

# Supervisory recurrent fuzzy neural network control for long-term ecological systems

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**Abstract-**This paper develops an intelligent ecological biomass management method called supervisory recurrent fuzzy neural network control (SRFNNC) to deal with the long-term management of ecological system, which is an uncertain nonlinear system subject to unpredictable but bounded disturbances. This SRFNNC system is composed of a recurrent fuzzy neural network (RFNN) controller and a supervisory controller. The RFNN controller is investigated to mimic an ideal controller and the supervisory controller is designed to compensate for the approximation error between the RFNN controller and the ideal controller. This SRFNNC is employed to keep the biomasses of an ecological system within a small neighborhood of the unique nontrivial optimal equilibrium state of the undisturbed ecosystem. By applying this controller, the accumulative yield of harvest is better than that obtained with state feedback control and no control.

**Keywords-** ecological system; recurrent fuzzy neural network

## 1. Introduction

For the analysis of ecosystems, which can include nonlinear phenomena such as predator switching, food limitations, and saturation of predator attack capacities, interaction in multi-species communities is a highly nonlinear affair [3]. The ecomodels also have to include explicitly possible effects of environmental disturbances. A great amount of effort has been devoted to the study of vulnerability and non-vulnerability of ecosystems subject to continual, unpredictable, but bounded disturbances due to changes in climatic conditions, diseases, and migrating species. [3,4,7]. The state feedback control method has been proposed by Lee and Leitmann [7] for controlling the disturbed ecological system.

In recent years, the neural network-based control technique has been proposed as an alternative design method for control of dynamic systems [5,11]. The most useful property of neural networks is their ability to approximate

linear or nonlinear mapping through learning. With this property, the neural network-based controllers have been developed to compensate the effects of nonlinearities and system uncertainties, so that the stability, convergence and robustness of the control system can be improved. The concept of incorporating fuzzy logic into a neural network has recently grown into a popular research topic [1,8]. The fuzzy neural network possesses advantages both of fuzzy systems and neural networks since it combines the fuzzy reasoning capability and the neural network on-line learning capability. However, the neural networks presented in [1,5,8,11] are static mapping networks of the feed-forward type. The recurrent neural network has been extensively presented since it has capabilities superior to the feedforward neural network, such as the dynamic response and the information storing ability [6,9,10]. Since a recurrent neural network has an internal feedback loop, it captures the dynamic response of a system with external feedback through delays. Thus, the recurrent neural network is a dynamic mapping network.

Few studies on management or control of ecological systems have employed intelligent methods such as fuzzy or neural network technologies. In the literature, only the biomass clustering design methodology has been used [2]. In this paper, an intelligent method called supervisory recurrent fuzzy neural network control (SRFNNC) is developed for the design of the ecological system management model. This SRFNNC system comprises a recurrent fuzzy neural network (RFNN) controller and a supervisory controller. The RFNN controller is used to mimic an ideal controller, and the supervisory controller is designed to compensate for the approximation error between the RFNN controller and the ideal controller. The RFNN is inherently a recurrent multilayered neural network for realizing fuzzy inference using dynamic fuzzy rules. Temporal relations are embedded in the network by adding feedback connections in the second layer of the fuzzy neural network. Moreover, an on-line parameter training methodology, using the gradient descent method and the Lyapunov stability theorem, is proposed to increase the learning capability. In

addition, to relax the requirement for the uncertain bound in the supervisory controller, an estimation mechanism is employed to observe the uncertain bound. Thus, the chattering phenomena of the control efforts can be relaxed. Finally, a comparison of ecological system design between the state feedback control and the proposed SRFNNC is presented to illustrate the effectiveness of the proposed design method. By applying this controller, the accumulative yield of harvest is also better than that obtained with state feedback control and no control.

## 2. Model of ecological system

Taking into consideration the complexity of the realistic ecological system, we obtain the exploited ecosystem model as a constant harvest matrix, which is a simplified model to represent the complex ecological behavior in reality [7].

