Bipanpositionable Bipancyclic of Hypercube*

Yuan-Kang Shih, Cheng-Kuan Lin, and Jimmy J. M. Tan
Department of Computer Science
National Chiao Tung University,
Hsinchu, Taiwan 30010, R.O.C.
{ykshih,cklin,jmtan}@cs.nctu.edu.tw

Lih-Hsing Hsu
Department of Computer Science and Information Engineering
Providence University
Taichung, Taiwan 43301, R.O.C.
lhhsu@pu.edu.tw

Abstract

A bipartite graph is *bipancyclic* if it contains a cycle of every even length from 4 to $|V(G)|$ inclusive. A hamiltonian bipartite graph G is *bipanpositionable* if, for any two different vertices x and y, there exists a hamiltonian cycle C of G such that $d_C(x, y) = k$ for any integer k with $d_G(x, y) \leq k \leq |V(G)|/2$ and $(k - d_G(x, y))$ being even. A bipartite graph G is *k-cycle bipanpositionable* if, for any two different vertices x and y, there exists a cycle of G with $d_C(x, y) = l$ and $|V(C)| = k$ and for any integer l with $d_G(x, y) \leq l \leq \frac{k}{2}$ and $(l - d_G(x, y))$ being even. A bipartite graph G is *bipanpositionable bipancyclic* if G is k-cycle bipanpositionable for every even integer k, $4 \leq k \leq |V(G)|$. We prove that the hypercube Q_n is bipanpositionable bipancyclic if and only if $n \geq 2$.

Keywords: bipanpositionable, bipancyclic, hypercube, hamiltonian.

*This work was supported in part by the National Science Council of the Republic of China under Contract NSC 96-2221-E-009-137-MY3.

1 Introduction

For the graph definitions and notations we follow [4]. Let $G = (V, E)$ be a *graph*, where V is a finite set and E is a subset of $\{(u, v) \mid (u, v)$ is an unorder pair of $V\}$. We say that V is the *vertex set* and E is the *edge set* of G. Two vertices u and v are *adjacent* if $(u, v) \in E$. A *path* is represented by $\langle v_0, v_1, v_2, \cdots, v_k \rangle$, where all vertices are distinct. The *length* of a path Q is the number of edges in Q. We also write the path $\langle v_0, v_1, v_2, \cdots, v_k \rangle$ as $\langle v_0, Q_1, v_i, v_{i+1} \cdots, v_j, Q_2, v_t, \cdots, v_k \rangle$, where Q_1 is the path $\langle v_0, v_1, \cdots, v_{i-1}, v_i \rangle$ and Q_2 is the path $\langle v_j, v_{j+1}, \cdots, v_{t-1}, v_t \rangle$. We use $d_G(u, v)$ to denote the distance between u and v in G, i.e., the shortest path joining u to v in G. A *cycle* is a path of at least three vertices such that the first vertex is the same as the last vertex. We use $d_c(u, v)$ to denote the distance between u and v in a cycle C, i.e., the length of the shortest path joining u to v in C. A *hamiltonian cycle* of G is a cycle that traverses every vertex of G exactly once. A *hamiltonian graph* is a graph with a hamiltonian cycle. A graph $G = (V_0 \cup V_1, E)$

is *bipaetite* if $V(G) = V_0 \cup V_1$ and $E(G)$ is a subset of $\{(u, v) \mid u \in V_0 \text{ and } v \in V_1\}$.

The *n-dimensional hypercube*, Q_n, consists of all n-bit binary strings as its vertices and two vertices $\mathbf{u}$ and $\mathbf{v}$ are adjacent if and only if their binary labels different in exactly one bit position. Let $\mathbf{u} = u_{n-1}u_{n-2}\dots u_1u_0$ and $\mathbf{v} = v_{n-1}v_{n-2}\dots v_1v_0$ be two n-bit binary strings. The Hamming distance $h(u, v)$ between two vertices u and v is the number of different bits in the corresponding strings of both vertices. The hypercubes Q_1, Q_2, and Q_3 are illustrated in Figure 1 and Q_4 is illustrated in Figure 2. Let Q_n^i be the subgraph of Q_n induced by $\{u_{n-1}u_{n-2}\dots u_1u_0 \mid u_{n-1} = i\}$ for $i = 0, 1$. Therefore, Q_n can be constructed recursively by taking two copies of Q_{n-1}, Q_n^0 and Q_n^1, and adding a perfect matching between these two copies. Let $\mathbf{u}$ be a vertex in Q_n^0 (resp. Q_n^1), we use $\bar{\mathbf{u}}$ to denote the unique neighbor of $\mathbf{u}$ in Q_n^1 (resp. Q_n^0). The *hypercube* is a widely used topology in computer architectures [8]. There are some interesting studies in hypercube [6, 10, 13].

