Construction Schemes of Hamiltonian Laceable and Bipancyclic Graphs *

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Abstract

In this paper, we study some *hamiltonian* properties of bipartite graphs. Every hamiltonian *bipartite graph* $G = (V_0 \cup V_1, E)$ satisfies $|V_0| = |V_1|$. Since the colors of a path in bipartite graphs alternate, any hamiltonian bipartite graphs cannot be *hamiltonian connected*. Thus, the concepts of *hamiltonian laceability*, *strongly hamiltonian laceability*, *hyper-hamiltonian laceability*, *bipancyclicity*, and *edge-bipancyclicity* for hamiltonian bipartite graphs are attractive topics in interconnection networks. In this paper, we propose several methods, which extend the result in [7], to construct hamiltonian laceable, strongly hamiltonian laceable, and hyper-hamiltonian laceable graphs. We also show that our construction schemes preserve bipancyclic and edge-bipancyclic properties.

Keywords–Hamiltonian laceable, Strongly hamiltonian laceable, Hyper-hamiltonian laceable, Bipancyclic, Edge-bipancyclic.

1 Introduction

For the graph definitions and notations we follow [2]. $G = (V, E)$ is a graph if V is a finite set and E is a subset of $\{(a, b) \mid (a, b)$ is an unordered pair of $V\}$. We say that V is the *node set* and E is the *edge set*. Two nodes a and b are *adjacent* if $(a, b) \in E$. A

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path is a sequence of adjacent nodes, written as $\langle v_0, v_1, v_2, \cdots, v_m \rangle$, in which all the nodes $v_0, v_1, v_2, \cdots, v_m$ are distinct except possibly $v_0 = v_m$. We also write the path $\langle v_0, P, v_m \rangle$ where $P = \langle v_0, v_1, v_2, \cdots, v_m \rangle$. A path is a *hamiltonian path* if its nodes are distinct and they span V. A cycle is a *hamiltonian cycle* if it traverses every node of G exactly once. A graph G is *hamiltonian* if it has a hamiltonian cycle; and G is *hamiltonian connected* if for any two nodes of G, there exists a hamiltonian path joining these two.

A graph $G = (V_0 \cup V_1, E)$ is *bipartite* if $V(G)$ is the union of two disjoint sets V_0 and V_1 such that each edge consists of one node from each set. We say that V_0 and V_1 are two different colored node sets. Any hamiltonian bipartite graph $G = (V_0 \cup V_1, E)$ satisfies $|V_0| = |V_1|$. We observe that the colors of a path in bipartite graphs alternate, any hamiltonian bipartite graph is not hamiltonian connected.

Simmons [15] introduced the concept of *hamiltonian laceability* for hamiltonian bipartite graphs. A bipartite graph is *equitable* if it has the same number of nodes in each of its two colors. If the number of nodes in each of its two color sets differ by exactly one, it is called *nearly equitable*. A bipartite graph is defined [15] to be *hamiltonian laceable* if (a) it is equitable and whenever x and y are nodes of opposite colors, there exists an x-y hamiltonian path; or else (b) it is nearly equitable and whenever x and y are nodes of the larger color set, there exists an x-y hamiltonian path. Hsieh et al. [8] extended this concept into *strongly hamiltonian laceability*. A hamiltonian laceable graph G is *strongly hamiltonian laceable* if (a) G is equitable and there is a simple path of length $|V(G)| - 2$ between any two nodes of the same color; or else (b) G is nearly equitable and there is a simple path of length $|V(G)| - 2$ between any two nodes of opposite colors. Lewinter et al. [11] further introduced the concept of *hyper-hamiltonian laceability*. A hamiltonian laceable graph G is *hyper-hamiltonian laceable* if (a) G is equitable and if v is any node of G, $G - v$ is hamiltonian laceable; or else (b) G is nearly equitable and if v is any node in the large color set, $G - v$ is hamiltonian laceable.