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{g}(\mathbf{x}(t)) - \mathbf{H}\mathbf{x}(t) \\ \mathbf{x}(t_0) &= \mathbf{x}^0\end{aligned}\quad (1)$$

where  $\mathbf{H} = \text{diag}(h_1, \dots, h_n)$  is a constant harvest matrix,

$$\mathbf{x}(t) = [x_1(t), \dots, x_n(t)]^T, \quad x_i > 0, \quad i = 1, \dots, n$$

where is an  $n$ -dimensional biomass vector, and its  $i$ th component represents the biomass of the  $i$ th species at time  $t$  and  $\mathbf{g}(\cdot)$  is continuous. A constant harvest effort vector  $\mathbf{h} = [h_1, \dots, h_n]^T$  is assumed to be unique of the corresponding non-trivial solution of

$$\mathbf{g}(\mathbf{x}(t)) - \mathbf{H}\mathbf{x} = 0. \quad (2)$$

Let  $h^*$  be the admissible constant harvest effort that maximizes the quantity  $\beta^T \mathbf{H}\mathbf{x}$  subject to (2), and let  $\mathbf{x}^*$  be the corresponding equilibrium state of (1), where  $\beta = [\beta_1, \dots, \beta_n]^T$  is a prescribed constant price vector, which is normalized as  $\beta = [1, \dots, 1]^T$  in this paper. Thus, under optimal steady state harvesting, the exploited ecosystem (1) becomes

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{g}(\mathbf{x}(t)) - \mathbf{H}^* \mathbf{x}(t) \\ \mathbf{x}(t_0) &= \mathbf{x}^0.\end{aligned}\quad (3)$$

If the exploited system (3) is not subjected to disturbances, then the harvest rate  $\mathbf{H}^* \mathbf{x}^*$  is optimal for the long-term management of the ecosystem, that is, in the steady state. However, real ecosystem in nature is continually disturbed by unpredictable forces such as diseases, migrating species and changes in climatic conditions. To include these important effects in the model, the model is modified to be

$$\dot{\mathbf{x}}(t) = \mathbf{g}(\mathbf{x}(t)) - \mathbf{H}^* \mathbf{x}(t) + \Delta \mathbf{g}(\mathbf{x}(t), \mathbf{v}(t)) \quad (4)$$

where  $\mathbf{v}(t) = [v_1(t), \dots, v_n(t)]^T$  is the uncertainty. It is assumed that only the possible sizes of the uncertain elements are known at time  $t$  and the function  $\Delta \mathbf{g}(\cdot)$  is continuous. In view of the presence of the continually acting unpredictable disturbances, the optimally exploited ecosystem may deviate from its equilibrium state and the constant harvest effort  $\mathbf{h}^*$  may no longer be optimal. To assure that the ecosystem with uncertainty is practically stabilizable, a control term is also included in the model. Thus, (4) becomes

$$\dot{\mathbf{x}}(t) = \mathbf{g}(\mathbf{x}(t)) - \mathbf{H}^* \mathbf{x}(t) + \Delta \mathbf{g}(\mathbf{x}(t), \mathbf{v}(t)) + \mathbf{u}(t) \quad (5)$$

where  $\mathbf{u}(t) = [u_1(t), \dots, u_n(t)]^T$  is the control input (this input may be negative which can be thought as to free captive fish). Besides the harvest rate  $\mathbf{H}^* \mathbf{x}$ , the control input  $\mathbf{u}(t)$  may be interpreted as the additional harvest rate of the exploited ecosystem. Only  $x_i > 0, i = 1, \dots, n$  are concerned, since they represent the biomasses.

Consider a species of animals whose population dynamics can be described as

$$\begin{aligned}\dot{x}(t) &= \frac{r}{K} x(t)(K - x(t)) \\ x(0) &= x^0\end{aligned}\quad (6)$$

where  $x(t)$  denotes the biomass of the species at time  $t$ ,  $r$  is the intrinsic growth rate and  $K$  is the environmental carrying capacity; both  $r$  and  $K$  are positive constants. Suppose system (6) is subjected to harvesting; let  $h$  denote a constant harvest effort and assume that the corresponding harvest rate at time  $t$  is given by  $hx(t)$ . Thus, the exploited system becomes

$$\dot{x}(t) = \frac{r}{K} x(t)(K - x(t)) - hx(t). \quad (7)$$

Using elementary calculus, we find that the value of  $h$ , which maximizes the harvest at equilibrium population  $x^*(t)$ , can be derived from

$$\dot{x}(t) = 0 \quad \text{and} \quad \frac{\partial hx(t)}{\partial x(t)} = 0, \quad \text{that is}$$

$$hx(t) = \frac{r}{K} x(t)(K - x(t)), \quad (8)$$

and

$$\frac{\partial hx(t)}{\partial x(t)} = r - \frac{2r}{K} x(t). \quad (9)$$

From (8) and (9), the biomass is  $x^* = \frac{K}{2}$  and

the corresponding value of  $h^*$  is given by  $h^* = \frac{r}{2}$ . Thus, under optimal (steady state) constant harvesting, (7) becomes

$$\dot{x}(t) = \frac{r}{K} x(t)(K - x(t)) - h^* x(t) \quad (10)$$

Assume that the contribution to the growth rate of the species from the unpredictable disturbances at time  $t$  is given by  $v(t)x(t)$ , where  $v(\cdot): R \rightarrow \Psi$  and  $\Psi \equiv \{v | -\alpha \leq v \leq \alpha, \alpha = \text{constant} > 0\}$ .

