Effective Learning Curve Model and Activate Media Learning Algorithm for Improving Learning Efficiency

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Abstract-In this world, technology developments are speeding. How to learn desired knowledge efficiently has become a complicated problem. In this paper, we introduced some learning phenomenon about people with learning curve and proposed an Effective Learning Curve Model to emulate this phenomenon. Using proposed learning function model, we can understand people's learning behavior and know every people has different learning functions on distinct courses. Different course learning sequence will cause distinct learning efficiency. In this view, we proposed Max Learning Slope First Algorithm (MLSFA) to give people some suggestions about course learning sequence. This algorithm can help us to understand how much time we have to spend on each course in order to get better learning efficiency under time limitation. Finally, we make some learning example and compare simulation results with other learning algorithm. From simulation result, we can see that our MLSFA algorithm has better learning efficiency than other methods.

Keywords: e-learning, efficient learning algorithm, learning function, learning model, learning efficiency

1. Introduction

Technology development progresses rapidly in the world. The things people deal with have become more and more complex. Many ten years ago, for the research of building airplane, Wright [1] had used math methods to create learning curve function and develop first thesis about learning curve. From that time, learning methods have been discussed for distinct application plan and different models also have been produced. If we set horizontal axis indicates learning time period on courses, while the vertical axis indicates learning efficiency, we call this Figure as learning curve. People's learning curves are different on learning distinct knowledge and will be changed because of many reasons such as difficulty of works, learning motivation, knowledge background of learners, and some other reasons. There are typical people's different learning curves describe as follow, and shown in Figure 1.

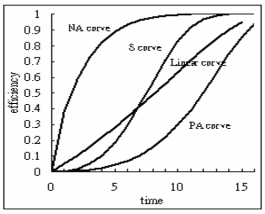
a. Negative Accelerating Curve

b. Positive Accelerating Curve

c. S Accelerating Curve

d. Linear Accelerating Curve

If there are some courses we prepare to learn, we can make courses relations into a learning graph as shown in Figure 2. In this Figure, nodes represent course name and arrows mean course learning sequence, for example, people can not learn course C till course A has passed or the learning efficiency will decrease, we set this decreasing parameter ∂. Of course, maybe there are some courses independent to others, such as course A and course B. That means we can start learning from either course A or course B. For this reason, we use ‘virtual start node’ as the beginning of learning graph.



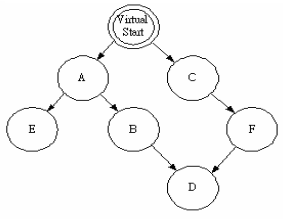


Fig. 1 Typical learning curve of people **Fig. 2** Courses learning sequence graph

Under this course learning graph, we hope there are some suggestions to understand how to get the maximum learning efficiency by spending minimum time on each course. For this reason, we introduce new learning function model to emulate people’s learning behavior on each course and propose max learning slope first algorithm (MLSFA) under score base and time base conditions to improve group courses learning efficiency. At the end, we compare simulation results of our proposed MLSFA algorithm with other learning algorithms. The simulation comparison is shown in Figure 6 and 7. From simulation result, we can see that our algorithm has better simulation result and can improve learner’s group courses learning efficiency.

The remainder of this paper is organized as follows. The section II, we introduce some related work and compare them with our proposed methods. In section III, we proposed Heuristic learning model to emulate people learning curve. Algorithms for improving Learning efficiency are described in section IV. The simulation results and comparisons are presented in section V. Finally, we provide conclusion in section VI.

2. Related work

Learning curve can be described in distinct math equation for different learning characteristic. Five commonly used learning curve are described In Yelle[2], and are introduced as following:

a. Log-linear model [1],, $f(x) = a_1 x^{-b}$

b. Standford-B model [3], $f(x) = a_1 (x+B)^{-b})$

c. S Curve Model [4], $f(x) = a_1 (M + (1-M) (x+B)^{-b})$

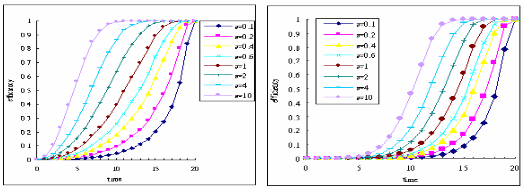
d. Time Constant Model [6], $Y(t)=Y_c + Y_f (1-e^{-t/\zeta})$

All the Learning model in Yelle[2] are suitable for special condition, but can’t cover all learning behavior we have introduced before, and will be limited in some learning application. In this paper, we proposed Effective Learning Curve Model try to emulate all learning behavior of people by tuning some function parameter. Then we raise Max Learning Efficiency Slope First Algorithm (MLESFA) to improve people’s learning efficiency under learning group courses, and make some example to prove that our MLESFA algorithm has better learning efficiency.