A graph is *pancyclic* if it contains a cycle of every length from 3 to $|V(G)|$ inclusive. The concept of pancyclic graphs is proposed by Bondy [3]. It is known that there is no odd cycle in any bipartite graph. For this reason, the concept of bipancyclic graph is proposed [7]. A bipartite graph is *bipancyclic* if it contains a cycle of every even length from 4 to $|V(G)|$ inclusive. It is proved that the hypercube Q_n is bipancyclic if $n \geq 2$ [9, 12]. A graph is *panconnected* if, for any two different vertices x and y, there exists a path of length l joining x and y with $d_G(x, y) \leq l \leq |V(G)| - 1$. The concept of panconnected graphs is proposed by Alavi and Williamson [1]. It is easy to see that any bipartite graph with at least 3 vertices is not panconnected. Therefore, the concept of bipanconnected graphs is proposed. A bipartite graph is *bipanconnected* if, for any two different vertices x and y, there exists a path of length l joining x and y with $d_G(x, y) \leq l \leq |V(G)| - 1$ and $(l - d_G(x, y))$ being even. It is proved that the hypercube is bipanconnected [9]. A hamiltonian graph G is *panpositionable* if for any two different vertices x and y of G and for any integer k with $d_G(x, y) \leq k \leq |V(G)|/2$, there exists a hamiltonian cycle C of G such that $d_C(x, y) = k$. A hamiltonian bipartite graph G is *bipanpositionable* if for any two different vertices x and y of G and for any integer k with $d_G(x, y) \leq k \leq |V(G)|/2$ and $(k - d_G(x, y))$ being even, there exists a hamiltonian cycle C of G such that $d_C(x, y) = k$. The concept of panpositionable and bipanpositionable are proposed by Kao et al. [11]. It is proved that the hypercube Q_n is bipanpositionable if $n \geq 2$ [11]. A bipartite graph G is *edge-bipancyclic* if for any edge in G, there is a cycle of every even length from 4 to $|V(G)|$ traversing through this edge. The concept of edge-bipancyclic is proposed by Alspach and Hare [2]. A bipartite graph G is *vertex-bipancyclic* if for any vertex in G, there is a cycle of every even length from 4 to $|V(G)|$ going through this vertex. The concept of vertex-bipancyclic is proposed by Hobbs [5]. Obviously, every edge-bipancyclic graph is vertex-bipancyclic. It is proved that the hypercube Q_n is edge-bipancyclic if $n \geq 2$ [9].

In this paper, we propose a more interesting property about hypercubes. A k cycle is a cycle of length k. A bipartite graph G is *k-cycle bipanpositionable* if for every different vertices x and y of G and for any integer l with $d_G(\mathbf{x}, \mathbf{y}) \leq l \leq \frac{k}{2}$ and $(l - d_G(x, y))$ being even, there exists a k cycle C of G such that $d_C(x, y) = l$. (Note that $d_C(x, y) \leq \frac{k}{2}$ for every cycle C of length k.) A bipartite graph G is *bipanpositionable bipancyclic* if G is k-cycle bipanpositionable for every even integer k with $4 \leq k \leq |V(G)|$. In this paper, we prove that the hypercube Q_n is bipanpositionable bipancyclic if and only if $n \geq 2$. As a consequence of this result, we can see that many previous results on hypercubes follows directly from ours. For example, the hypercube is bipancyclic, bipanconnected, bipanpositionable, edge-bipancyclic