In the system in (10), assume that the disturbances are given as

$$v(t) = -0.1 \cos(t). \quad (11)$$

To assure that the exploited ecosystem is practically stabilizable, we also include a control term  $u(t)$  in (10), where  $u(\cdot): R \rightarrow R$ . The control  $u(t)$  corresponds to decreasing the harvest rate at time  $t$  if its value is positive (if  $u(t) > h^*x(t)$ , harvesting is replaced by replenishing) and increasing the harvest rate if its value is negative. Thus the uncertain system becomes

$$\dot{x}(t) = \frac{r}{K}x(t)(K-x(t)) - h^*x(t) + v(t)x(t) + u(t). \quad (12)$$

An accumulative yield can be defined as

$$Y_{ac}(\tau) = \int_0^\tau (h^*x(t) - u(t))dt. \quad (13)$$

Our goal is to find an optimal control  $u^*(t)$  such that at time  $\tau$ ,  $x_1(\tau) = x_2(\tau)$  and the accumulative yield ( $Y_{ac}(\tau)$ ) is maximized.

The biomasses of the ecosystem (12) with initial conditions  $(x^0) = 2.039$  and without disturbances and controls are shown in Fig. 1. When the disturbances in (11) are included in the system in (12), the biomasses without the controls are shown in Fig. 2.

### 3. State feedback control [7]

Defining the transformation

$$z(t) = \ln\left(\frac{x(t)}{x^*}\right), \quad (14)$$

then if  $z(t) \rightarrow 0$  the state  $x(t) \rightarrow x^*$  exponentially. Consider the single biomass ecological system (12), it becomes

$$\dot{z}(t) = (r - h^*) - \frac{rx^*}{K}e^{z(t)} + v(t) + \frac{u(t)}{x^*e^{z(t)}}. \quad (15)$$

From (8) and (9), (15) becomes

$$\dot{z}(t) = \frac{r}{2}(1 - e^{z(t)}) + v(t) + \frac{2u(t)}{Ke^{z(t)}}. \quad (16)$$

The uncontrolled nominal system is

$$\dot{z}(t) = \frac{r}{2}(1 - e^{z(t)}). \quad (17)$$

Consider the Lyapunov function given by

$$V(z) = e^z - 1 - z \quad (18)$$

so that

$$\dot{V}(z) = -\frac{r}{2}(e^z - 1)^2 \leq 0. \quad (19)$$

The stability of system (17) is guaranteed. From the *Theorem* of [7], we have  $m(z, v) = \frac{K}{2}ve^z$  so

that  $|m(z, v)| \leq \frac{K}{2}\alpha e^z$ . Furthermore, in view of (18), the following equation can be found

$$\mu(z) = \alpha(e^z - 1). \quad (20)$$

Finally, the state feedback control law is achieved as

$$u_{fb1}(z) = \begin{cases} -[\text{sgn}(e^z - 1)]\frac{K}{2}\alpha e^z, & \text{if } |\alpha(e^z - 1)| > \varepsilon \\ -[\frac{\alpha(e^z - 1)}{\varepsilon}]\frac{K}{2}\alpha e^z, & \text{if } |\alpha(e^z - 1)| \leq \varepsilon \end{cases}. \quad (21)$$

Thus, if  $\varepsilon \rightarrow 0$ , the  $u_{fb1}$  approaches a switching (discontinuous) control.

### 4. Supervisory recurrent fuzzy neural network design

In the following discussion, we focus on the single biomass ecological system model to illustrate the effectiveness of the SRFNNC. The SRFNNC system for single biomass ecological system is shown in Fig. 3.

From the ecological system model (16), an integrated sliding function is defined as

$$s(t) = \dot{z}(t) + k_1z(t) + k_2\int_0^t z(\tau)d\tau \quad (22)$$

where  $k_1$  and  $k_2$  are positive constants.