3. Heuristic Learning Model

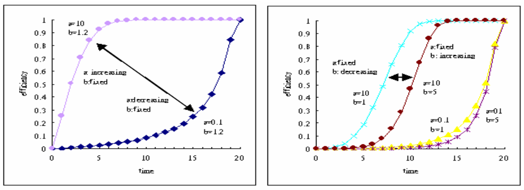
In this section, we want to find a question can emulate all the learning behavior of people, as in Figure 1. At first, we choose $1-e^{-at}$ as our base function. We all know $1-e^{-at}=1$, when $t\rightarrow\infty$, and $1-e^{-at}=1$, if $t\rightarrow 0$. From the characteristic of exponential function, we can see if parameter ‘a’ changed decreasingly or increasingly between 0 to ∞, and time increasing at the same time, it will produced some behavior curves like in Figure 1. Under experimentally testing, we take $\eta(t) = c(1 - \exp(-\frac{a(nt)^b}{1-(nt)}))$ as our learning function model, in that, t is time sequence, ‘a’ and ‘b’ are function parameter in order to emulate learner’s learning behavior, n is simulation time slot range, used to 0.01, c is function coefficient indicating learning speed of emulated learning function.

Using function $f = \eta(t)$, we tune parameter 'a', 'b' and 'c'. In the following, we set parameter 'a' from 0.1 to 10, 'b' from 0.7 to 5 and parameter 'c', the speeding coefficient of learning function, to 1. We can get relative function curves shown in Figure 3. Compare these figures, the effects of parameter 'a', 'b' and 'c' to function model are shown in Figure 4.



a. 'a' = 0.1 to 10, 'b'=2, 'c'=1   b. 'a' = 0.1 to 10, 'b'=5, 'c'=1

Fig. 3 Emulating learning curve



a. curve effect of parameter 'a'   b. curve effect of parameter 'b'

Fig. 4 Effects of learning function parameter

From Figure 3, if we want to make a emulation of learner's learning curve, we can only set the range of parameter 'a' between 0.1 to 10 , parameter 'b' between o.7 to 5 and c=1. If we want the emulation curves more precisely, we can just tune parameter 'c' from 0.8 to 2.0. From experiments, effects of parameter 'a' on learning function are shown in Figure 4.a, effects of parameter 'b' are in Figure 4.b respectively. Therefore, we can emulate some different learning behavior curves under combinations of parameter 'a', 'b' and 'c' between different range, as described in Table 1, and the emulating learning behavior curves are shown in Figure 5.

Table 1 Learning behavior under parameter 'a', 'b', 'c'

parameter	PA Curve	LA Curve	SA Curve	NA Curve
a	2.0– 10	0.8 – 1.2	2.0 - 10	0.1 – 0.5
b	0.7 – 1.5	0.8 – 1.2	1.5 - 5	0.7 – 5.0
c	1	1	1	1

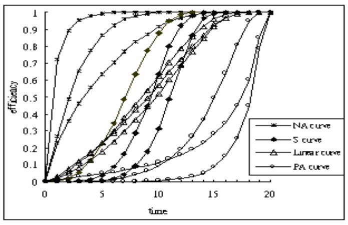


Fig. 5 Emulating people's learning behavior

But how can we get the parameter 'a', 'b' and 'c' of each user? We can collect and store much database about the relations of learning efficiency with learner's personality such as knowledge background, learning attitude, course difficulty, etc. Before learning, we can make some pre-testing to predict suitable parameter 'a', 'b' and 'c' of each course respectively in order to get proper learning function to emulate learner's learning behavior. Under learning group courses, there are some learning sequence relations between courses. So, if we want to get the max learning efficiency in some condition, the model can be formulated as follow.

$$\text{Max}\ \left(\sum_{i=1}^{m}\sum_{j=1}^{n} W_i * \frac{d\eta_i(t_{ij})}{dt}\right)$$

Where, m: courses number

n: time slots of studying

W_i: weights of course i

$\eta_i(t)$: learning function on course i

$d\eta_i(t_{ij})/dt$: learning efficiency of spending Δ time j on course i

Subject to $t_{ij} \geqq 0$, $T \geqq 0$, $\sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij} = T$

$W_i \geqq 0$, $\sum_{i=1}^{m} W_i = 1$

$\frac{d\eta_i(t_{ij})}{dt} \ge 0$,

$\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{d\eta_i(t_{ij})}{dt} \le 100$

In time t_j, we choose max

$\left(w_1 \frac{d\eta_1(t_j)}{dt}, w_2 \frac{d\eta_2(t_j)}{dt}, .., w_n \frac{d\eta_n(t_j)}{dt} \right)$ as first course reading priority.