Hamiltonian laceability, which deals with embedding a hamiltonian path in a given graph, is an important topic in interconnection networks. The ring embedding problem, which deals with all the possible lengths of cycles in a given graph, is investigated in the interconnection networks [4, 1, 6, 9, 5]. A graph is called *pancyclic* if it contains a cycle of every length from 3 to $|V(G)|$ inclusive [3]. The concept of pancyclicity has been extended to *bipancyclicity* [13]. Bipancyclicity is essentially a restriction of the concept of pancyclicity to bipartite graphs whose cycles are necessarily of even length. A bipartite graph is *edge-bipancyclic* [13] if every edge lies on a cycle of every even length from 4 to $|V(G)|$ inclusive.

Recent studies have proposed several operations performing on hamiltonian laceable graphs to yield several attractive properties. In [7], Harary and Lewinter proposed some recursively defined hamiltonian laceable graphs, denoted by $J(G)$, and asked that whether there are additional operations performing on hamiltonian laceable graphs to yield hamiltonian laceability. In 1993, Lewinter [11] proposed an operation performing on hamiltonian laceable graphs to yield hyper-hamiltonian laceability. In 1996, Liu [12] proposed another recursively construction scheme to construct hamiltonian-type graphs. In Section 2, we show that the construction scheme proposed in [7] for $J(G)$ will make hamiltonian laceable graphs to be both hamiltonian laceability and strongly hamiltonian laceability. We also propose some recursively construction schemes to construct hamiltonian-type graphs. On the other hand, bipancyclic property is also an attractive topic in interconnection networks. In 1988, Saad and Schultz [14] proved that hypercube, Q_n, is bipancyclic if and only if $n \geq 2$. In 1991, Jwo et al. [10] also showed that all the even cycles with length l such that $6 \leq l \leq n!$ can be embedded in star graph S_n. Hence, in Section 3, we show that these recursively construction schemes performing on bipancyclic and edge-bipancyclic graphs can yield bipancyclicity and edge-bipancyclicity, respectively. Section 4 provides some concluding remarks.

2 Hamiltonian Laceability, Strongly Hamiltonian Laceability, and Hyper-Hamiltonian Laceability

Given a hamiltonian laceable graph G, Harary and Lewinter [7] proposed a construction scheme to extend G to a larger graph $J(G)$, while maintaining the hamiltonian laceable property as follows. *Let G be a bipartite graph with white and black node sets $\{x_1, x_2, \cdots, x_m\}$ and $\{y_1, y_2, \cdots, y_n\}$, respectively. Let $J(G)$ be the graph obtained from G by adding a white node w, a black node b, and all the edges (b, x_i) and (w, y_j) for $1 \le i \le m$ and $1 \le j \le n$.* See Figure 1.

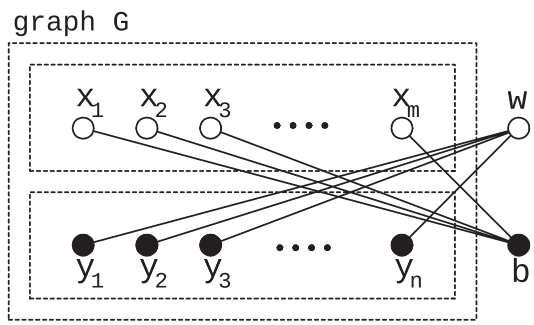

Figure 1: Graph $J(G)$.

In [7], Harary and Lewinter showed that if G is an equitable hamiltonian laceable graph with at least four nodes, then $J(G)$ is hamiltonian laceable. In the following, we have a further result that $J(G)$ is not only hamiltonian laceable, but also strongly hamiltonian laceable.

Theorem 1 *Let G be an equitable hamiltonian laceable graph with at least four nodes. Then $J(G)$ is strongly hamiltonian laceable. Moreover, $J(G) - b$ and $J(G) - w$ are both hamiltonian laceable.*

Proof. By [7], $J(G)$ is hamiltonian laceable. To show that $J(G)$ is strongly hamiltonian laceable, it is sufficient to prove that both $J(G)-b$ and $J(G)-w$ are hamiltonian laceable. We shall prove the case for $J(G) - b$. As for $J(G) - w$, the case is similar.