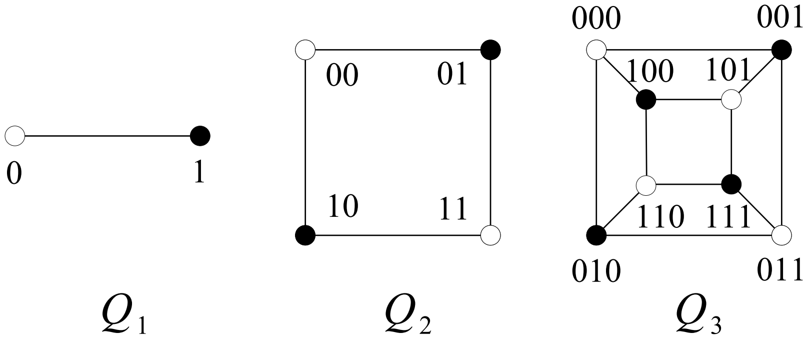

Figure 1: The graphs Q_1, Q_2 and Q_3

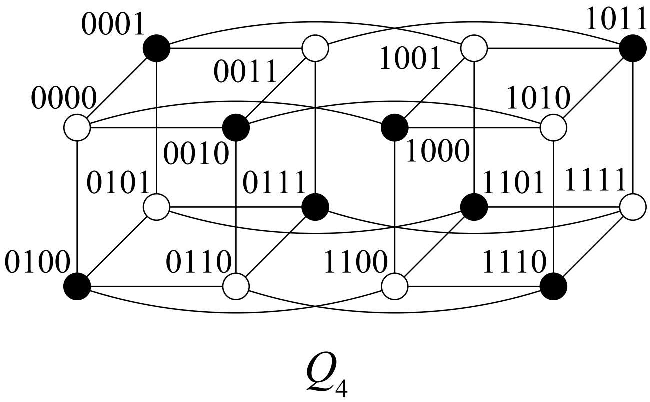

Figure 2: The 4-dimensional hypercube

and vertex-bipancyclic. Therefore, our result unify theses results in a general sense.

2 Bipanpositionable Pancyclic Property

We prove our main result by induction as stated in Lemma 1 and Theorem 1 below.

Lemma 1. *The Q_3 is bipanpositionable bipancyclic.*

Proof. Let $\mathbf{x}$ and $\mathbf{y}$ be two different vertices in Q_3. Obviously, $d_{Q_3}(\mathbf{x},\mathbf{y}) = 1, 2$ or 3. Since the hypercube is vertex symmetric, without loss of generality, we may assume that $\mathbf{x} = 000$.

Case 1: Suppose that $d_{Q_3}(\mathbf{x},\mathbf{y}) = 1$. Since Q_3 is edge symmetric, we assume that $\mathbf{y} = 001$. See Table 1

Case 2: Suppose that $d_{Q_3}(\mathbf{x},\mathbf{y}) = 2$. We have $\mathbf{y} \in \{011, 101, 110\}$. See Table 1

Case 3: Suppose that $d_{Q_3}(\mathbf{x},\mathbf{y}) = 3$. We have $\mathbf{y} = 111$. See Table 1

Thus, Q_3 is bipanpositionable bipancyclic. □

Theorem 1. *The Q_n is bipanpositionable bipancyclic if and only if $n \geq 2$.*

Proof. We observe that Q_1 is not bipanpositionable bipancyclic. So we start with $n2 \geq 2$. We prove Q_n is bipanpositionable bipancyclic by induction on n. It is easy to see that Q_2 is bipanpositionable bipancyclic. By Lemma 1, this statement holds for $n = 3$. Suppose that Q_{n-1} is bipanpositionable bipancyclic for some $n \geq 4$. Let $\mathbf{x}$ and $\mathbf{y}$ be two distinct vertices in Q_n, and let k be an even integer with $k \geq \max\{4, 2d_{Q_n}(\mathbf{x},\mathbf{y})\}$ and $k \leq 2^n$. For every integer l with $d_{Q_n}(\mathbf{x},\mathbf{y}) \leq l \leq \frac{k}{2}$ and $(l - d_{Q_n}(\mathbf{x},\mathbf{y}))$ being even, we need to construct a k-cycle C of Q_n with $d_C(\mathbf{x},\mathbf{y}) = l$.

