Blending Operations with an Blending Range Control on Every Primitive's Subsequent Blend for Non-Zero Implicit Surfaces

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Abstract

Among geometric modeling techniques, non-zero implicit surface modeling, such as soft object modeling, has been applied wide for 3D computer graphics and animation due to its lower computing complexity of blending. However, existing blending methods still cannot give non-zero implicit surfaces the ability to individually adjust their primitives' subsequent blends in sequential blends. Hence, this paper proposes new blending operations that can have an individual blending range control on every primitive's subsequent blend for non-zero implicit surfaces, such as soft object modeling and Ricci's constructive geometry. Furthermore, this paper proposes a generalized method to transform some of existing blending operations into the newly proposed blending operations stated above. Base on the generalized method, this paper has transformed conic and hyper-ellipsoidal blends into the proposed blends.

Keywords: Implicit surfaces, Soft objects, Constructive geometry, Field functions, Blending operations.

1. Introduction

In implicit surface modeling, an implicit surface is defined as a level surface of a three dimensional function. Implicit surface modeling can easily construct a complex object by a successive composition of blending operations on some simple primitive implicit surfaces, such as planes, sphere, cone,…, etc. Blending operations can connect intersecting objects smoothly with transitions automatically generated to avoid unwanted sharp edges, kinks, and creases. Generally, implicit surfaces can be categorized into (1) zero implicit surfaces, i.e. a level surface of a zero value, and (2) non-zero implicit surfaces, i.e. a level surface of a non-zero value.

In the literature of zero implicit surfaces, R-functions [19] give Boolean set operations with C^n, $n \geq 1$, continuity, but without blending range control. Blends with blending range control parameters to adjust the size of the resulting blending surface without deforming the overall shapes of blended primitives were proposed in [3, 4, 5, 9, 10, 11, 13, 15, 16, 19, 20, 21, 22, 24, 25, 26, 27]. The scale method [13] and the displacement method [25] can develop high dimensional blending operators for sequential blends and multiple blends. The method in [14] can especially offer an individual blending range controls on every primitive's subsequent blend for zero implicit surfaces blending.

In non-zero implicit surfaces, soft objects modeling were proposed and very popular for low-degree computing complexity of blending. Primitives of soft objects are defined as a level surface of field functions, which enable soft objects to be blended easily by performing addition operations only. Existing field functions and blending operations for soft objects can be found in [1, 2, 5, 7, 8, 12, 14, 17, 18, 28, 29]. Among these, boolean set operations [18, 23] can not adjust the shape of the blends. Super-ellipsoidal blends with a curvature parameter to adjust the shape of transition of blending surfaces were proposed in Constructive Geometry [23]. The blends in [5, 13] can offer blending range parameters. The scale method [13] particularly can develop high dimensional blending operators for generating sequential blends and multiple blends. Field functions with blending range parameters were proposed in [14].

Although non-zero implicit surfaces are attracting much attention and applied wide in 3D computer graphics due to its lower computing complexity of blending, their existing blends, denoted as $B_k(f_1,\ldots,f_k)$ with primitives $f_1,\ldots,f_k$, to be blended, still face the following difficulty:

When $B_k(f_1,\ldots,f_k)$ is used as a new primitive in other blends, such as $B_2(B_k(f_1,\ldots,f_k), f_{k+1})$, primitives $f_1,\ldots,f_k$, always have similar subsequent blending surfaces with f_{k+1} because they always have the same blending range to blend with f_{k+1} in B_2. That is, $B_k(f_1,\ldots,f_k)$ still can not provide an individual blending range control on every primitive's subsequent blend.

Therefore, to conquer the above difficulty of soft objects, this paper proposes new blends that can individually and controllably adjust the blending range of every primitive's subsequent blending surface in sequential blends, without deforming the original blending surfaces. Precisely, the proposed blends, denoted as $B_{CAk}(f_1,...,f_k)$, can

(1). Provide parameters m_i, i=1,...,k, for primitives $f_1,...,f_k$, to adjust the blending ranges of primitives' subsequent blends, without deforming the original blending surface of $B_{Ck}(f_1,...,f_k)$, when $B_{Ck}(f_1,...,f_k)$ is applied as a new primitive in sequential blends.

(2). Offer blending range parameters r_i, i=1,...,k, for primitives $f_1,\ldots,f_k$, to adjust the size of the transition of the blending surface of $B_{Ck}(f_1,...,f_k)$ without changing the overall shapes of blended primitives.

(3). Be used to generate smooth sequential and multiple blends.

Furthermore, this paper also proposes a generalized method to transform some of existing blending operators into the newly proposed blending operations stated above. Based on the generalized method, two dimensional conic blends and high dimensional hyper-ellipsoidal blends are developed in this paper.

The remainder of this paper is organized as follows. Section 2 reviews previous works and describes their difficulty. Section 3 presents the generalized method. Section 4 derives new blending operators from the generalized method. Section 5 demonstrates some examples. Conclusions are given in Section 6.

2. Previous works

This section defines non-zero implicit surfaces first. Then, the scale method [13] is reviewed and its problem is described.

2.1. Definitions of non-zero implicit surfaces

In non-zero implicit surfaces, a simple object can be defined using primitive defining functions $f_i(X)$, i=1,...,k, such as sphere, planes, ellipsoids, by

$$S(f_i, \mathrm{C}) \equiv \{X\in R^3 | f_i(X) \leq \mathrm{C}\}, \text{ or}$$

$$L(f_i, \mathrm{C}) \equiv \{X\in R^3 | f_i(X) \geq \mathrm{C}\},$$

where C is a threshold larger than 0, $f_i(X)$ maps R^3 to R_+, and R_+ means the set $\{x \leq 0 | x\in R\}$. The boundary surface or the shape of the object $S(f_i, C)$ or $L(f_i, C)$ is given by f_i=C .