If the system uncertainties are well known and measurable, an ideal controller can be obtained from (16)

$$u_i(t) = u_{id}(t) = \frac{K}{2}e^{z(t)}\left[-\frac{r}{2}(1 - e^{z(t)}) - v(t) - \frac{1}{k_1}\ddot{z}(t) - \frac{k_2}{k_1}z(t)\right] \quad (23)$$

Substituting (23) into (16) gives

$$\ddot{z}(t) + k_1\dot{z}(t) + k_2z(t) = 0. \quad (24)$$

If  $k_1$  and  $k_2$  are chosen to correspond to the coefficients of a Hurwitz polynomial, that is a polynomial whose roots lie strictly in the open left half of the complex plane, then  $\lim_{t \rightarrow \infty} z(t) = 0$ .

Since the system parameters may be unknown or perturbed, the ideal controller  $u_{id}$  cannot be implemented. To overcome this, a recurrent fuzzy neural network (RFNN) controller will be designed to approximate this ideal controller. In addition, a supervisory controller is designed to compensate for the approximation error between the RFNN controller and the ideal controller in

(23). Thus, the block diagram of the supervisory fuzzy neural network control (SRFNNC) system is shown in Fig. 4 where the inputs of the RFNN controller are  $s(t)$  and its derivative. The SRFNNC is assumed to take the following form:

$$u_i(t) = u_{srn}(t) = u_{rn}(t) + u_{sp}(t) \quad (25)$$

where  $u_{rn}(t)$  is the RFNN controller and  $u_{sp}(t)$  is the supervisory controller.

#### 4.1. Recurrent Fuzzy Neural Network Controller

Figure 4 shows a four-layer neural network comprising the input (the  $i$  layer), membership (the  $j$  layer), rule (the  $k$  layer), and output (the  $o$  layer) layers. This network is adopted to implement the proposed RFNN. The recurrent feedback is embedded in the network by adding feedback connections in the second layer of the fuzzy neural network. Since the recurrent neuron has an internal feedback loop, it captures the dynamic mapping network. The signal propagation and the basic function in each layer are introduced as follows:

Layer 1 - Input layer: For every node  $i$  in this layer, the net input and the net output are represented as

$$net_i^1(N) = x_i^1 \quad (26)$$

$$y_i^1(N) = f_i^1(net_i^1(N)) = net_i^1(N), \quad i = 1, 2 \quad (27)$$

where  $x_i^1$  represents the  $i$ th input to the node of layer 1 and  $N$  denotes the number of iterations.

Layer 2 - Membership layer: In this layer, each node performs a membership function. The Gaussian function is adopted as the membership function. For the  $j$ th node

$$net_j^2(N) = -\frac{(x_i^2 + y_j^2(N-1)\theta_{ij}^2 - m_{ij}^2)^2}{(\sigma_{ij}^2)^2} \quad (28)$$

$$y_j^2(N) = f_j^2(net_j^2(N)) = \exp(net_j^2(N)) \quad (29)$$

$j = 1, 2, \dots, m$

where  $m_{ij}^2$  is the mean,  $\sigma_{ij}^2$  is the standard deviation and  $\theta_{ij}^2$  is the feedback gain of the Gaussian function in the  $j$ th term of the  $i$ th input linguistic variable  $x_i^2$  to the node of layer 2, respectively, and  $m$  is the total number of linguistic variables with respect to the input nodes.

Layer 3 - Rule layer: Each node  $k$  in this layer is denoted by  $\Pi$ , which multiplies the incoming signal and outputs the product. For the  $k$ th rule node

$$net_k^3(N) = \prod_j w_{jk}^3 x_j^3 \quad (30)$$

$$y_k^3(N) = f_k^3(net_k^3(N)) = net_k^3(N), \quad k = 1, 2, \dots, n$$

(31)

where  $x_j^3$  represents the  $j$ th input to the node of layer 3, the weights  $w_{jk}^3$  between the membership layer and the rule layer are assumed to be unity.

Layer 4 - Output layer: The single node  $o$  in this layer is labeled as  $\Sigma$ , which computes the overall output as the summation of all incoming signals

$$net_o^4(N) = \sum_k w_{ko}^4 x_k^4 \quad (32)$$

$$y_o^4(N) = f_o^4(net_o^4(N)) = net_o^4(N), \quad o = 1 \quad (33)$$

where the link weight  $w_{ko}^4$  is the output action strength of the  $o$ th output associated with the  $k$ th rule,  $x_k^4$  represents the  $k$ th input to the node of layer 4, and  $y_o^4$  is the output of the recurrent fuzzy neural network controller.