Case 1: $d_{Q_n}(\mathbf{x},\mathbf{y}) = 1$. Without loss of generality, we may assume that both $\mathbf{x}$ and $\mathbf{y}$ are

Table 1: Proof of Lemma 1

Case 1	$\mathbf{y} = 001$	4-cycle	$d_C(\mathbf{x}, \mathbf{y}) = 1$	$\langle 000, 001, 011, 010, 000 \rangle$
		6-cycle	$d_C(\mathbf{x}, \mathbf{y}) = 1$	$\langle 000, 001, 101, 111, 110, 100, 000 \rangle$
			$d_C(\mathbf{x}, \mathbf{y}) = 3$	$\langle 000, 100, 101, 001, 011, 010, 000 \rangle$
		8-cycle	$d_C(\mathbf{x}, \mathbf{y}) = 1$	$\langle 000, 001, 101, 111, 011, 010, 110, 100, 000 \rangle$
			$d_C(\mathbf{x}, \mathbf{y}) = 3$	$\langle 000, 100, 101, 001, 011, 111, 110, 010, 000 \rangle$
Case 2	$\mathbf{y} = 011$	4-cycle	$d_C(\mathbf{x}, \mathbf{y}) = 2$	$\langle 000, 001, 011, 010, 000 \rangle$
		6-cycle	$d_C(\mathbf{x}, \mathbf{y}) = 2$	$\langle 000, 001, 011, 010, 110, 100, 000 \rangle$
		8-cycle	$d_C(\mathbf{x}, \mathbf{y}) = 2$	$\langle 000, 001, 011, 010, 110, 111, 101, 100, 000 \rangle$
			$d_C(\mathbf{x}, \mathbf{y}) = 4$	$\langle 000, 001, 101, 111, 011, 010, 110, 100, 000 \rangle$
	$\mathbf{y} = 101$	4-cycle	$d_C(\mathbf{x}, \mathbf{y}) = 2$	$\langle 000, 001, 101, 100, 000 \rangle$
		6-cycle	$d_C(\mathbf{x}, \mathbf{y}) = 2$	$\langle 000, 001, 101, 111, 110, 100, 000 \rangle$
		8-cycle	$d_P(\mathbf{x}, \mathbf{y}) = 2$	$\langle 000, 001, 101, 111, 011, 010, 110, 100, 000 \rangle$
			$d_C(\mathbf{x}, \mathbf{y}) = 4$	$\langle 000, 001, 011, 111, 101, 100, 110, 010, 000 \rangle$
	$\mathbf{y} = 110$	4-cycle	$d_C(\mathbf{x}, \mathbf{y}) = 2$	$\langle 000, 010, 110, 100, 000 \rangle$
		6-cycle	$d_C(\mathbf{x}, \mathbf{y}) = 2$	$\langle 000, 100, 110, 111, 101, 001, 000 \rangle$
		8-cycle	$d_C(\mathbf{x}, \mathbf{y}) = 2$	$\langle 000, 100, 110, 010, 011, 111, 101, 001, 000 \rangle$
			$d_C(\mathbf{x}, \mathbf{y}) = 4$	$\langle 000, 100, 101, 111, 110, 010, 011, 001, 000 \rangle$
Case 3	$\mathbf{y} = 111$	6-cycle	$d_C(\mathbf{x}, \mathbf{y}) = 3$	$\langle 000, 001, 011, 111, 110, 100, 000 \rangle$
		8-cycle	$d_C(\mathbf{x}, \mathbf{y}) = 3$	$\langle 000, 001, 011, 111, 101, 100, 110, 010, 000 \rangle$

in Q_n^0. $(l - d_{Q_n}(\mathbf{x}, \mathbf{y}))$ is even, so l is an odd number.