Besides, to construct a complex object from primitives $S(f_i, C)$ or $L(f_i, C)$, i=1,…, k, a blend can be written using a blending operator $B_k(x_1,\ldots,x_k)$: $R_+^{\,k}\rightarrow R$ by

$$S(B_k \circ F_k, C), \text{ or}$$

$$L(B_k \circ F_k, C)$$

where $(B_k \circ F_k)$ denotes the blending operation $B_k(f_1,\ldots,f_k)$:$R^3\rightarrow R_+$, and $(B_k \circ F_k)$=C is called the blending surface. The symbol F_k means $(f_1,\ldots, f_k)$:$R^3\rightarrow R_+^{\,k}$. The blending operator $B_k(x_1,\ldots,x_k)$ is to smoothly connect blended primitives $S(f_i, C)$ or $L(f_i, C)$, i=1,…, k, with transitions generated automatically. Furthermore, $(B_k \circ F_k)$ can be reused a new primitive in sequential blends, such as $S(B_2(B_k \circ F_k, f_{k+1}), C)$.

Depending on the threshold C, non-zero implicit surfaces can be categorized as follows. When the threshold C in $S(f_i, C)$ is always set to be 1, then

$$S(f_i, 1)$$

is called the constructive geometry [23], denoted CG for short. Super-ellipsoidal blending operators [23],

$$B_k(x_1,\ldots,x_k)=(x_1^{\,p}+\ldots+x_k^{\,p})^{1/p}$$

can derive a full family of Boolean set operators on $S(f_i, 1)$ with the parameter p to adjust the shape of the resulting blending surface. Super-ellipsoidal blends on $S(f_i, 1)$, i=1,…, k, can be written as

$$S((f_1^{\,p}+\ldots+f_k^{\,p})^{1/p}, 1).$$

However, super-ellipsoidal blends always deform the overall shapes of primitives $S(f_i, 1)$ because of lacking blending range parameters to controllably adjust the size of the transition of the resulting blending surface.

On the other hand, when C in $L(f_i, C)$ is always set to be 0.5 and primitive defining functions f_i are field functions in [1, 2, 7, 8, 12, 14, 18, 28, 29], which map R^3 to the interval [0, 1] and are given by

$$f_i(X)=(P \circ d_i)(X),$$

where $d_i(X)$:$R^n\rightarrow R_+$ is a distance function, and $P(d)$ is a potential function that can map $d\in R_+$ to [0,1] and is decreasing monotonically by d, then

$$L(f_i, 0.5)$$

is called a soft object. Because field functions are required to be used as primitive defining functions, soft objects $L(f_i, 0.5)$, i=1,…, k, can be blended easily by the following soft blend,

$$L(f_1+\ldots+f_k, 0.5).$$

But, similar to super-ellipsoidal blends, the soft blend can not provide blending range parameter, either. To develop blending operators with blending range parameters for non-zero implicit surfaces, the scale method was proposed in [13] and is described in the next section.

2.2. The scale method

In the scale method, given an existing union blending operator $H_k(x_1,\ldots,x_k)=0$ on zero implicit surfaces $S(f_i, 0)$, $i=1,\ldots,k$, with blending range parameters r_i, $i=1,\ldots,k$, whose 2D shape is shown in Figure 1, then:

(1). A scale function $B_{Ak}:R^k \rightarrow R_+$ can be given by

$$B_{Ak}(x_1,\ldots,x_k)=h_p \qquad \textbf{(1)}$$

where h_p is the root h of the equation

$$T(h)=H_k(x_1/h-1,\ldots,x_k/h-1)=0;$$

(2). A scale function $B_{Sk}:R^k \rightarrow R_+$ can be given by

$$B_{Sk}(x_1,\ldots,x_k)=h_p \qquad \textbf{(2)}$$

where h_p is the root h of the equation

$$T(h)=H_k(1-x_1/h,\ldots,1-x_k/h)=0.$$

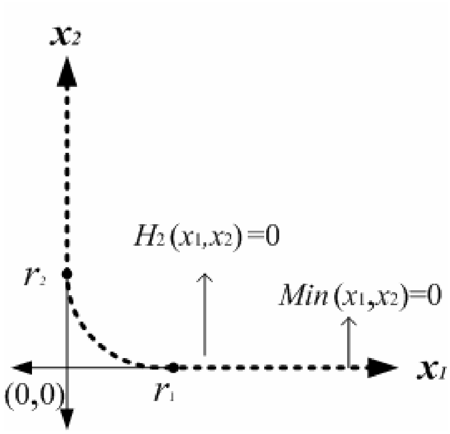


Figure 1. A 2D base curve $H_2(x_1, x_2)=0$.

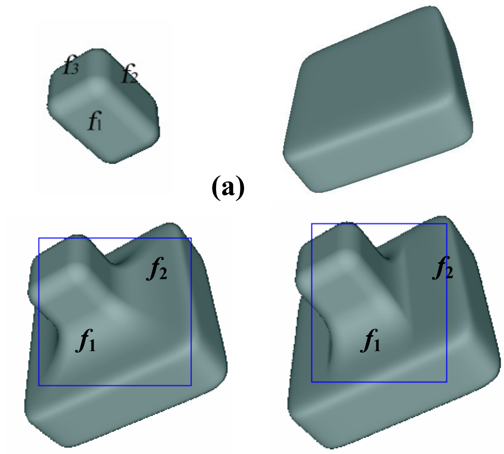
Thus, B_{Ak} and B_{Sk} in Eqs. (1) and (2) can be used as a union and an intersection operators, respectively, on $S(f_i, 1)$ in CG; In addition, B_{Sk} and B_{Ak} can be used as a union and an intersection operators, respectively, on $L(f_i, 0.5)$ in soft object modeling. In [13], conic and hyper-ellipsoidal blends were developed.