#### 4.2. On-Line Learning Algorithm

The on-line learning algorithm is a gradient descent algorithm in the space of network parameters and aims to minimize  $s(t)\dot{s}(t)$ . Therefore,  $s(t)\dot{s}(t)$  is selected as the error function. Taking the first derivative of  $s(t)$  and using (16) yields

$$\begin{aligned} \dot{s}(t) &= \ddot{z}(t) + k_1 \dot{z}(t) + k_2 z(t) \\ &= k_1 \left[ \frac{r}{2} (1 - e^{z(t)}) + v(t) + \frac{2}{Ke^{z(t)}} u(t) + A_d(\ddot{z}, z) \right] \end{aligned} \quad (34)$$

where  $A_d(\ddot{z}, z) \equiv \frac{1}{k_1} \ddot{z}(t) + \frac{k_2}{k_1} z(t)$ . Substituting (25) into (34) and multiplying both sides by  $s(t)$  gives

$$\begin{aligned} s(t)\dot{s}(t) &= s(t)k_1 \left[ \frac{r}{2} (1 - e^{z(t)}) + v(t) \right. \\ &\quad \left. + \frac{2}{Ke^{z(t)}} (u_{rn}(t) + u_{sp}(t)) + A_d(\ddot{z}, z) \right] \end{aligned} \quad (35)$$

According to the gradient descent method, the weights in the output layer are updated by the following:

$$\begin{aligned} \dot{w}_{ko}^4 &\equiv -\eta_w \frac{\partial s(t)\dot{s}(t)}{\partial w_{ko}^4} \\ &= -\eta_w \frac{\partial s(t)\dot{s}(t)}{\partial u_{srn}} \frac{\partial u_{srn}}{\partial w_{ko}^4} \\ &= -\eta_w \frac{\partial s(t)\dot{s}(t)}{\partial u_{rn}} \frac{\partial u_{rn}}{\partial w_{ko}^4} \\ &= \eta_w k_1 \frac{s(t)}{Ke^{z(t)}} x_k^4 \equiv \eta_{w'} \frac{s(t)}{e^{z(t)}} x_k^4 \end{aligned} \quad (36)$$

where  $\eta_w$  is the learning rate with a positive constant and  $\eta_{w'} \equiv \frac{\eta_w k_1}{K}$ . Since the weights in

the rule layer are unity, only the approximation error term needs to be calculated and propagated by the following:

$$\begin{aligned}\delta_k^3 &\equiv -\frac{\partial s(t)\dot{s}(t)}{\partial u_{srn}} \frac{\partial u_{srn}}{\partial net_o^4} \frac{\partial net_o^4}{\partial y_k^3} \frac{\partial y_k^3}{\partial net_k^3} \\ &= \frac{s(t)}{Ke^{z(t)}} k_1 w_{ko}^4.\end{aligned}\quad (37)$$

The multiplication is done in the membership layer and the error term is computed as follows:

$$\begin{aligned}\delta_j^2 &\equiv -\frac{\partial s(t)\dot{s}(t)}{\partial u_{srn}} \frac{\partial u_{srn}}{\partial net_o^4} \frac{\partial net_o^4}{\partial y_k^3} \frac{\partial y_k^3}{\partial net_k^3} \frac{\partial net_k^3}{\partial y_j^2} \frac{\partial y_j^2}{\partial net_j^2} \\ &= \sum_k \delta_k^3 y_k^3.\end{aligned}\quad (38)$$

The update laws of  $m_{ij}^2$ ,  $\sigma_{ij}^2$  and  $\theta_{ij}^2$  can also be obtained by the gradient search algorithm, i.e.,

$$\begin{aligned}\dot{m}_{ij}^2 &\equiv -\eta_m \frac{\partial s(t)\dot{s}(t)}{\partial m_{ij}^2} \equiv -\eta_m \delta_j^2 \frac{\partial net_j^2}{\partial m_{ij}^2} \\ &= -\eta_m \delta_j^2 \frac{2(x_i^2 + y_j^2(N-1)\theta_{ij}^2 - m_{ij}^2)}{(\sigma_{ij}^2)^2}\end{aligned}\quad (39)$$

$$\begin{aligned}\dot{\sigma}_{ij}^2 &\equiv -\eta_\sigma \frac{\partial s(t)\dot{s}(t)}{\partial \sigma_{ij}^2} \equiv -\eta_\sigma \delta_j^2 \frac{\partial net_j^2}{\partial \sigma_{ij}^2} \\ &= -\eta_\sigma \delta_j^2 \frac{2(x_i^2 + y_j^2(N-1)\theta_{ij}^2 - m_{ij}^2)^2}{(\sigma_{ij}^2)^3}\end{aligned}\quad (40)$$

$$\begin{aligned}\dot{\theta}_{ij}^2 &\equiv -\eta_\theta \frac{\partial s(t)\dot{s}(t)}{\partial \theta_{ij}^2} \equiv -\eta_\theta \delta_j^2 \frac{\partial net_j^2}{\partial \theta_{ij}^2} \\ &= \eta_\theta \delta_j^2 \frac{2(x_i^2 + y_j^2(N-1)\theta_{ij}^2 - m_{ij}^2)y_j^2(N-1)}{(\sigma_{ij}^2)^2}\end{aligned}\quad (41)$$

where  $\eta_m$ ,  $\eta_\sigma$  and  $\eta_\theta$  are the learning rates with positive constants.