Case 1: For any x_k, find a hamiltonian path of $J(G) - b$ joining w and x_k. (See Figure 2(a).)

Since G is hamiltonian laceable, G has a hamiltonian path joining x_k and y_i for any i. We arbitrarily choose a y_i and let $\langle x_k, P, y_i\rangle$ be a hamiltonian path of G. By definition, $(w, y_i) \in E(J(G))$. Thus, $J(G) - b$ has a hamiltonian path $\langle x_k, P, y_i, w\rangle$.

Case 2: For any $x_k \neq x_{k'}$, find a hamiltonian path of $J(G) - b$ joining x_k and $x_{k'}$. (See Figure 2(b).)

Since G is hamiltonian laceable, for any y_i, G has a hamiltonian path joining x_k and y_i. We arbitrarily choose a y_i and let y_r,y_s be the two nodes adjacent to $x_{k'}$ on this hamiltonian path. Hence, we may label this path as$\langle x_k, P_1, y_r, x_{k'}, y_s, P_2, y_i\rangle$ without loss of generality. By definition, $(w, y_r), (w, y_i) \in E(J(G))$. So $\langle x_k, P_1, y_r, w, y_i, P_2, y_s, x_{k'}\rangle$ is a hamiltonian path in $J(G) - b$ joining x_k and $x_{k'}$. Therefore, the proof of this theorem is complete. □

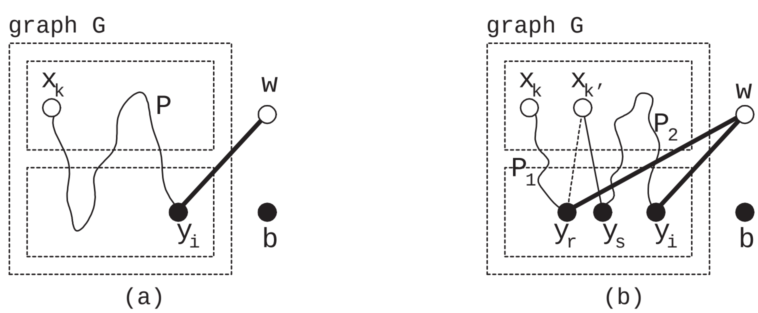

Figure 2: (a) Case 1: For any x_k, find a hamiltonian path of $J(G) - b$ joining w and x_k. (b) Case 2: For any $x_k \neq x_{k'}$, find a hamiltonian path of $J(G) - b$ joining x_k and $x_{k'}$.

With a similar argument as above, we have the following result.

Theorem 2 *Let G be a nearly equitable hamiltonian laceable graph with at least five nodes. Then, $J(G)$ is strongly hamiltonian laceable.*

Now, we define a new graph $J'(G)$ with $V(J'(G)) = V(J(G))$ and $E(J'(G)) = E(J(G)) \cup \{(w,b)\}$. With this additional edge (w,b) in $J'(G)$, we have another stronger result that $J'(G)$ is hyper-hamiltonian laceable if G is hamiltonian laceable with at least four nodes. We note, however, that $J(G)$ is not necessarily hyper-hamiltonian laceable. For example, let G be the complete bipartite graph $K_{2,2}$, $J(G)$ is strongly hamiltonian laceable but not hyper-hamiltonian laceable. In order to prove that $J'(G)$ is hyper-hamiltonian laceable, we show a lemma first.