However, because B_{Ak} and B_{Sk} behaves like $Min(x_1,\ldots,x_k)$ and $Max(x_1,\ldots,x_k)$ on non-blending regions after blending, primitives $f_1,\ldots,f_k$ oF the blends $B_{Ak}(f_1,\ldots,f_k)$ and $B_{Sk}(f_1,\ldots,f_k)$ always remain their original properties after blending. It leads to the following difficulty that

In sequential blends of $B_{Ak}(f_1,\ldots,f_k)$ and $B_{Sk}(f_1,\ldots,f_k)$, such as $B_{A2}(B_{Ak}(f_1,\ldots,f_k), f_{k+1})$, the primitives $f_1,\ldots,f_k$, always have similar subsequent blending surfaces with f_{k+1}. That is, in non-zero implicit surfaces, B_{Ak} and B_{Sk} can not provide an individual blending range control on every primitive's subsequent blending surface, without deforming the original blending surfaces $B_{Ak}(f_1,\ldots,f_k)=C$ and $B_{Sk}(f_1,\ldots,f_k)=C$.

For example, as shown from Figure 2(b), primitives f_1 and f_2 of the left object $S(B_{S3}(f_1,f_2,f_3), 1)$ in Figure 2(a) always have similar subsequent union blending surfaces with the super-ellipsoid. $B_{S3}(f_1,f_2,f_3)$ can not behave like the object in Figure 2(c), where primitives f_1 and f_2 of $B_{CS3}(f_1,f_2,f_3)$ in Section 3 can be adjusted to have different subsequent union blending surfaces with the super-ellipsoid.

Although the method in [15] can develop blending operators with an individual blending range control on every primitive's subsequent blend, unfortunately it is designed only for developing blends only on zero implicit surfaces, not on non-zero implicit surfaces. This is because the range of the developed operators in [15] is always on a real space R and can not be on a non-negative space R_+, and the range of the blending operators of non-zero implicit surfaces is required to be on R_+.



(b) (c)

Figure 2. (a) Left: an intersection $S(B_3(f_1, f_2, f_3), 1)$ on 3 pairs of parallel planes; Right: a super-ellipsoid. (b) The union of the two objects in Figure 2(a), where f_1 and f_2 always have similar subsequent union blending surfaces with the super-ellipsoid. (c) Sequential blends of $B_{CS3}(f_1,f_2, f_3)$, in Section 3, with different subsequent union blending surfaces of f_1 and f_2 with the super-ellipsoid.

3. A generalized method

In order to solve the difficulty stated in the above section, this section proposes a generalized method to develop new blending operators that can provide an individual blending range control on every primitive's subsequent blends for non-zero implicit surfaces. The generalized method is described by the following two steps:

Step (1): Choose a base surface that is a k-dimensional union blending operator $H_k(x_1,...,x_k)$ =0 on zero implicit surfaces $S(f_i, 0)$, i=1,...,k, and offers blending range parameters r_i, i=1,...,k. Precisely, the shape of the base surface must be the same as the shape $Min(x_1,...,x_k)$=0 on non-blending regions, and its 2D shape is similar to that of the scale method in Figure 1.

Step (2): Develop two new blending operators $B_{CAk}(x_1,...,x_k):R^k\rightarrow R_+$ and $B_{CSk}(x_1,..., x_k):R^k\rightarrow R_+$ on non-zero implicit surfaces using the base surface $H_k(x_1,...,x_k)$ =0 by:

$$B_{CAk}(x_1,...,x_k)=h_p \quad (3)$$

where h_p is the root h of the equation

$$T(h)=H_k(x_1/(C^{(1-m_1)}h^{m_1})-1,\ldots,x_k/(C^{(1-m_k)}h^{m_k})-1)=0;$$

$$B_{CSk}(x_1,...,x_k)=h_p \quad (4)$$

where h_p is the root h of the equation

$$T(h)=-H_k(1-x_1/(C^{(1-m_1)}h^{m_1}),\ldots,1-x_k/(C^{(1-m_k)}h^{m_k}))=0;$$

and $r_i\leq 1$ must hold for i=1,...,k, in Eq. (4) and m_i>0 must hold for i=1,...,k, in Eqs. (3)-(4).

Some properties of $B_{CAk}(x_1,...,x_k)$ and $B_{CSk}(x_1,..., x_k)$ from Eqs. (3)-(4) are discussed as follows.

3.1. When C=1 for CG

(1). When C=1, then whatever positive values $m_1,\ldots,m_k$ are set, $S(B_{CAk}(f_1,...,f_k)$, 1) and $S(B_{CSk}(f_1,...,f_k)$, 1) can always be a union blend and an intersection blend, respectively, on $S(f_i$, 1), i=1,…, k, with blending ranges r_i, i=1,...,k, for CG. And the shapes $B_{CAk}(f_1,...,f_k)$=1 and $B_{CSk}(f_1,...,f_k)$=1 never change whatever positive values $m_1,\ldots, m_k$ are set.

The reason why $S(B_{CAk}(f_1,...,f_k)$, 1) is a union blend on $S(f_i$, 1), i=1,…, k, is explained as follows. Whatever positive values $m_1,\ldots, m_k$, are set and C=1, the set $S(B_{CAk}(f_1,...,f_k)$, 1) is always equivalent to $S(H_k(f_1-1,...,f_k-1), 0)$. This can be derived by setting 0.5 to both C and h of the equation $T(h)$=0 in Eq. (3). The set $S(H_k(f_1-1,...,f_k-1), 0)$ means the union blend of $S(f_i-1, 0)$, i=1,...,k, by $H_k(x_1,...,x_k)$=0. Since the set $S(f_i-1,0)$ is equivalent to $S(f_i,1)$, this implies the set $S(B_{CAk}(f_1,...,f_k)$, 1) is the union blend of $S(f_i,1)$, i=1,...,k, completing the proof. As for $S(B_{CSk}(f_1,...,f_k)$, 1), it is equivalent to the set $S(-H_k(1-f_1,...,1-f_k), 0)$, which is the complement of the union blend of $S(1-f_i,0) \equiv L(f_i,1)$, i=1,...,k, by $H_k(x_1,...,x_k)$. Since the set $L(f_i,1)$ is the complement of $S(f_i,1)$, it follows form De Morgan's law that $S(B_{CSk}(f_1,...,f_k)$, 1) is the intersection blend of $S(f_i,1)$, i=1,...,k. In fact, the shapes of two dimensional $B_{CA2}(x_1,x_2)$=1 and $B_{CS2}(x_1,x_2)$=1 in Figure 3 can tell the reason, too.

