A Nonlinearly Weighted Sobel Edge Detector

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Abstract - *In this paper, a modified Sobel edge detector (SED) is proposed where the gradient is nonlinearly weighted. The modified SED is called nonlinearly weighted SED (NWSED). In the SED, only the difference of neighbor pixels is considered in the gradient calculation. However, in the NWSED the ratio of neighbor pixels is put into account. The NWSED is then analyzed and found of nonlinear weighting property. With the property, the difference of low gradients and high gradients is increased. To justify the proposed NWSED, simulations are performed. The results indicate that the NWSED has better performance than the SED in the preserving of edge information.*

Keywords: Edge Detection, Sobel, Nonlinearly Weighted Gradient

1. Introduction

Edge detection is an important preprocessing scheme in the image processing such as image segmentation and analysis [1]. Several approaches to edge detection have been reported. Some proposed schemes are Sobel [2], Canny [3], Marr and Hildreth [4], and so forth. Because of its simplicity in computation, the Sobel edge detector (SED) is one of popular approaches. In a digital image, the gradient in the SED is found by approximating the differences of neighbor pixels. Note that the ratio of neighbor pixels also provides useful information in edge detection. This paper presents a modified SED, which is called nonlinearly weighted SED (NWSED). In the proposed NWSED, weighs are nonlinearly added on the gradients calculated from the SED. This paper is organized as follows. In Section 2, a brief review of the SED is given. In Section 3, the proposed NWSED is described and the nonlinear weighting property is derived as well. In Section 4, simulation results are given to justify the proposed approach. Finally, conclusive remarks are made in Section 5.

2. Review of the SED

In this section, the SED is briefly reviewed. For details, one may consult [1, 2]. Given image $\boldsymbol{I}$, the magnitude of gradient at the (j,k) pixel is approximated as

$$|G(j,k)| = |G_x(j,k)| + |G_y(j,k)| \tag{1}$$

where $G_x(j,k)$ and $G_y(j,k)$ are the gradients along x-axis and y-axis, respectively. Denote a 3×3 sub-image of $\boldsymbol{I}$ as

$$\boldsymbol{I}_{3\times3} = \begin{bmatrix} I(j-1,k-1) & I(j-1,k) & I(j-1,k+1) \\ I(j,k-1) & I(j,k) & I(j,k+1) \\ I(j+1,k-1) & I(j+1,k) & I(j+1,k+1) \end{bmatrix} \tag{2}$$

The gradient $G_x(j,k)$ is calculated by the following two steps. First, a new 3×3 sub-image $\boldsymbol{I}_x$ is found as

$$\boldsymbol{I}_x = \boldsymbol{M}_x .* \boldsymbol{I}_{3\times3} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} .* \begin{bmatrix} I(j-1,k-1) & I(j-1,k) & I(j-1,k+1) \\ I(j,k-1) & I(j,k) & I(j,k+1) \\ I(j+1,k-1) & I(j+1,k) & I(j+1,k+1) \end{bmatrix} \tag{3}$$

where

$$\boldsymbol{M}_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \tag{4}$$

is the mask for $G_x(j,k)$ and .* is an element-to-element multiplication. Then $G_x(j,k)$ is obtained as

$$G_x(j,k) = \sum_{m=1}^{3}\sum_{n=1}^{3} I_x(m,n) \tag{5}$$

where $I_x(m,n)$ are elements of $\boldsymbol{I}_x$. Similarly, the gradient $G_y(j,k)$ is found as in $G_x(j,k)$ except the mask used in (3) is $\boldsymbol{M}_y = (\boldsymbol{M}_x)^T$. By (1), the gradient image of $\boldsymbol{I}$ is obtained through $G_x(j,k)$ and $G_y(j,k)$. Note that in the SED only the difference of neighbor pixels is involved in the calculation of gradient.