Lemma 1 *Let G be a bipartite graph. Suppose that $G - v$ is hamiltonian laceable for every $v \in V(G)$ if G is equitable or else $G - v$ is hamiltonian laceable for every node v in the larger color set if G is nearly equitable, then G is hamiltonian laceable.*

Proof. We shall prove that G is hamiltonian laceable by finding a hamiltonian path (1) joining any two distinct nodes x and y with different colors if G is equitable, or else (2) joining any two distinct nodes x and y in the larger color set if G is nearly equitable. Let a be a node adjacent to y. Since $G - y$ is hamiltonian laceable, $G - y$ has a hamiltonian path joining x and a. Thus, there is a hamiltonian path of G joining x and y since $(a, y) \in E(G)$. So G is hamiltonian laceable. □

Theorem 3 *If G is an equitable hamiltonian laceable graph with at least four nodes, then $J'(G)$ is hyper-hamiltonian laceable.*

Proof. We shall prove the following two statements: (1) $J'(G)$ is hamiltonian laceable; and (2) $J'(G) - f$ is hamiltonian laceable for any $f \in V(J'(G))$. By Lemma 1, (1) is correct if (2) is. Thus, we need only to check whether (2) holds.

By Theorem 1, $J'(G) - f$ is hamiltonian laceable, if $f = w$ or $f = b$. Now, consider the case that $f \neq w$ and $f \neq b$, we may without loss of generality assume that $f = y_m$ for some m. Then, $J'(G) - y_m$ contains a subgraph isomorphic to $J'(G) - b$, where node b replaces the node y_m. $J'(G) - b$ is hamiltonian laceable, so is $J'(G) - y_m$. Thus, the theorem follows. □

In a like manner as above, we have the following result.

Theorem 4 *If G is a nearly equitable hamiltonian laceable graph with at least five nodes, then $J'(G)$ is hyper-hamiltonian laceable.*

Therefore, we may use the above two operations, J and J', to recursively construct infinitely many hamiltonian-type graphs. In the following, we have yet another construction scheme for $X(G)$, to construct hamiltonian laceable graphs. With operation X, we can recursively construct hamiltonian laceable graphs by adding less edges to G than J.

Definition 1 *Let G be a hamiltonian laceable graph with white node set $\{x_1, x_2, \cdots, x_m\}$ and black node set $\{y_1, y_2, \cdots, y_n\}$. Let $X(G)$ be the graph resulting from adding a white node w and a black node b. And $E(X(G)) = E(G) \cup (w,b) \cup (w,y_n) \cup (b,x_m) \cup \bigcup_{(x_m,y_i)\in E(G)} (w,y_i) \cup \bigcup_{(y_n,x_i)\in E(G)} (b,x_i)$.* In other words, we arbitrarily choose a white node x_m and a black node y_n, make a new copy for each, say w and b, and add necessarily edges together with three more edges (w,b), (w,y_n), and (b,x_m). See Figure 3.

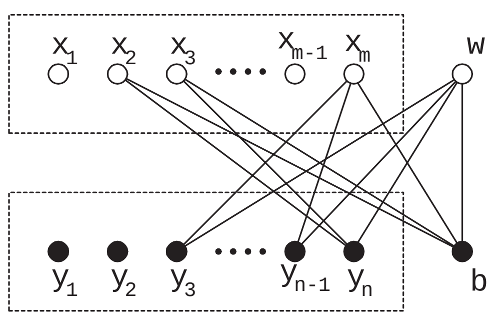

Figure 3: Graph $X(G)$.

By the definition of $X(G)$, we have the following result.

Theorem 5 *If G is an equitable hamiltonian laceable graph with at least four nodes, then $X(G)$ is also hamiltonian laceable.*

Proof. Let $V(G)$ be the union of white node set $\{x_1, x_2, \cdots, x_n\}$ and black node set $\{y_1, y_2, \cdots, y_n\}$. To prove that $X(G)$ is hamiltonian laceable, for any two nodes with different colors, we need to find a hamiltonian path joining these two. We divide the proof into three cases.

Case 1: Find a hamiltonian path of $X(G)$ joining w and b. (See Figure 4(a).)

Since G is hamiltonian laceable, G has a hamiltonian path $\langle x_n, P_1, y_n\rangle$. $(b, x_n), (w, y_n) \in E(X(G))$ by definition. Thus, we have a hamiltonian path $\langle b, x_n, P_1, y_n, w\rangle$ of $X(G)$.