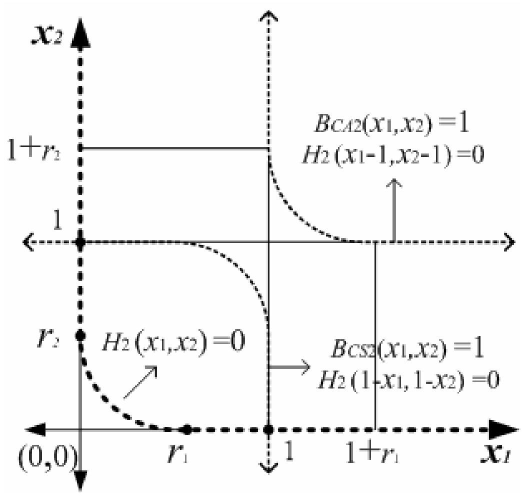


Figure 3. The shapes of 2D blending curves $B_{CA2}(x_1,x_2)$=1 and $B_{CS2}(x_1,x_2)$=1 as C=1 for CSG.

(2). When C=1, $B_{CAk}(f_1,...,f_k)$ and $B_{CSk}(f_1,...,f_k)$ can be applied to generate sequential blends because for any h>0, $B_{CAk}(f_1,...,f_k)=h$ and $B_{CSk}(f_1,...,f_k)=h$ both are smooth blending surfaces of the objects $S(f_i/h^{m_i},1)$, i=1,...,k.

This is because level surfaces $B_{CAk}(x_1,...,x_k)$ =h and $B_{CSk}(x_1,...,x_k)=h$ can be viewed as the surface $H_k(x_1-1,...,x_k-1)=0 \equiv B_{CAk}(x_1,...,x_k)=1$ and $H_k(1-x_1,...,1-x_k)=0 \equiv B_{CSk}(x_1,...,x_k)=1$, scaled, respectively, by $(h^{m_1},\ldots, h^{m_k})$. So for any h>0 $B_{CAk}(f_1,...,f_k)=h$ and $B_{CSk}(f_1,...,f_k)=h$ are the union and the intersection blending surfaces of the objects $S(f_i/h^{m_i}$, 1), i=1,...,k, by the operators $B_{CAk}(x_1,...,x_k)$=1 and $B_{CSk}(x_1,...,x_k)$=1.

(3). When C=1, then in non-blending regions, $B_{CAk}(x_1,...,x_k)$ and $B_{CSk}(x_1,...,x_k)$ can behave like

$$Min(x_1^{1/m_1},...,x_k^{1/m_k}) \text{ and}$$

$$Max(x_1^{1/m_1},...,x_k^{1/m_k}).$$

Since every primitive f_i can become f_i^{1/m_i} after the blend $S((B_{CAk} \circ F_k)$, 1) or $S((B_{CSk} \circ F_k)$, 1), it follows that varying parameter m_i, i=1,...,k, can adjust the sizes of the subsequent blending surface of primitives F_k. Precisely, in sequential blends $S(B_{CA2}(B_{CAk} \circ F_k), f_{k+1})$, 1) with blending ranges r_a and r_b for B_{CA2} , primitives f_i, i=1,...,k, respectively, have blending ranges

$$((1+r_a)^{m_i}-1),\ i=1,...,k,$$

to blend with f_{k+1}. As m_i >1, the blending range of f_i with f_{k+1} gets larger than r_a; as m_i <1, the blending range of f_i with f_{k+1} becomes smaller than r_a. Similarly, $B_{CSk}(x_1,...,x_k)$ has the same property, too.

Since $H_k(x_1,...,x_k)=0$ is $Min\ (x_1,...,x_k)=0$ in non-blending regions, from the requirement of **Step (1),** and C=1, solving the root h of the equations

$$T(h)=Min(x_1/(C^{(1-m_1)}h^{m_1})-1,\ldots,x_k/(C^{(1-m_k)}h^{m_k})-1)=0 \text{ and}$$
$$T(h)=Min(1-x_1/(C^{(1-m_1)}h^{m_1}),\ldots,1-x_k/(C^{(1-m_k)}h^{m_k}))=0$$

in Eqs. (3)-(4) yields the roots $h_p=Min(x_1^{1/m_1},\ldots, x_k^{1/m_k})$ and $h_p=Max(x_1^{1/m_1},\ldots,x_k^{1/m_k})$. This explains the reason why $B_{CAk}(x_1,...,x_k)$ and $B_{CSk}(x_1,..., x_k)$ can behave like $Min(x_1^{1/m_1},\ldots,x_k^{1/m_k})$ and $Max\ (x_1^{1/m_1},\ldots,x_k^{1/m_k})$ in non-blending regions .

(4). When C=1, then by deriving the dual forms,

A. Union: $S(B_{CAk}(f_1,...,f_k), 1)$,

B. Interaction: $S(B_{CSk}(f_1,..., f_k), 1)$, and

C. The difference of $S(f_1,1)$ from $S(f_2,1),\ldots, S(f_k, 1)$: $S(B_{CSk}(f_1,1/f_2...,1/f_k), 1)$,

can offer a full family of boolean set operations on $S(f_i, 1)$, $i=1,\ldots, k$, for CG..

3.2. When C=0.5 for soft objects

(1). When C=0.5, then whatever positive values $m_1,\ldots,m_k$ are set, $L(B_{CAk}(f_1,...,f_k),0.5)$ and $L(B_{CSk}(f_1, ...,f_k), 0.5)$ can always be an intersection blend and a union blend, respectively, on $L(f_i, 0.5)$, $i=1,\ldots, k$, with blending ranges $r_i/2$, $i=1,...,k$, for soft objects modeling. The shapes $B_{CAk}(f_1,...,f_k)=0.5$ and $B_{CSk}(f_1,...,f_k)=0.5$ never change no matter what positive values $m_1,\ldots, m_k$, are set.