The most useful property of a neural network is its ability to approximate linear or nonlinear mapping through learning. According to the universal approximation theorem, there exists an optimal RFNN such that

$$u_{id}(t) = u_{rn}(\mathbf{w}^*, t) + \varepsilon(t) \quad (42)$$

where  $\mathbf{w}^* = [w_{ko}^4 \ m_{ij}^{2*} \ \sigma_{ij}^{2*} \ \theta_{ij}^{2*}]^T$  is the ideal weight vector of the recurrent neural network controller, and  $\varepsilon(t)$  denotes the approximation error and is assumed to be bounded by  $0 \leq |\varepsilon(t)| \leq E$  where  $E$  is a positive constant. The error bound is assumed to be a constant during the observation; however, it is difficult to measure in practical applications. Therefore, a bound estimation is developed to observe the bound of the approximation error. Define the

estimation error of the bound

$$\tilde{E}(t) = E - \hat{E}(t) \quad (43)$$

where  $\hat{E}(t)$  is the estimated error bound. The supervisory controller is designed to compensate for the effect of approximation error and is chosen as

$$u_{sp}(t) = \hat{E}(t) \text{sgn}(s(t)). \quad (44)$$

Substituting (25) into (16) reveals the following:

$$\dot{z}(t) = \frac{r}{2}(1 - e^{z(t)}) + v(t) + \frac{2}{Ke^{z(t)}}(u_{rn}(t) + u_{sp}(t)) \quad (45)$$

After some straightforward manipulation, the error equation governing the system can be obtained through (16), (23) and (25) as follows:

$$\dot{s}(t) = \ddot{q}(t) + k_1 \dot{q}(t) + k_2 q(t) = u_{id}(t) - u_{rn}(\mathbf{w}^*, t) - u_{sp}(t). \quad (46)$$

### 4.3. Stability analysis of algorithm

Define a Lyapunov function as

$$V_2(s(t), \tilde{E}(t)) = \frac{1}{2}s^2(t) + \frac{1}{2\eta_E}\tilde{E}^2(t) \quad (47)$$

where  $\eta_E$  is the learning rate with a positive constant. Differentiating (47) with respect to time and using (42), (43), (44) and (46) yields

$$\begin{aligned}\dot{V}_2(s(t), \tilde{E}(t)) &= s(t)\dot{s}(t) + \tilde{E}(t)\dot{\tilde{E}}(t) / \eta_E \\ &= s(t)(\varepsilon(t) - u_{sp}(t)) + \tilde{E}(t)\dot{\tilde{E}}(t) / \eta_E \\ &= \varepsilon(t)s(t) - \hat{E}|s(t)| + \tilde{E}(t)\dot{\tilde{E}}(t) / \eta_E\end{aligned}\quad (48)$$

If the adaptive law for the supervisory controller is chosen as

$$\dot{\tilde{E}}(t) = -\dot{\hat{E}}(t) = -\eta_E |s(t)| \quad (49)$$

then (48) can be rewritten as

$$\begin{aligned}\dot{V}_2(s(t), \tilde{E}(t)) &= \varepsilon(t)s(t) - \hat{E}|s(t)| - (E - \hat{E}(t))|s(t)| \\ &= \varepsilon(t)s(t) - E|s(t)| \leq |\varepsilon(t)||s(t)| - E|s(t)| \\ &= -(E - |\varepsilon(t)|)|s(t)| \leq 0.\end{aligned}\quad (50)$$

Since  $\dot{V}_2(s(t), \tilde{E}(t))$  is negative semi-definite, that is  $V_2(s(t), \tilde{E}(t)) \leq V_2(s(0), \tilde{E}(0))$ , it implies that  $s(t)$  and  $\tilde{E}(t)$  are bounded. Let function  $\Omega(t) \equiv (E - |\varepsilon(t)|)|s(t)| \leq -\dot{V}_2(s(t), \tilde{E}(t))$ , and integrate  $\Omega(t)$  with respect to time, we can then obtain

$$\int_0^t \Omega(\tau) d\tau \leq V_2(s(0), \tilde{E}(0)) - V_2(s(t), \tilde{E}(t)). \quad (51)$$