4. Improving learning efficiency algorithm

In this section, our objective is to provide a mathematical analysis in learning courses and to define behavioral strategies that lead the learning efficiency to the optimal operating point. Several implementation aspects will be briefly discussed in these works. We assumed there are six courses A, B, C, D, E, F and their weights of each course is normalized to be $W_i/\Sigma W_i$, as in table 2. Using pre-testing, we can obtain users' learning curve of each course and get discrete learning efficiency of each unit time by differential course learning functions $D\eta_i(t)|_{t=tj}$, and multiplying these values with course weight W_i, we get the results in Table 2. Next, sorting Table 2 on learning efficiency field by descending, we get value as in Table 3.

Table 2 Learning efficiency*normalized weight

Course	Weight	Learning efficiency											
		j=1	2	3	4	5	6	7	8	9	10	11	12
A	0.1	3.0	2.5	2.0	1.0	0.5	0.4	0.3	0.2	0.1	0	0	0
B	0.1	4.0	3.0	1.5	1.0	0.5	0	0	0	0	0	0	0
C	0.2	4.0	3.0	3.0	3.0	2.0	1.0	1.0	1.0	1.0	0.6	0.4	0
D	0.3	5.4	5.1	4.5	3.0	3.0	2.4	2.1	1.5	1.5	0.9	0.6	0
E	0.2	5.0	3.0	3.0	3.0	2.0	2.0	1.0	0.6	0.4	0	0	0
F	0.1	2.0	2.0	1.5	1.0	1.0	1.0	1.0	0.5	0	0	0	0

Table 3. Learning efficiency by sorting from Table 2

Time	j=1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
LS	d1	d2	e1	d3	c1	b1	a1	b2	c2	c3	c4	d4	d5	e2	e3
US	5.4	5.1	5.0	4.5	4.0	4.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0

.. etc.

In the following discussion, first, we have made the assumption that some courses are dependent to others, as shown in Figure 7. In this condition, there are two questions we must face to solve. The first question is how can we know the minimum spending time on each course for the purpose of getting 60 score to pass the courseware, and the second question is how to obtain maximum course learning efficiency in order to get best score under given time limitation. For these reason, we propose score base algorithm 1.1 to solve first question, to know the minimum spending time on each course for the purpose of getting 60 score to pass all the courses, and time base algorithm 1.2 to solve the second question, to get the best score under given time limitation.

But because some courses are dependent to each other, if course n-1 has not passed, the learning efficiency of courses n will be influent. We raise parameter ∂ as this effect, and learning efficiency will be changed to learning efficiency*(∂^m), in that $0 \leqq \partial \leqq 1$, m is the fail or unlearn courses before learned course n. To these question, we propose score base algorithm 1.1 to solve first question, to know the minimum spending time on each course for the purpose of getting 60 score to pass all the courses, and time base algorithm 1.2 to solve the second question, to get the best score under given time limitation. At first, we made a course learning sequence choosing principle, such that,

a. high level learning course first
b. high course weight first with the same level
c. effecting more learning courses first
d. learning from left to right

Algorithm 1.1 Score base

1. To do the same step 0 to 4 as in algorithm 1.1.
2. Using course learning sequence choosing principle, we record course learning sequence in array CS, and relative course weight in CW.
3. Get courses number cn
4. For i=1 to cn
5. Do while (total score (i) < wanted score)

6. Get max chapter learning efficiency from course i and record obtained score from score Table 2
7. Record course chapter learning sequence to LS
8. Increase and record course chapter learning time
9. End Do
10. Next i
11. Print wanted results

Algorithm 1.2 Time base

1. To do the same step 0 to 4 as in algorithm 1.1.
2. Using course learning sequence choosing principle, we record course learning sequence in array CS, and relative course weight in CW.
3. Get courses number cn
4. For i=1 to cn
5. Do while (total score (i)<60 and total learning time < time limited)
6. Get max chapter learning efficiency from course i and record obtained score from score Table 2
7. Record course chapter learning sequence to LS
8. Increase course chapter learning time and record
9. End Do
10. Next i
11. Do while (total learning time < time limited)
12. Get max learning efficiency course chapter and record obtained score from score Table 3
13. Record course chapter learning sequence to LS
14. Increase course chapter learning time and record
15. End Do
16. Print wanted results