The reason why $L(B_{CSk}(f_1,...,f_k), 0.5)$ is a union blend of $L(f_i, 0.5)$, $i=1,\ldots, k$, is explained as follows. Whatever positive values $m_1,\ldots, m_k$, are set and C= 0.5, the set $L(B_{CSk}(f_1,...,f_k), 0.5)$ is always equivalent to $L(-H_k(1-f_1/0.5,...,1-f_k/0.5), 0)$. This can be derived by setting 0.5 to both C and h of the equation $T(h)=0$ in Eq. (4). The set $L(-H_k(1-f_1/0.5,...,1-f_k/0.5), 0)$ equals $S(H_k(1-f_1/0.5,...,1-f_k/0.5), 0)$, which is the union blend of $S(1-f_i/0.5, 0)$, $i=1,...,k$, by $H_k(x_1,...,x_k)=0$. Since $S(1-f_i/0.5,0)$ equals $L(f_i,0.5)$, this implies $L(B_{CSk}(f_1,...,f_k), 0.5)$ is the union blend of $L(f_i, 0.5)$, $i=1,...,k$. This completes the proof. As for the reason of $L(B_{CAk}(f_1,...,f_k), 0.5)$, it can be proved similarly. Besides, the shapes of two dimensional $B_{CA2}(x_1,x_2)$ =0.5 and $B_{CS2}(x_1,x_2)=0.5$ in Figure 4 can explain the reason, too.

(2). When C=0.5, $B_{CAk}(f_1,...,f_k)$ and $B_{CSk}(f_1,...,f_k)$ can be applied to generate sequential blends because for any h>0, $B_{CAk}(f_1,...,f_k)=h$ and $B_{CSk}(f_1,...,f_k)=h$ both are

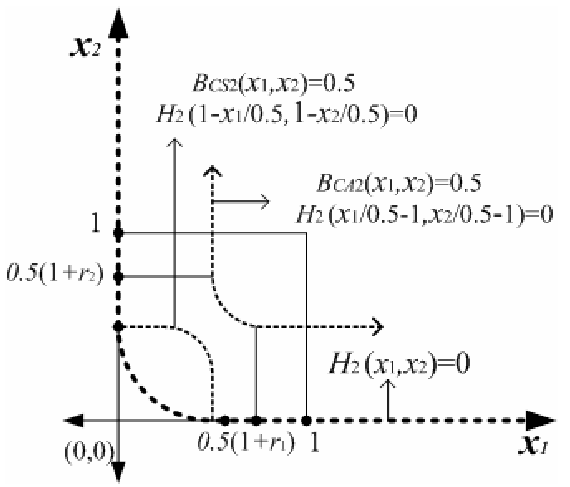


Figure 4. The shapes of 2D blending curves $B_{CA2}(x_1,x_2)=0.5$ and $B_{CS2}(x_1,x_2)=0.5$ as C=0.5 for blending soft objects.

smooth blending surfaces the objects $L(f_i/(h/0.5)^{m_i},0.5)$, $i=1,...,k$.

This is because level surfaces $B_{CAk}(x_1,...,x_k)=h$ and $B_{CSk}(x_1,...,x_k)=h$ can be viewed as the surfaces $H_k(x_1/0.5-1,...,x_k/0.5-1)=0 \equiv B_{CAk}(x_1,...,x_k)=0.5$ and $H_k(1-x_1/0.5,...,1-x_k/0.5)=0 \equiv B_{CSk}(x_1,...,x_k)=0.5$, scaled, respectively, by $((h/0.5)^{m_1},\ldots,(h/0.5)^{m_k})$. So for any h>0, $B_{CAk}(f_1,...,f_k)=h$ and $B_{CSk}(f_1,...,f_k)=h$ are the intersection and the union blending surfaces of objects $L(f_i/(h/0.5)^{m_i},0.5)$, $i=1,...,k$, by the operators $B_{CAk}(x_1,...,x_k)=0.5$ and $B_{CSk}(x_1,...,x_k)=0.5$.

(3). As C=0.5, in non-blending regions, $B_{CAk}(x_1,...,x_k)$ and $B_{CSk}(x_1,..., x_k)$ can behave like

$$Min(0.5(x_1/0.5)^{1/m_1},\ldots, 0.5(x_k/0.5)^{1/m_k}) \text{ and}$$
$$Max(0.5(x_1/0.5)^{1/m_1},\ldots, 0.5(x_k/0.5)^{1/m_k}).$$

Since every primitive f_i can become $0.5(f_1/0.5)^{1/mi}$ after the blend $L((B_{CAk} \circ F_k), 0.5)$ or $L((B_{CSk} \circ F_k), 0.5)$, it follows that varying m_i, $i=1,...,k$, can be used to adjust the size of the subsequent blending surface of primitives f_i, $i=1,...,k$. For example, in sequential blends $S(B_{CA2}(B_{CSk} \circ F_k), f_{k+1}), 1)$ with blending ranges r_a and r_b for B_{CA2}, primitives f_i, $i=1,...,k$, respectively, have blending ranges

$$((1+r_a)^{m_i}-1)/2,\ i=1,...,k,$$

to blend with f_{k+1}. As m_i >1, the blending range of f_i with f_{k+1} is larger than $r_a/2$; as m_k <1, the blending range of f_i with f_{k+1} gets smaller than $r_a/2$. $B_{CAk}(x_1,...,x_k)$ has the same property, too.