Because  $V_2(s(0), \tilde{E}(0))$  is bounded, and  $V_2(s(t), \tilde{E}(t))$  is nonincreasing and bounded, the following result can be obtained:

$$\lim_{t \rightarrow \infty} \int_0^t \Omega(\tau) d\tau < \infty. \quad (52)$$

Moreover,  $\dot{\Omega}(t)$  is bounded, so using Barbalat's Lemma [8] yields  $\lim_{t \rightarrow \infty} \Omega(t) = 0$ . That is,  $s(t) \rightarrow 0$  as  $t \rightarrow \infty$ . As a result, the supervisory recurrent fuzzy neural network control system is asymptotically stable.

## 5. Simulation results

To compare between the proposed SRFNNC and the state feedback control [7] for the ecological system, the same parameters are used for these simulations. The simulation results of the single biomass case for these design methods are shown in Fig. 5 for the state trajectories and control inputs. These simulation results demonstrate that the state trajectories are controlled to achieve fast response and stable steady state using the proposed SRFNNC.

From (13), the comparison of accumulative yield of the single biomass case for SRFNNC, state feedback control and no control is shown in Fig. 6. As can be seen, the proposed SRFNNC can obtain the best accumulative yield.

Ecological systems are very different depending on the geographical area, climatic conditions, and type of biomass considered. In this paper, a recurrent fuzzy neural network approach is proposed to cope with a specific case study (such as the relation between pollution and environment protection). The disturbance term has been taken into consideration, the proposed approach can highlight how to face these limitations. The control of more species ecosystem will be our future research.

## 6. Conclusion

This paper considers the control of ecological system subject to continual, unpredictable, but bounded disturbance due to changes in climatic conditions, diseases, and migrating species. A SRFNNC is developed and is then employed to control the biomasses within a small neighborhood of the unique nontrivial optimal equilibrium state of the undisturbed exploited ecosystem. This model is simplified and can be thought as a relation between pollution and environment protection, the control goal is to maintain the species equilibrium and maximize the accumulative yield. The positive harvest input stands for the gain in species (or for pollution) while the negative harvest input stands free captive species (or for environment protection). Under the disturbed system, the accumulative yield with SRFNNC is better than that obtained

with the state feedback control and no control.

## References

- [1] Y. C. Chen, C. C. Teng, A Model Reference Control Structure Using A Fuzzy Neural Network, *Fuzzy Sets and Systems* 73 (1995) 291-312.
- [2] M. Equihua, Fuzzy clustering of ecological data, *Journal of ecology* 78 (1990) 519-534.
- [3] B. S. Goh, *Management and analysis of biological populations*, Elsevier Science, Amsterdam, 1980.
- [4] B. S. Goh, G. Leitmann, T. L. Vincent, Optimal control of a prey-predator systems, *Mathematical Biosciences* 19 (1974) 263-286.
- [5] S. S. Ge, C. C. Hang, T. Zhang, Adaptive Neural Network Control of Nonlinear Systems By State and Output Feedback, *IEEE Transactions on System, Man and Cybernetic-Part B: Cybernetic* 29 (1999) 818-828.
- [6] C. C. Ku, K. Y. Lee, Diagonal Recurrent Neural Networks for Dynamic Systems Control, *IEEE Transactions on Neural Networks* 6 (1995) 144-156.
- [7] C. S. Lee, G. Leitmann, On optimal long-term management of some ecological systems subject to uncertain disturbances, *International Journal of Systems Science* 14 (1983) 979-994.
- [8] C. T. Lin, C. S. G. Lee, *Neural Fuzzy Systems: A Neural-Fuzzy Synergism to Intelligent Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1996.
- [9] C. H. Lee, C. C. Teng, Identification and Control of Dynamic Systems Using Recurrent Fuzzy Neural Networks, *IEEE Transactions on Fuzzy Systems* 8 (2000) 349-366.
- [10] F. J. Lin, R. J. Wai, Hybrid Control Using Recurrent Fuzzy Neural Network for Linear-Induction Motor Servo Drive, *IEEE Transactions on Fuzzy Systems* 9 (2001) 102-115.
- [11] M. Zhihong, H. R. Wu, M. Palaniswami, An Adaptive Tracking Controller Using Neural Networks for A Class Of Nonlinear Systems, *IEEE Transactions on Neural Networks* 9 (1998) 947-1031.

