cost  $c_i < a_i$ , and its profit function is  $\pi_i = (p_i - c_i)x_i$ . Without loss of generality, we let firms 1 and 2 be the leader and the follower respectively in any Stackelberg game. Given  $b_1$ ,  $b_2 > 0$ , and  $b_1b_2 > r^2$ , the profit functions are strictly concave in all five markets and are maximized when the first-order conditions hold.

To make our comparison of the five market structures meaningful, we need to ensure that in every market equilibrium both firms produce positive outputs and all equilibrium prices are higher than the marginal costs. This is guaranteed when goods are complements, but when goods are substitutes we need the following assumption.

# Assumption 1. We assume $b_i(a_i - c_i) - r(a_j - c_j) \ge 0$ for $i, j = 1, 2, i \ne j$ .

This is equivalent to assuming that the JPM outputs are positive. It ensures

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positive output and price margins in every market equilibrium; this can be verified using (3) and (4) below. This assumption is identical to the necessary condition pointed out by Amir and Jin (2001) for the result in Singh and Vives (1984), i.e., Bertrand outputs are always higher than Cournot outputs. Hence it is also needed for any price or output ranking among the five market structures.

In order to solve the equilibrium prices and outputs in the five market structures we use the following first-order conditions for firm i = 1, 2:

Cournot: 
$$a_i - c_i - 2b_i x_i - rx_j = 0$$
,  
Bertrand:  $b_j a_i - ra_j + b_j c_i - 2b_j p_i + rp_j = 0$ ,  
JPM:  $b_j a_i - ra_j + b_j c_i - rc_j - 2b_j p_i + 2rp_j = 0$ ,  
C-S leader:  $a_1 - c_1 - 2b_1 x_1 - rx_2 + 0.5r^2 x_1 = 0$ ,  
C-S follower:  $a_2 - c_2 - 2b_2 x_2 - rx_1 = 0$ ,  
B-S leader:  $b_2 a_1 - ra_2 + b_2 c_1 - 2b_2 p_1 + rp_2 + 0.5r^2 (p_1 - c_1) = 0$ ,  
B-S follower:  $b_1 a_2 - ra_1 + b_1 c_2 - 2b_1 p_2 + rp_1 = 0$ .

From these conditions and the demand functions (1) and (2), we can solve the equilibrium prices and outputs. Let the superscripts C, B, CS, BS, and J stand for Cournot, Bertrand, C-S, B-S, and JPM, respectively. Then, the equilibrium prices for each of the market structures are:

$$p_i^C = c_i + \frac{b_i [2b_j (a_i - c_i) - r(a_j - c_j)]}{4b_i b_2 - r^2},$$
(3a)

$$p_i^B = c_i + \frac{(2b_i b_2 - r^2)(a_i - c_i) - rb_i(a_j - c_j)}{4b_i b_2 - r^2},$$
(3b)

$$p_i^J = c_i + \frac{1}{2}(a_i - c_i),$$
 (3c)

$$p_1^{CS} = c_1 + \frac{2b_2(a_1 - c_1) - r(a_2 - c_2)}{4b_2},$$
(3d)

$$p_2^{CS} = c_2 + \frac{(4b_1b_2 - r^2)(a_2 - c_2) - 2b_2r(a_1 - c_1)}{4(2b_1b_2 - r^2)},$$
(3e)

$$p_1^{BS} = c_1 + \frac{(2b_1b_2 - r^2)(a_1 - c_1) - rb_1(a_2 - c_2)}{2(2b_1b_2 - r^2)},$$
(3f)

$$p_2^{BS} = c_2 + \frac{b_1(4b_1b_2 - 3r^2)(a_2 - c_2) - r(2b_1b_2 - r^2)(a_1 - c_1)}{4b_1(2b_1b_2 - r^2)}.$$
 (3g)