Case 1.1: $l = 1$. Suppose that $k \le 2^{n-1}$. By induction, there is a k-cycle C of Q_n^0 with $d_C(\mathbf{x}, \mathbf{y}) = 1$. Suppose that $k \ge 2^{n-1} + 2$. By induction, there is a 2^{n-1}-cycle C' of Q_n^0 with $d_C(\mathbf{x}, \mathbf{y}) = 1$. Without loss of generality, we write $C' = \langle \mathbf{x}, P, \mathbf{z}, \mathbf{y}, \mathbf{x} \rangle$ such that $d_P(\mathbf{x}, \mathbf{z}) = k - 2$. Suppose that $k - 2^{n-1} = 2$. Then $C = \langle \mathbf{x}, P, \mathbf{z}, \bar{\mathbf{z}}, \bar{\mathbf{y}}, \mathbf{y}, \mathbf{x} \rangle$ forms a $(2^{n-1} + 2)$-cycle with $d_C(\mathbf{x}, \mathbf{y}) = 1$. Suppose that $k - 2^{n-1} \ge 4$. By induction, there is a $(k - 2^{n-1})$-cycle C'' of Q_n^1 such that $d_{C''}(\bar{\mathbf{z}}, \bar{\mathbf{y}}) = 1$. We write $C'' = \langle \bar{\mathbf{z}}, R, \bar{\mathbf{y}}, \bar{\mathbf{z}} \rangle$ with $d_R(\bar{\mathbf{z}}, \bar{\mathbf{y}}) = k - 2^{n-1} - 1$. Then $C = \langle \mathbf{x}, P, \mathbf{z}, \bar{\mathbf{z}}, R, \bar{\mathbf{y}}, \mathbf{y}, \mathbf{x} \rangle$ forms a k-cycle of Q_n with $d_C(\mathbf{x}, \mathbf{y}) = l$.

Case 1.2: $l \ge 3$. Suppose that $k - l - 1 \le 2^{n-1}$. By induction, there is a $(l + 2)$-cycle C' of Q_n^0 with $d_{C'}(\mathbf{x}, \mathbf{y}) = 1$. We write $C' = \langle \mathbf{x}, P, \mathbf{y}, \mathbf{x} \rangle$ where $d_P(\mathbf{x}, \mathbf{y}) = l$. By induction, there is a $(k - l - 1)$-cycle C'' of Q_n^1 with $d_{C''}(\bar{\mathbf{x}}, \bar{\mathbf{y}}) = 1$. We then write $C'' = \langle \bar{\mathbf{y}}, R, \bar{\mathbf{x}}, \bar{\mathbf{y}} \rangle$ such that $d_R(\bar{\mathbf{y}}, \bar{\mathbf{x}}) = k - l - 1$. Then $C = \langle \mathbf{x}, P, \mathbf{y}, \bar{\mathbf{y}}, R, \bar{\mathbf{x}}, \mathbf{x} \rangle$ forms a k-cycle of Q_n with $d_C(\mathbf{x}, \mathbf{y}) = l$. Suppose that $k - l - 2 \ge 2^{n-1} + 1$. By induction, there is a $(k - 2^{n-1})$-cycle C' of Q_n^0 with $d_{C'}(\mathbf{x}, \mathbf{y}) = l$. We write $C' = \langle \mathbf{x}, P, \mathbf{y}, \mathbf{u}, R, \mathbf{x} \rangle$ with $d_P(\mathbf{x}, \mathbf{y}) = l$ and $d_R(\mathbf{u}, \mathbf{x}) = k - (2^{n-1} - 1) - l - 2$. By induction, there is a (2^{n-1})-cycle C'' of Q_n^1 with $d_{C''}(\bar{\mathbf{x}}, \bar{\mathbf{u}}) = 1$. We write $C'' = \langle \bar{\mathbf{x}}, \bar{\mathbf{u}}, S, \bar{\mathbf{x}} \rangle$ with $d_S(\bar{\mathbf{u}}, \bar{\mathbf{x}}) = 2^{n-1} - 1$. Then $C = \langle \mathbf{x}, P, \mathbf{y}, R, \mathbf{u}, \bar{\mathbf{u}}, S, \bar{\mathbf{x}}, \mathbf{x} \rangle$ forms a k-cycle of Q_n with $d_C(\mathbf{x}, \mathbf{y}) = l$.