The reason why $B_{CAk}(x_1,...,x_k)$ and $B_{CSk}(x_1,..., x_k)$ can behave like $Min(0.5(x_1/0.5)^{1/m_1},\ldots,0.5(x_k/0.5)^{1/m_i})$ and $Max(0.5(x_1/0.5)^{1/m_1},\ldots,0.5(x_k/0.5)^{1/m_k})$ is showed as follows. Because $H_k(x_1,...,x_k)=0$ must be $Min(x_1,...,x_k)=0$ on non-blending regions and $C=0.5$, then solving the root h of the equations $T(h)=$

$$Min(x_1/(0.5^{(1-m_1)}h^{m_1})-1,\ldots,x_k/(0.5^{(1-m_k)}h^{m_k})-1)=0 \text{ and}$$

$$Min(1-x_1/(0.5^{(1-m_1)}h^{m_1}),\ldots,1-x_k/(0.5^{(1-m_k)}h^{m_k}))=0,$$

in Eqs. (3)-(4) yields the roots $h_p=Min(0.5(x_1/0.5)^{1/m_1}, ...,0.5(x_k/0.5)^{1/m_k})$ for Eq. (3) and $h_p=Max(0.5(x_1/0.5)^{1/m_1},...,0.5(x_k/0.5)^{1/m_k})$ for Eq. (4). This gives the proof.

3.3. Gradients of B_{CAk} and B_{CSk}

Calculating the gradients of B_{CAk} and B_{CSk} is usually needed in a shading process, and hence is described below. Let both equations $T(h)=0$ in Eqs. (3) and (4) be viewed as a equation of variables $h,x_1,...,x_k$, and written by

$$T(h) \Rightarrow G(h,x_1,...,x_k)=0.$$

Then, from the implicit theorem [6], both the gradients of $B_{CAk}(x_1,...,x_k)$ and $B_{CSk}(x_1,...,x_k)$ in Eqs. (3)-(4) can be calculated using the values of the root h_p, $x_1..., x_k$, and written, respectively, by

$$B_{CAk}^{(x_i)}(x_1,...,x_k)=$$

$$-G^{(x_i)}(h_p,x_1,...,x_k)/G^{(h)}(h_p,x_1,...,x_k),\ i=1,...,k,$$

where $G(h, x_1,...,x_k)$ is $T(h)$ in Eq. (3);

$$B_{CSk}^{(x_i)}(x_1,...,x_k)=$$

$$-G^{(x_i)}(h_p,x_1,...,x_k)/G^{(h)}(h_p,x_1,...,x_k),\ i=1,...,k,$$

where $G(h, x_1,...,x_k)$ is $T(h)$ in Eq. (4).

4. Deriving real blending operators

Section 3 presents a generalized method only. This section derives some real operators based on the generalized method.

In fact, many existing union blending operators on zero implicit surfaces can be used as the base surface of **Step (1)** in Section 3, such as Hoffmann's conics [9], Middleditch's ellipses [16], super-ellipsoids [26] and hyper-ellipsoids [13]. In the following, hoffmann's conics and hyper-ellipsoids are applied to develop B_{CAk} and B_{CSk}.

4.1. Two-dimensional blending operators

In fact, 2D $H_2(x_1, x_2)$ in **Step (1)** can be defined piecewise. That is, $H_2(x_1, x_2)=0$ is the union of the curve $Min(x_1, x_2)=0$ and an arc tangent to $Min(x_1, x_2)=0$. When conics $H_H(x_1,x_2)=0$ [9] are used as the arc, 2D base curve $H_2(x_1, x_2)=0$ can be written by

$$\begin{cases} H_H(x_1,x_2)=0 & \text{blending region} \\ Min(x_1,x_2)=0 & \text{non-blending region} \end{cases}$$

where $H_H(x_1,x_2)=r_2^2x_1^2+r_1^2x_2^2+r_2^2r_1^2-2r_2^2r_1x_1-2r_1^2r_2x_2+2px_1x_2$, $-\infty<p<r_1r_2$, then conic blending operators B_{CA2} and B_{CS2}, with blending range parameters r_1 and r_2 and a curvature parameter p, for binary blends can be given, respectively, by:

$B_{CA2}(x_1, x_2)=$

$$\begin{cases} (x_1/C^{1-m_1})^{1/m_1} & x_2 \ge (1+r_2)C^{1-m_2}(x_1/C^{1-m_1})^{m_2/m_1} \\ (x_2/C^{1-m_2})^{1/m_2} & x_1 \ge (1+r_1)C^{1-m_1}(x_2/C^{1-m_2})^{m_1/m_2} \\ h_p & \text{otherwise} \end{cases} \quad \mathbf{(5)}$$

where $m_1>0$, $m_2>0$, $h_p\in T^{-1}(0)$, and

$$T(h)=H_H(x_1/(C^{(1-m1)}h^{m1})-1,x_k/(C^{(1-m2)}h^{m2})-1);$$

$B_{CS2}(x_1, x_2)=$

$$\begin{cases} (x_1/C^{1-m_1})^{1/m_1} & x_2 \le (1-r_2)C^{1-m_2}(x_1/C^{1-m_1})^{m_2/m_1} \\ (x_2/C^{1-m_2})^{1/m_2} & x_1 \le (1-r_1)C^{1-m_1}(x_2/C^{1-m_2})^{m_1/m_2} \\ h_p & \text{otherwise} \end{cases} \quad \mathbf{(6)}$$

where $m_1>0$, $m_2>0$, $r_1\le1$ and $r_2\le1$, $h_p\in T^{-1}(0)$, and

$$T(h)=-H_H(1-x_1/(C^{(1-m_1)}h^{m_1}),\ 1-x_k/(C^{(1-m_2)}h^{m_2})).$$

To solve the root h_p of the equation $T(h)=0$ in Eqs. **(5)-(6)**, one can choose a numerical method, such as Newton-Ralphson method where

$$h=Min((x_1/C^{(1-m_1)})^{1/m_1}, (x_2/C^{(1-m_2)})^{1/m_2})$$

can be used as the initial guess for Eq. (5), and

$$h=Max((x_1/C^{(1-m_1)})^{1/m_1}, (x_2/C^{(1-m_2)})^{1/m_2})$$

can be used as the initial guess for Eq. (6).