The equilibrium outputs are:

$$x_i^C = \frac{2b_j(a_i - c_i) - r(a_j - c_j)}{4b_1 b_2 - r^2},$$
(4a)

$$x_i^B = \frac{b_j [(2b_1b_2 - r^2)(a_i - c_i) - rb_i(a_j - c_j)]}{(b_1b_2 - r^2)(4b_1b_2 - r^2)},$$
(4b)

$$x_i^J = \frac{b_j(a_i - c_i) - r(a_j - c_j)}{2(b_1 b_2 - r^2)},$$
(4c)

$$x_1^{CS} = \frac{2b_2(a_1 - c_1) - r(a_2 - c_2)}{2(2b_1b_2 - r^2)},$$
(4d)

$$x_2^{CS} = \frac{(4b_1b_2 - r^2)(a_2 - c_2) - 2rb_2(a_1 - c_1)}{4b_2(2b_1b_2 - r^2)},$$
(4e)

$$x_1^{BS} = \frac{(2b_1b_2 - r^2)(a_1 - c_1) - rb_1(a_2 - c_2)}{4b_1(b_1b_2 - r^2)},$$
(4f)

$$x_{2}^{BS} = \frac{b_{1}(4b_{1}b_{2} - 3r^{2})(a_{2} - c_{2}) - r(2b_{1}b_{2} - r^{2})(a_{1} - c_{1})}{4(b_{1}b_{2} - r^{2})(2b_{1}b_{2} - r^{2})}.$$
 (4g)

It is easy to check that Assumption 1 guarantees that all the outputs are positive and the prices are higher than the marginal costs.

### 3. Price and Output Rankings

In this section we compare the equilibrium outputs and prices between different market structures, given any possible values of parameters  $a_1$ ,  $a_2$ ,  $c_1$ ,  $c_2$ ,  $b_1$ , and  $b_2$  and subject to the conditions assumed earlier. The substitute and complementary goods cases will be separately considered.

## **3.1 Substitute Goods** ( $r \ge 0$ )

(i) *Output comparison*: When goods are substitutes, no clear ranking can be found for output comparison. For instance, even if firms are symmetric, the output comparison for two firms is indeterminate, e.g.,  $x_1^{BS} \le x_1^B$  but  $x_2^{BS} \ge x_2^B$ . When firms are asymmetric, the output comparison is indeterminate for the same firm, such as in the JPM versus Cournot case.

(ii) *Price comparison*: Given  $r \ge 0$ , we use expression (3) and Assumption 1 to compare equilibrium prices. Contrary to the output comparison and despite firm asymmetries in demand and costs, the following proposition shows a more general result.

*Proposition 1.* When goods are substitutes, there is a clear price ranking for the five market structures:

$$p_i^J \ge p_i^C \ge p_i^{CS} \ge p_i^{BS} \ge p_i^B.$$
(5)

Intuitively, this clear ranking in prices, in contrast to outputs, seems to be related to the strategic complementarity of prices when the goods are substitutes. This strategic complementarity ensures that one firm's higher price encourages the other firm to raise its price, leading to a clear price ranking. We will further explain how this works using CV in the next section.

(iii) **Consumer surplus and social welfare comparison**: As lower prices make consumers better off, the ranking for consumer surplus is exactly the opposite of the one in (5). The social welfare, however, is only comparable when outputs can be ranked, e.g.,  $SW^c$  is lower than  $SW^B$ ,  $SW^{BS}$ , and  $SW^{CS}$ . Otherwise the welfare comparison is indeterminate. For instance, when  $b_i = 1$ ,  $c_i = 0$ , r = 1/2,  $a_1 = 1$ , and  $a_2 = 2$ , the social welfare is slightly lower in Bertrand than in B-S markets.

#### **3.2 Complementary Goods** (r < 0)

(i) *Price comparison*: When goods are complements, no clear ranking can be found for price comparison. In some cases, the comparisons are indeterminate even if firms are symmetric, e.g.,  $p_1^B \le p_1^{BS}$  but  $p_2^B \ge p_2^{BS}$ . When firms are asymmetric, the price comparison is indeterminate, as in the JPM versus Bertrand case.

(ii) *Output comparison*: Using Assumption 1 and expression (4) and despite firm asymmetry in demand and costs we get the following proposition.

*Proposition 2.* When goods are complements, there is a clear output ranking for the five market structures:

$$x_i^J \ge x_i^B \ge x_i^{CS} \ge x_i^{CS} \ge x_i^C.$$
(6)

Intuitively, this clear ranking in outputs, in contrast to prices, seems to be related to the strategic complementarity of outputs when the goods are complements. This strategic complementarity ensures that one firm's higher output encourages the other firm to raise its output, leading to a clear output ranking. We will further explain how this works using CV in the next section.