From the example of learning graph as in Figure 7, using algorithm 1.1 and 1.2, we can get course learning sequence such as course C, A, E, B, F, and D. If we want to pass all courses, the time we must spend on each course is TA=3, TB=2, TC=4, TD=4, TE=4, TF=4 and each course score we will get is as follow, SA=75, score of course A, SB=70, SC=65, SD=60, SE=70, SF=65, respectively. Therefore if we want to pass all the courses, we must spend at least to 21 unit times, and will obtain final score equal to 66 with multiplying each course score by course weighting. Another question is that, if we have unit time >21, for example 30, what score we can get max? First, we use algorithm 1.1 to make all courses pass, and then use algorithm 1.2. We can obtain the time we spend on each course, TA=3, TB=3, TC=5, TD=9, TE=6, TF=4 and get related scores as follow, SA=75, SB=85, SC=75, SD=95, SE=90, SF=65. At last, we obtain final score equal to 84. And the course learning sequence suggestion is stored in variable LS.

5. Efficiency comparison

For proving our learning function and algorithms having better learning efficiency, we use learning graph, Figure 6 and Figure 7, as our simulation example. Table 2 is learner's learning efficiency of each course by differential course learning function from discrete data sampling. At the first, we take Figure 6 into consideration that courses are independent, and we compare MLESFA1 simulation result with Depth First algorithm (DFA), Bread First algorithm (BFA) and Random algorithm (RA). DFA learns all chapters of course A by sequence, and then course B, C, D, E and F. BFA learns chapter 1of course A next chapter 1 of course B, C, D, E, and F, after that, learns chapter 2 of course A, B, C,D,E,F and will not stop till all chapters have been learned completely. RA means random choosing course chapter to learn. Because of course chapter having its learning sequence, random choosing course chapter to learn will affect chapter learning efficiency, we assume effect parameter η =0.9. The comparison learning efficiency curves are shown in Figure 6 under course learning graph Figure 6.

Next, we think Figure 2 that courses are dependent as our simulation graph. The courses learning sequence under choosing course learning sequence principle are course A, C, B, E, F, D of algorithm MLESFA, course A, E, B, D, C, F of algorithm DFA , course A, C, E, B, F, D of algorithm BFA respectively. As to algorithm RA, we get randomly courses learning sequence is course F, B, D, C, E, A. Because courses are dependent, if parent course haven't passed, and we insist on learning following courses, the course learning efficiency will be affected. This effect will bigger than courses dependent and we assumed this effect parameter η =0.8. If learning course have m parent courses not passed, the effect parameter will be changed to η^m. The comparison learning efficiency curves are shown in Figure 7 under condition Figure 2. From above discussion, we can see our algorithm has better learning efficiency result both in Figure 6 under Figure 6 of courses independence, and in Figure 7 under Figure 2 of courses dependence.

6. Conclusion

In this paper, we proposed new effective heuristic learning curve function f= η (t) to emulate people's learning behavior. From tuning function parameter we can emulate all people's learning behavior curves. Under time limitation every one wants to understand how to learn will get the best result and how much time we must spend on each course. For obtain better learning efficiently, we raised two different course learning algorithm, score-based algorithm and time-based algorithm. Through simulation, we can get suggestions about the courses learning sequence, the times we spend on each course in order to get the maximum learning efficiency under time limitation. From the result comparison in Figure 6 and Figure 7, we see that our proposed algorithm has better learning efficiency.

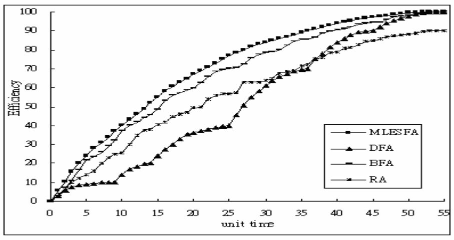


Fig. 8 Efficiency comparison under courses independence.

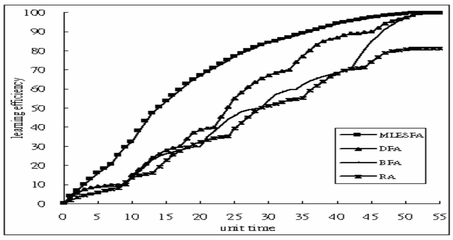


Fig. 9 Efficiency comparison under courses dependence.

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