4.2. High-dimensional blending operators

When the base surface $H_k(x_1,\ldots,x_k)$ in **Step (1)** is given by hyper-ellipsoids [13],

$$H_k(x_1,\ldots,x_k)=\sum_{i=1}^{k}[(r_i-x_i)/r_i]_+^{p_i}-1=0,$$

where $[*]_+\equiv Max(0, *)$ and both $r_i>0$ and $p_i>1$ hold for $i=1,\ldots,k$, then from Eqs. (3)-(4) hyper-ellipsoidal blending operators B_{CAk} and B_{CSk}, with blending range parameters $r_1,\ldots,r_k$, and curvature parameters $p_1,\ldots,p_k$, for simultaneous multiple blends can be given, respectively, by

$$B_{CAk}(x_1,...,x_k)=h_p \quad \mathbf{(7)}$$

where $h_p\in T^{-1}(0)$,

$$T(h)=\sum_{i=1}^{k}[(r_i-x_i/(C^{(1-m_i}h^{m_i})+1)/r_i]_+^{p_i}-1,$$

and m_i>0 for i=1,...,k;

$$B_{CSk}(x_1,...,x_k)=h_p \qquad \textbf{(8)}$$

where $h_p \in T^{-1}(0)$,

$$T(h)=1-\sum_{i=1}^{k}[(r_i+x_i/(C^{(1-m_i)}h^{m_i})-1)/r_i]_+^{p_i},$$

m_i>0 for i=1,...,k, and r_i≤1 for i=1,...,k.

To solve the root h_p of the equation $T(h)$=0 in Eqs. **(7)-(8)**, one can utilize a numerical method, such as Newton-Ralphson method where

$$h=Min(\,(x_1/C^{(1\text{-}m_1)})^{1/m_1},\ldots,(x_k/C^{(1\text{-}m_k)})^{1/m_k}\,)$$

can be used as the initial guess for Eq. (7), and

$$h=Max(\,(x_1/C^{(1\text{-}m_1)})^{1/m_1},\ldots,(x_k/C^{(1\text{-}m_k)})^{1/m_k}\,)$$

can be used as the initial guess for Eq. (8).

5. Demonstration

This section demonstrates the individual blending range control on primitives' subsequent blends by the proposed blends of Eqs. (5)-(8) in Section 3.

Example 1: When reused as a primitive in a union blend, the intersection operations ($B_{CSk} \circ F_k$)=1 for CG and ($B_{CAk} \circ F_k$)=0.5 for soft objects can vary parameters $m_1,\ldots,m_k$, to individually adjust the blending range of the subsequent union blends of primitives $f_1, \ldots, f_k$, and the original blending surfaces ($B_{CSk} \circ F_k$)=1 and ($B_{CAk} \circ F_k$)=0.5 always keep unchanged. This can be seen from the sequential blends $S(B_{CA2}(B_{CS3} \circ F_3, f_4), 1)$ in Figure 5(b), where when m_1 for f_1 of B_{CS3} is increased from 0.25, 0.5, 0.75, 1, 1,25 to 1.5, only the subsequent union blending surface of f_1=1 with the super-ellipsoid f_4=1 gets bigger and bigger and the shape ($B_{CS3} \circ F_3$)=1 remains unchanged, as shown on the objects from top left to bottom right.

Example 2: When reused as a primitive in a difference blend, the intersection operation ($B_{CSk} \circ F_k$)=1 for CG can individually vary parameters $m_1,\ldots,m_k$, to adjust the blending range of the subsequent blend of primitives $f_1, \ldots, f_k$, the original blending surface ($B_{CSk} \circ F_k$)=1 always remains unchanged. This can be seen from the sequential blends $S(B_{CS2}(f_5, 1/(B_{CS4} \circ F_4)), 1)$ in Figure 6(b), where when m_2 for f_2 of B_{CS4} is increased from 0.25, 0.5, 0.75, 1, 2 to 3, only the subsequent union blending surface of f_2=1 with the super-ellipsoid f_5=1 is adjusted to get bigger and bigger, as shown from the objects from top left to bottom right.

Example 3: When reused as a primitive in a union blend, the union operations ($B_{CAk} \circ F_k$)=1 for CG, and ($B_{CSk} \circ F_k$)=0.5 for soft objects, can vary parameters $m_1,\ldots, m_k$, to individually adjust the blending range of the subsequent difference blend of primitives $f_1,\ldots, f_k$, the original blending surfaces ($B_{CAk} \circ F_k$)=1 and ($B_{CSk} \circ F_k$)=0.5 always remain unchanged. This can be seen from the sequential blends $S(B_{CA2}(B_{CA2} \circ F_2, f_3), 1)$ in Figure 7(b), where when m_1 for f_1 of B_{CA2} is increased from 0.3, 0.6, 1 to 2, only the subsequent union blending surface of f_1=1 with the toroid f_3=1 gets rounded gradually, as shown on the objects from top left to bottom right.

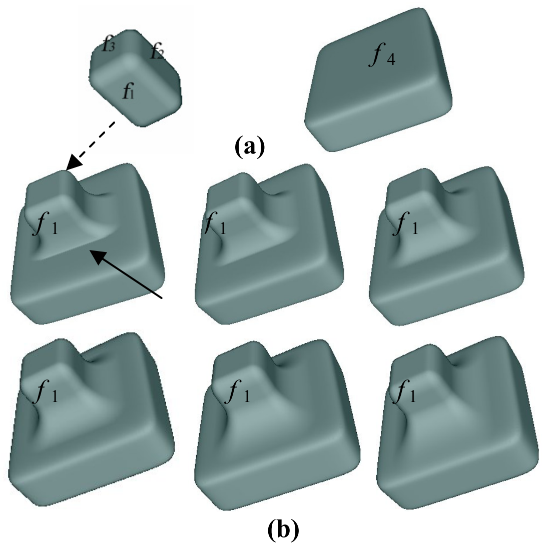


Figure 5. (a) Left: An intersection ($B_{CS3} \circ F_3$)=1 on 3 pairs of parallel planes; Right: A super-ellipsoid f_4=1. (b) The union of the two objects in Figure 5(a), where only the subsequent blending surface, pointed by a arrow, of f_1=1 with the super-ellipsoid f_4=1 are enlarged gradually, but the shape ($B_{CS3} \circ F_3$)=1 remains unchanged, pointed by a dotted arrow, as m_1 for f_1 is increased from 0.25, 0.5, 0.75, 1, 1.25 to 1.5 for the objects from top left to bottom right.