3. The Proposed NWSED

In this section, the proposed NWSED modified from the SED is described. Then the nonlinear weighting property of NWSED in the gradient calculation is derived. Note that the ratio of neighbor pixels also provides useful information for edge detection. In the proposed NWSED, the ratio information of neighbor pixels is put into the gradient calculation where masks $\boldsymbol{M}_x$ and $\boldsymbol{M}_y$ are, respectively, modified as

$$\boldsymbol{M}_{x,NW} = \begin{bmatrix} -1 & 0 & 1 \\ -2\alpha & 0 & 2\alpha \\ -1 & 0 & 1 \end{bmatrix} \tag{6}$$

and

$$\boldsymbol{M}_{y,NW} = \begin{bmatrix} -1 & -2\beta & -1 \\ 0 & 0 & 0 \\ 1 & 2\beta & 1 \end{bmatrix} \tag{7}$$

In (6), α is the ratio obtained from $\max\{I(j,k-1)/I(j,k+1), I(j,k+1)/I(j,k-1)\}$ and in (7) the ratio β is found as $\max\{I(j-1,k)/I(j+1,k), I(j+1,k)/I(j-1,k)\}$. Note that α and β are no less than one. By masks $\boldsymbol{M}_{x,NW}$ and $\boldsymbol{M}_{y,NW}$, the gradients obtained from the SED are nonlinearly weighted. This property is derived in the following.

In the derivation, only $\boldsymbol{M}_{x,NW}$ is considered since the derivation is equally well for $\boldsymbol{M}_{y,NW}$. Note that the difference of gradient $G_x(j,k)$ with masks $\boldsymbol{M}_{x,NW}$ and $\boldsymbol{M}_x$ is given as

$$\Delta G_x(j,k) = 2(\alpha-1)[I(j,k+1) - I(j,k-1)] \tag{8}$$

Without loss of generality, assume $I(j,k+1) \geq I(j,k-1)$. Then $\alpha - 1$ can be expressed as

$$\alpha - 1 = \frac{I(j,k+1)}{I(j,k-1)} - 1 = \frac{I(j,k+1) - I(j,k-1)}{I(j,k-1)} \tag{9}$$

Therefore, (8) can be written as

$$\Delta G_x(j,k) = \frac{2[I(j,k+1) - I(j,k-1)]^2}{I(j,k-1)} \tag{10}$$

which is always positive with $I(j,k-1) > 0$. From (10), it is obvious that $\Delta G_x(j,k)$ is a nonlinear function of the difference $I(j,k+1) - I(j,k-1)$ and $I(j,k-1)$. It implies two things in (10). First, for a given $I(j,k-1)$ the difference $I(j,k+1) - I(j,k-1)$ will be nonlinearly increased by the square law. Second, for a fixed $I(j,k+1) - I(j,k-1)$ a smaller $I(j,k-1)$ results in a larger increase in $\Delta G_x(j,k)$.

With masks $\boldsymbol{M}_{x,NW}$ and $\boldsymbol{M}_{y,NW}$, in the NWSED the magnitude of gradient at the (j,k) pixel of $\boldsymbol{I}$ is calculated as

$$\begin{aligned} |G_{NW}(j,k)| &= |G_x(j,k) + \Delta G_x(j,k)| \\ &\quad + |G_y(j,k) + \Delta G_y(j,k)| \end{aligned} \tag{11}$$

where

$$\begin{aligned} \Delta G_y(j,k) &= 2(\beta-1)[I(j+1,k) - I(j-1,k)] \\ &= \frac{2[I(j+1,k) - I(j-1,k)]^2}{I(j-1,k)} \end{aligned} \tag{12}$$

is the added weight on $G_y(j,k)$. In (12), $I(j+1,k) \geq I(j-1,k)$ is assumed. Note that in (12) gradient $G_x(j,k)$ may be reduced by $\Delta G_x(j,k)$, which is always positive, if $G_x(j,k)$ is negative. This is also true for $G_y(j,k)$. To avoid the problem, (11) is modified as

$$\begin{aligned} |G_{NW}(j,k)| &= |G_x(j,k) + \mathrm{sgn}[G_x(j,k)]\Delta G_x(j,k)| \\ &\quad + |G_y(j,k) + \mathrm{sgn}[G_y(j,k)]\Delta G_y(j,k)| \end{aligned} \tag{13}$$

where $\mathrm{sgn}(x)$ is the signum function whose value is +1 if $x \geq 0$ and −1 otherwise.

To understand the nonlinearity of $\Delta G_x(j,k)$ in the NWSED, an example is given in the following. Let $I(j,k-1) = 20$ and $|I(j,k+1) - I(j,k-1)|$ be 10, 20, 30, 40, and 50, respectively. Then, by (10) $\Delta G_x(j,k)$ is 10, 40, 90, 160, and 250, respectively. As in the example, $\Delta G_x(j,k)$ is nonlinearly increased as $|I(j,k+1) - I(j,k-1)|$ increases. Note that the difference between low gradients and high gradients increases. Since the gradient at edge is high in general, the NWSED, therefore, has better ability to preserve edge information than the SED. These ideas are verified by examples in the following section.