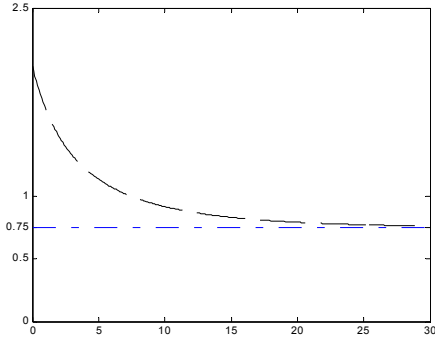


Fig. 1. Biomasses of single biomass ecological system (undisturbed and uncontrolled)

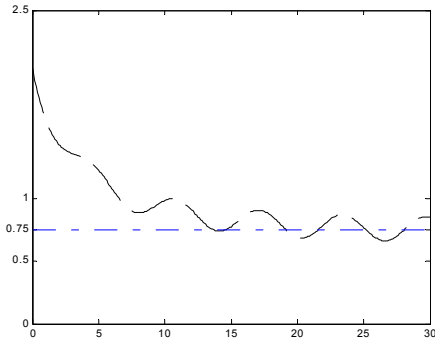


Fig. 2. Biomasses of single biomass ecological system (disturbed and uncontrolled)

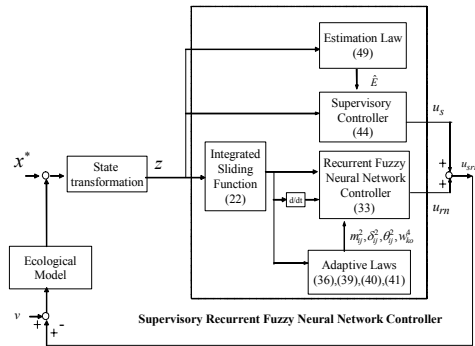


Fig. 3. Supervisory-recurrent-fuzzy-neural network control for ecological system

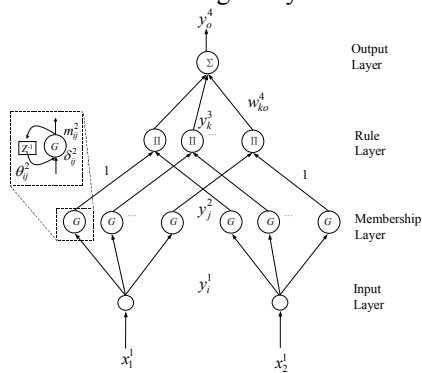


Fig. 4. Network structure of a recurrent fuzzy neural network.

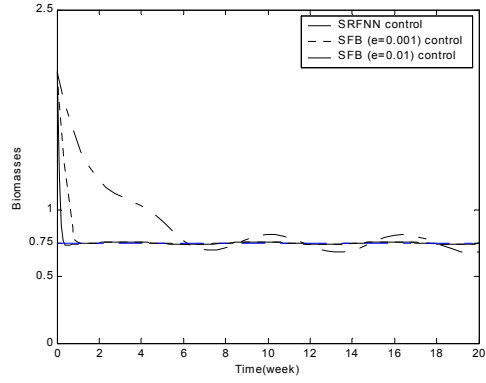


Fig. 5(a). Biomasses of single biomass ecological system

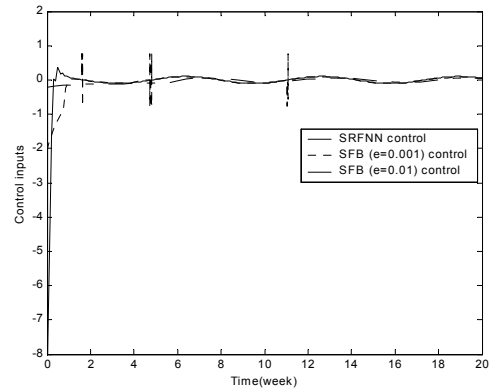


Fig. 5(b). Control inputs of single biomass ecological system

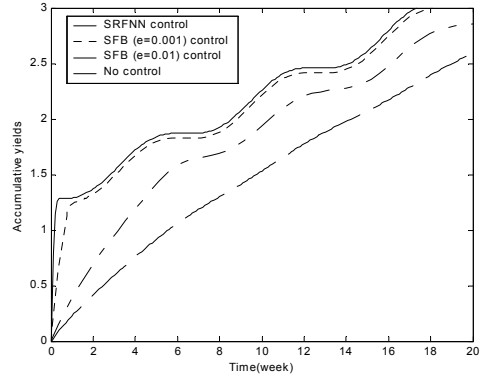


Fig. 6. Accumulative yields of single biomass ecological system