Case 2: $d_{Q_n}(\mathbf{x}, \mathbf{y}) \ge 2$ and $l = 2$. Since $d_{Q_n}(\mathbf{x}, \mathbf{y}) \le l$ and $l = 2$, so $d_{Q_n}(\mathbf{x}, \mathbf{y}) = 2$. Without loss of generality, we may assume that $\mathbf{x}$ is in Q_n^0 and $\mathbf{y}$ is in Q_n^1. Then $d_{Q_n}(\bar{\mathbf{x}}, \mathbf{y}) = 1$ and $d_{Q_n}(\bar{\mathbf{y}}, \mathbf{x}) = 1$.

Suppose that $k = 4$. Then $C = \langle \mathbf{x}, \bar{\mathbf{x}}, \mathbf{y}, \bar{\mathbf{y}}, \mathbf{x} \rangle$ forms a 4-cycle of Q_n with $d_{Q_n}(\mathbf{x}, \mathbf{y}) = 2$. Suppose that $6 \le k \le 2^{n-1}+2$. By induction, there is a $(k-2)$-cycle $C' = \langle \mathbf{x}, P, \bar{\mathbf{y}}, \mathbf{x} \rangle$ of Q_n^0 such that $d_P(\mathbf{x}, \bar{\mathbf{y}}) = k - 3$. Then $C = \langle \mathbf{x}, P, \bar{\mathbf{y}}, \mathbf{y}, \bar{\mathbf{x}}, \mathbf{x} \rangle$ forms a k-cycle of Q_n with $d_C(\mathbf{x}, \mathbf{y}) = 2$. Suppose that

$k \geq 2^{n-1}+4$. By induction, there is a 2^{n-1}-cycle C' of Q_n^0 with $d_{C'}(\mathbf{x}, \bar{\mathbf{y}}) = 1$. We write $C' = \langle \mathbf{x}, P, \mathbf{z}, \bar{\mathbf{y}}, \mathbf{x} \rangle$ with $d_P(\mathbf{x}, \mathbf{z}) = 2^{n-1} - 2$. By induction, there is a $(k - 2^{n-1})$-cycle C'' of Q_n^1 with $d_{C''}(\mathbf{y}, \bar{\mathbf{z}}) = 1$. We write $C'' = \langle \mathbf{y}, \bar{\mathbf{z}}, R, \mathbf{y} \rangle$ with $d_R(\mathbf{y}, \bar{\mathbf{z}}) = k - 2^{n-1} - 1$. Then $C = \langle \mathbf{x}, P, \mathbf{z}, \bar{\mathbf{z}}, R, \mathbf{y}, \bar{\mathbf{y}}, \mathbf{x} \rangle$ forms a k-cycle of Q_n with $d_C(\mathbf{x}, \mathbf{y}) = 2$.