4. Simulation Results

In this section, two examples are used to compare the NWSED and the SED. To make $\Delta G_x(j,k)$ and $\Delta G_y(j,k)$ calculable, the pixel value is set to one if it is zero. In the first example, image Cameraman shown in Figure 1 is used in the simulation. By (13), the gradient image of $|G_{NW}(j,k)|$ for the proposed NWSED is shown in Figure 2 and by (1) the gradient image of $|G(j,k)|$ for the SED is given in Figure 3. The difference image of $|G_{NW}(j,k)|-|G(j,k)|$ is shown in Figure 4. For better observation, Figures 2 to 4 are displayed by the same scale and in the inverted mode. From Figure 4, it can be seen that the gradients on edges in image Cameraman have been intensified and that the parts with larger contrast, e.g. the contour of coat, have larger difference. This is consistent with the nonlinear weighting property in the NWSED.

In the second example, image Tank shown in Figure 5 is used in the simulation. As in the first example, by (13) and (1) $|G_{NW}(j,k)|$ and $|G(j,k)|$ are found and then the difference image of $|G_{NW}(j,k)|-|G(j,k)|$ is acquired. These images are displayed in Figures 6, 7, and 8, respectively. As before, the displaying scale is identical and in the inverted mode. In Figure 8, the parts of high gradients, such as the shadow on the ground and the star in the middle of image Tank, have been intensified more. Consequently, the nonlinear weighting property in the NWSED is verified. Both examples indicate that the proposed NWSED preserves better edge information than the SED because of the nonlinear weighting property. By the simulation results, the proposed NWSED is justified.

5. Conclusions

In this paper, the ratio of neighbor pixels is incorporated into the SED. The proposed edge detector is termed as nonlinearly weighted SED (NWSED). The proposed approach is shown that the gradient of the SED is nonlinearly weighted. That is, a larger gradient in the image has a larger weight. Since the gradient at edge is generally large, thus the magnitude of the gradient is intensified. The idea is justified by examples in the simulation. As expected, the results indicate that the proposed NWSED has better performance than the SED in the preserving the edge information.

References

[1] R.C. Gonzalez and R.E. Woods, *Digital Image Processing*, 2nd Edition, Prentice Hall, 2002.

[2] I.E. Sobel, *Camera Models and Machine Perception*, Ph.D. dissertation, Stanford University, USA, 1970.

[3] J. Canny, “A Computational Approach to Edge Detection,” *IEEE Trans. on PAMI*, Vol. 8, No. 6, pp. 679-698, 1986.

[4] D. Marr and E. Hildreth, “Theory of Edge Detection,” Proc. Royal Society of London, B, 207, pp. 187-207, 1981.

Fig. 1 Original Cameraman

Fig. 2 Gradient image of $|G_{NW}(j,k)|$ **(Cameraman)**

Fig. 3 Gradient image of $|G(j,k)|$ **(Cameraman)**

Fig. 4 Difference image of $|G_{NW}(j,k)|-|G(j,k)|$ **(Cameraman)**

Fig. 5 Original Tank

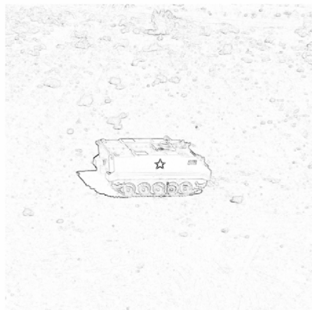

Fig. 6 Gradient image of $|G_{NW}(j,k)|$ **(Tank)**

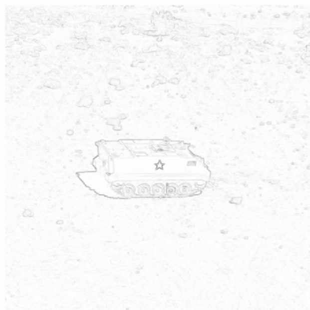

Fig. 7 Gradient image of $|G(j,k)|$ **(Tank)**

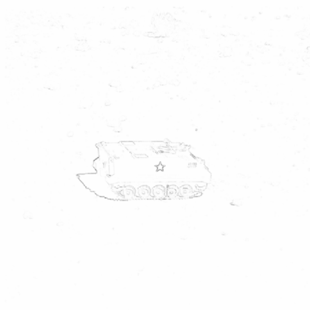

Fig. 8 Difference image of $|G_{NW}(j,k)|-|G(j,k)|$ **(Tank)**