(iii) Consumer surplus and social welfare comparison: When the output can be ranked, so can the social welfare (SW). As  $SW = u(\mathbf{x}) - \mathbf{c'x}$ , where  $\mathbf{x} = (x_1, x_2)$  and  $\mathbf{c} = (c_1, c_2)$ , we have  $\partial SW/\partial \mathbf{x} = u'(\mathbf{x}) - \mathbf{c'} = \mathbf{p} - \mathbf{c}$ . Then, when prices are higher than marginal costs, which is the case for all five markets, higher outputs guarantee higher social welfare. Hence we have a clear welfare ranking identical to that of outputs. Consumer surplus is comparable when prices can be ranked, e.g.,  $CS^c$  is lower than  $CS^B$ ,  $CS^{BS}$ , and  $CS^J$ . Otherwise the consumer surplus comparison is indeterminate. For instance, when  $b_i = 1$ ,  $c_i = 0$ , r = -1,  $a_1 = 1$ , and  $a_2 = 2$ , the consumer surplus is higher in Cournot than in C-S markets.

In the following section, we explain the output and price rankings using CV.

# 4. Ranking and CV

As mentioned in the introduction, the five market structures differ in three

aspects: (i) sequential versus simultaneous moves, (ii) price versus quantity competition, and (iii) cooperative versus non-cooperative equilibrium. Given this diversity, we use CV as a single framework to explain the price and output rankings, i.e., to compare all equilibrium outcomes. Once again we give separate consideration to the cases of substitute and complementary goods.

### **4.1 Substitute Goods** ( $r \ge 0$ )

First, we show that all five market equilibrium outcomes can be obtained by price competition with various CV. The reason for using price competition is that when goods are substitutes, prices are strategic complements, which is crucial to generate a clear ranking. We posit that each firm *i* has a conjecture  $\sigma_i = \partial p_j / \partial p_i$  on its rival's response to its own price change. Given  $\sigma_i$ , the first-order condition for profit maximization for firm *i* is:

$$x_{i} - \frac{b_{j}(p_{i} - c_{i})}{b_{1}b_{2} - r^{2}} + \frac{r\sigma_{i}(p_{i} - c_{i})}{b_{1}b_{2} - r^{2}} = 0.$$
(7)

From (7) we can solve  $\sigma_i$  as a function of firm *i* price and output:

$$\sigma_{i} = \frac{1}{r} \left[ b_{j} - \frac{(b_{l}b_{2} - r^{2})x_{i}}{p_{i} - c_{i}} \right].$$
(8)

Substituting the equilibrium price and output from each market structure, (3) and (4), into (8) we find their corresponding  $\sigma_i$ . Obviously, for Bertrand competition we have  $\sigma_i^B = 0$ . Using (3a) and (4a) we find  $\sigma_i^C = r/b_i$  in a Cournot market. For JPM we use (3c) and (4c) to get  $\sigma_i^J = (a_j - c_j)/(a_i - c_i)$ . For B-S,  $\sigma_2^{BS} = 0$ , (3f) and (4f) imply  $\sigma_1^{BS} = r/2b_1$ . For C-S, (3d) and (4d) imply  $\sigma_1^{CS} = rb_2/(2b_1b_2 - r^2)$ , while (3e) and (4e) imply  $\sigma_2^{CS} = r/b_2$ . In spite of our asymmetry assumptions, the comparison of these values leads to the following clear ranking:  $\sigma_i^J \ge \sigma_i^C \ge \sigma_i^{CS} \ge \sigma_i^B$ .

Clearly, this ranking is identical to the ranking in (5). To explain this identity it is sufficient to show that an increase in  $\sigma_i$  results in higher equilibrium prices for both firms. Let us write the first-order condition for firm *i* as  $b_j a_i - 2b_j p_i + r(a_j - p_j) + b_j c_i + r\sigma_i(p_i - c_i) = 0$ . Then, we get the response function for firm *i*:

$$p_{i} = c_{i} + \frac{b_{j}(a_{i} - c_{i}) - r(a_{j} - p_{j})}{2b_{i} - r\sigma_{i}}.$$
(9)

Given (9) and  $r \ge 0$ , we see that the response function is upward sloping and shifts upwards with any rise in  $\sigma_i$ . Hence, increases in  $\sigma_1$  and  $\sigma_2$  shift response curves rightward/upward and must result in higher equilibrium prices.