6. Conclusions

In non-zero implicit surfaces, such as soft object modeling and CG, most of the existing blends always keep unchanged every primitive's property on non-blending regions after blending. This causes that when reused as a new primitive in other blends, they can not adjust their primitives' subsequent blending surfaces. To solve this problem, this paper has proposed new blends that can provide parameters to adjust their primitives' subsequent blending surfaces, respectively, without deforming the original blending surface. On the contrary, other existing blends of non-zero implicit surfaces do not have this kind of ability.

Furthermore, this paper has proposed a generalized method to transform some existing union

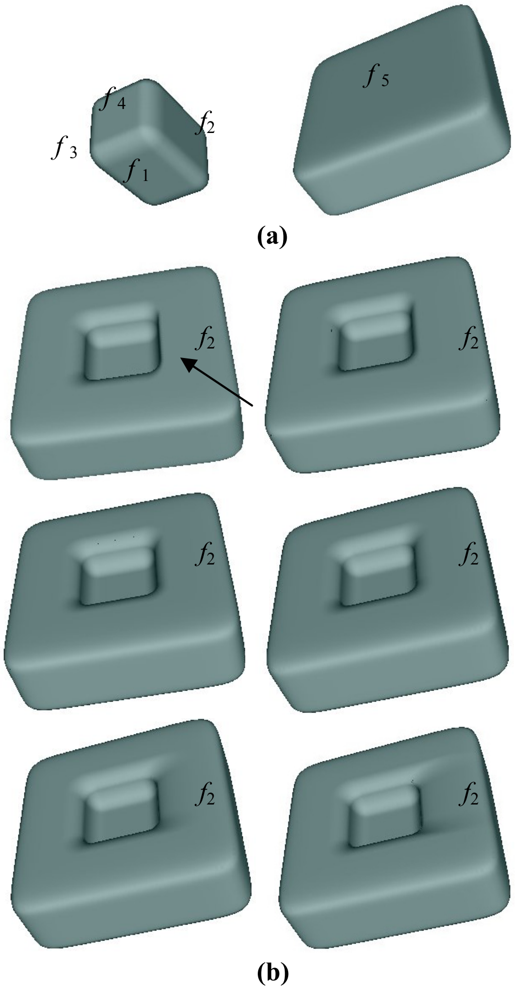


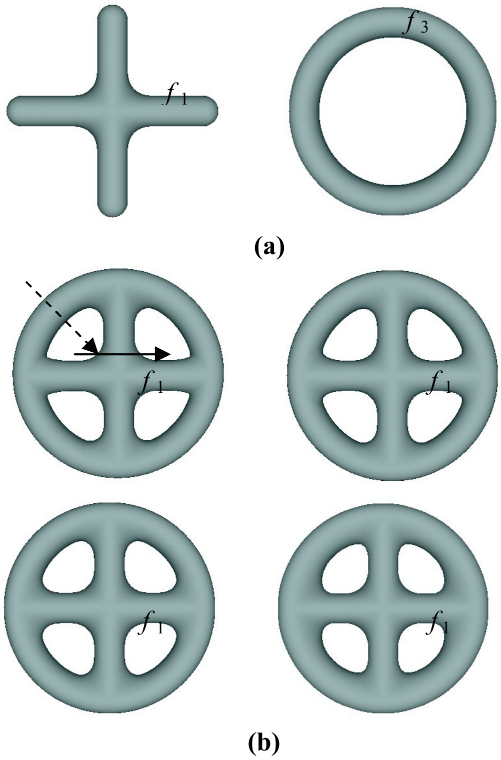
Figure 6. (a) Left: An intersection ($B_{CS4} \circ F_4$)=1 on six planes; Right: A super-ellipsoid f_5=1. (b) The difference of the super-ellipsoid from ($B_{CS4} \circ F_4$)=1 above. Only the subsequent blending surface, pointed by a arrow, of f_2=1 with the super-ellipsoid f_5=1 gets rounded gradually but the shape ($B_{CS4} \circ F_4$)=1 remains unchanged, as m_2 for f_2 is increased from 0.25, 0.5, 0.75, 1, 2 to 3 for the objects from top left to bottom right.

blending operators into a new blending operator that has an individual blending range control on every primitive's subsequent blend. In this paper, conic and hyper-ellipsoidal blends have been developed by the generalized method. Precisely, the developed blending operators can:

(1). Provide blending range parameters to adjust the size of the resulting blending surface, without deforming the overall shapes of blended primitives.

(2). Provide parameters to adjust their primitives' subsequent blending surfaces, respectively, in sequential blends without deforming the original blending surface.

(3). Provide curvature parameters to adjust the shape of the transition of the resulting blending surface.



(b)

Figure 7. (a) Left: An union ($B_{CA2} \circ F_2$)=1 on crossing cylinders; Right: A toroid f_3=1 (b) The union of the two objects in Figure 7(a), where only the subsequent blending surface, pointed by a arrow, of f_1=1 with the toroid f_3=1 enlarges gradually, but the joint of the crossing cylinders can remain unchanged, pointed by a dotted arrow, as m_1 for f_1 is increased from 0.3, 0.6, 1 to 2 for the objects from top left to bottom right.

(4). Generate smooth sequential blending surfaces with overlapping blending regions.

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