Case 3: $d_{Q_n}(\mathbf{x}, \mathbf{y}) \geq 2$ and $l \geq 3$. Without loss of generality, we may assume that $\mathbf{x}$ is in Q_n^0 and $\mathbf{y}$ is in Q_n^1. Suppose that $k - l - d_{Q_n}(\mathbf{x}, \mathbf{y}) + 2 \leq 2^{n-1}$. By induction, there is a $(l + d_{Q_n}(\mathbf{x}, \mathbf{y}) - 2)$-cycle $C' = \langle \mathbf{x}, P, \bar{\mathbf{y}}, \mathbf{u}, R, \mathbf{x} \rangle$ of Q_n^0 such that $d_P(\mathbf{x}, \bar{\mathbf{y}}) = l - 1$ and $d_R(\mathbf{u}, \mathbf{x}) = d_{Q_n}(\mathbf{x}, \mathbf{y}) - 2$. By induction, there is a $(k - l - d_{Q_n}(\mathbf{x}, \mathbf{y}) + 2)$-cycle C'' of Q_n^1 with $d_{C''}(\mathbf{y}, \bar{\mathbf{u}}) = 1$. We write $C'' = \langle \mathbf{y}, S, \bar{\mathbf{u}}, \mathbf{y} \rangle$ with $d_S(\mathbf{y}, \bar{\mathbf{u}}) = k - l - d_{Q_n}(\mathbf{x}, \mathbf{y}) + 1$. Then $C = \langle \mathbf{x}, P, \bar{\mathbf{y}}, \mathbf{y}, S, \bar{\mathbf{u}}, \mathbf{u}, R, \mathbf{x} \rangle$ forms a k-cycle of Q_n with $d_C(\mathbf{x}, \mathbf{y}) = l$. Suppose that $k - l - d_{Q_n}(\mathbf{x}, \mathbf{y}) + 4 \geq 2^{n-1}$. By induction, there is a $(k - 2^{n-1})$-cycle $C' = \langle \mathbf{x}, P, \bar{\mathbf{y}}, \mathbf{u}, R, \mathbf{x} \rangle$ of Q_n^0 such that $d_P(\mathbf{x}, \bar{\mathbf{y}}) = l - 1$ and $d_R(\mathbf{u}, \mathbf{x}) = k - 2^{n-1} - l$. By induction, there is a 2^{n-1}-cycle C'' of Q_n^1 with $d_{C''}(\mathbf{y}, \bar{\mathbf{u}}) = 1$. We write $C'' = \langle \mathbf{y}, S, \bar{\mathbf{u}}, \mathbf{y} \rangle$ with $d_S(\mathbf{y}, \bar{\mathbf{u}}) = 2^{n-1} - 1$. Then $C = \langle \mathbf{x}, P, \bar{\mathbf{y}}, \mathbf{y}, S, \bar{\mathbf{u}}, \mathbf{u}, R, \mathbf{x} \rangle$ forms a k-cycle of Q_n with $d_C(\mathbf{x}, \mathbf{y}) = l$.

The theorem is proved. □

References

[1] Y. Alavi and J. E. Williamson, "Panconnected Graphs", Studia Scientiarum Mathematicarum Hungarica, Vol. 10, pp.19–22, 1975.

[2] B. Alspach and D. Hare, "Edge-pancyclic Block-intersection Graphs", Discrete Mathematics, Vol. 97, pp.17–24, 1997.

[3] J. A. Bondy, "Pancyclic Graphs", Journal of Combinatorial Theory, Series B, Vol. 11, pp.80–84, 1971.

[4] J. A. Bondy and U. S. R Murty, "Graph Theory with Applications", North-Holland, New York, 1980.

[5] A. Hobbs, "The Square of a Block is Vertex Pancyclic", Journal of Combinatorial Theory, Series B, Vol. 20, pp.1–4, 1976.

[6] L.-H. Hsu, S.-C. Liu, and Y.-N. Yeh, "amiltonicity of Hypercubes with a Constraint of Required and Faulty Edges", Journal of Combinatorial Optimization, Vol. 14, pp.197-204, 2007.

[7] J. Mitchem and E. Schmeichel, "Pancyclic and Bipancyclic Graphs - a Survey", Graphs and Applications, pp.271–278, 1982.

[8] F.T. Leighton, "Introduction to Parallel Algorithms and Architecture: Arrays · Trees · Hypercubes", Morgan Kaufmann, San Mateo, CA, 1992.

[9] T.-K. Li, C.-H. Tsai, J. J.-M. Tan, and L.-H. Hsu, "Bipanconnected and Edge-fault-tolerant Bipancyclic of Hypercubes", Information Processing Letters, Vol. 87, pp.107–110, 2003.

[10] C.-K. Lin, J.J.M. Tan, D. Frank Hsu, L.-H. Hsu, "On the spanning commectively and spanning laceability of hypercube-like networks", Theoretical Computer Science, Vol. 381, pp.218-229, 2007.

[11] S.-S. Kao, C.-K. Lin, H.-M. Huang, and L.-H. Hsu, "Panpositionable Hamiltonian Graph", Ars Combinatoria, Vol. 81, pp.209–223, 2006.

[12] Y. Saad and M. H. Schultz, "Topological Properties of Hypercubes", IEEE Transactions on Computers, Vol. 37, pp.867–872, 1988.

[13] M.-C. Yang, J.J.M. Tan, and L.-H. Hsu, "Highly Fault-Tolerant Cycle Embeddings of Hypercubes", Journal of Sys-

tems Architecture, Vol. 53, pp.227-232, 2007.