Figure 1. The Impact of CV on the Equilibrium with Upward Sloping Reaction Curves



However, this conclusion may not hold when r < 0 because the response functions are downward sloping. Notice that a rise in  $\sigma_i$  shifts the reaction curves downwards, which makes at least one firm's price lower, but not necessarily both.

Figure 2. The Impact of CV on the Equilibrium with Downward Sloping Reaction Curves



# **4.2** Complementary Goods (r < 0)

We now explain the output ranking when goods are complements. As was the

case with substitute goods, the exploitation of strategic complementarity is required. With complementary goods this can be obtained by reformulating the model in terms of quantity competition and quantity CV parameters. We first show that the ranking of quantity CV parameters always coincides with that of price CV parameters, so we can use our  $\sigma_i$  to obtain the former.

Let  $\theta_i = \partial x_i / \partial x_i$  be the conjecture by firm *i* of its rival's response to its own output change. Notice that we are looking for the  $\theta_i$  that generate the same equilibrium outcome as the  $\sigma_i$ . To find the relation between  $\theta_i$  and  $\sigma_i$ , we use the first-order condition for firm *i* in quantity competition:

$$p_i - c_i - b_i x_i - r\theta_i x_i = 0.$$
<sup>(10)</sup>

Combining (10) with (8) for the same equilibrium  $x_i$  and  $p_i$ , we find that:

$$\theta_i = \frac{1}{r} \left( \frac{b_1 b_2 - r^2}{b_j - r \sigma_i} - b_i \right)$$

It is obvious that  $\theta_i$  is increasing in  $\sigma_i$ . Thus its ranking must be identical to that of  $\sigma_i$ . When r < 0, the previous ranking of  $\sigma_i$  is no longer valid. Nonetheless, using the same  $\sigma_i$  as before, it is straightforward to check that when r < 0 the ranking of  $\sigma_i$  is  $\sigma_i' \ge \sigma_i^B \ge \sigma_i^{BS} \ge \sigma_i^{CS} \ge \sigma_i^C$  and so we have:

$$\theta_i^J \ge \theta_i^B \ge \theta_i^{BS} \ge \theta_i^{CS} \ge \theta_i^C$$

Notice that the ranking of  $\theta_i$  is identical to the output ranking in (6). Thus we can use the former and the response function to explain the latter. Rewriting the first-order condition (10) as  $a_i - c_i - 2b_i x_i - rx_j - r\theta_i x_i = 0$ , we solve for the response function for firm *i*:

$$x_i = \frac{a_i - c_i - rx_j}{2b_i + r\theta_i}$$

For r < 0, the response curve is upward sloping and shifts upwards when  $\theta_i$  rises. Increases in  $\theta_1$  and  $\theta_2$  shift both response curves upward and result in higher equilibrium outputs, similar to the case of price competition with substitute goods (see Figure 1). Therefore, a ranking of  $\theta_i$  implies an output ranking. Again, this argument does not work when r > 0. In that case, the reaction curves become downward sloping and shift downwards with any rise in  $\theta_i$ . As shown in Figure 2, a rise in  $\theta_i$  results in a lower output for at least one firm, but not necessarily both.

Therefore, when the levels of CV can be ranked, we can rank the strategic complementary variables, i.e., prices of substitute goods and outputs of complementary goods. In the context of the five considered markets, the level of CV is a good indicator of market competitiveness.

## 5. Final Remarks

The present paper deals with two issues in asymmetric duopoly. First, given substitute (complement) goods we obtain a clear price (output) ranking across five asymmetric and linear duopoly structures: Cournot, Bertrand, Cournot-Stackelberg, Bertrand-Stackelberg, and joint profit maximization. Second, we obtain an identical ranking in the level of price or output conjecture variation in the five market structures. These simple results suggest some internal connections between seemingly unrelated market structures and firms' strategic reactions. Intuitively, to the extent that the CV ranking reflects the toughness of firm behavior, it tends to influence market competitiveness.

There are further issues worth exploring. First, the linear model has several limitations in spite of its wide usage in teaching and in theoretical and empirical research. Thus, looking at the existence of price and output rankings in non-linear models is a natural area of future research. Second, as we only examined duopoly, asymmetric oligopoly may also be a fruitful extension for future investigation.

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