Case 2: For any y_i and x_i, find a hamiltonian path of $X(G)$ joining w and y_i, and a hamiltonian path joining b and x_i. (See Figure 4(b).)

We shall show the case for w and y_i, the other case is similar. Since G is hamiltonian laceable, we have a hamiltonian path of G joining x_n and y_i, say $\langle x_n, P_2, y_i\rangle$. By definition, $(b, x_n), (w, b) \in E(X(G))$. Hence, $\langle w, b, x_n, P_2, y_i\rangle$ forms a hamiltonian path of $X(G)$.

Case 3: For any x_i and y_j, find a hamiltonian path of $X(G)$ joining x_i and y_j. (See Figure 4(c).)

Since G is hamiltonian laceable, we have a hamiltonian path of G joining x_i and y_j, say $\langle x_i, P_3, y_n, P_4, y_j\rangle$. Let x_k be the node adjacent to y_n on path P_3. We remark that if $y_j = y_n$, then P_4 is an empty path. Thus, we may relabel $\langle x_i, P_3, y_n, P_4, y_j\rangle$ as $\langle x_i, P_3', x_k, y_n, P_4, y_j\rangle$. By definition, edges (x_k, b), (b, w), and (w, y_n) belong to $E(X(G))$. Therefore, $\langle x_i, P_3', x_k, b, w, y_n, P_4, y_j\rangle$ is a hamiltonian path of $X(G)$. Therefore, the proof of this theorem is complete. □

With a similar argument as above, we have the following result.

Theorem 6 *If G is a nearly equitable hamiltonian laceable graph with at least five nodes, then $X(G)$ is also hamiltonian laceable.*

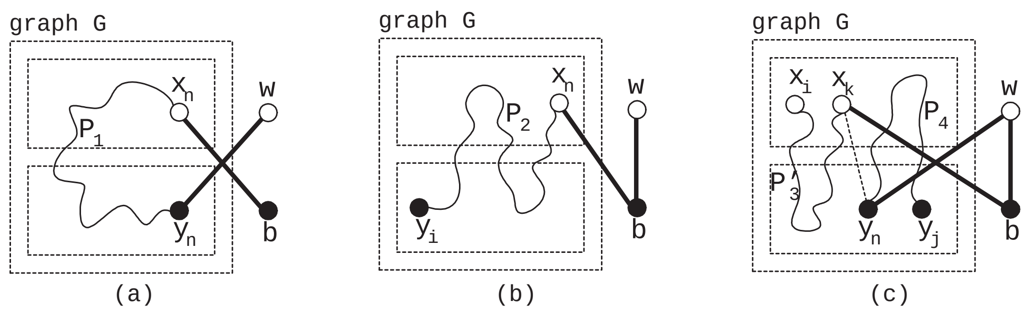

Figure 4: (a) Case 1: Find a hamiltonian path of $X(G)$ joining w and b; (b) Case 2: For any y_i and x_i, find a hamiltonian path of $X(G)$ joining w and y_i, and a hamiltonian path joining b and x_i; (c) Case 3: For any x_i and y_j, find a hamiltonian path of $X(G)$ joining x_i and y_j.

3 Bipancyclicity and Edge-Bipancyclicity

Theorem 7 *Let G be an equitable bipartite graph with at least four nodes. If G is bipancyclic, then $J(G)$ is also bipancyclic.*

Proof. We show this result by finding cycles of every even length from 4 to $|V(G)|+2$ in $J(G)$. Since G is bipancyclic, there are cycles of every even length from 4 to $|V(G)|$ in $J(G)$. Hence, we need only to find a cycle of length $|V(G)|+2$ in $J(G)$. Let a_1, a_2, a_3, and a_4 be four consecutive nodes in the hamiltonian cycle of G; the color of a_1,a_3 be white, and a_2,a_4 be black. Thus, we may label this cycle as $\langle a_1, a_2, a_3, a_4, P, a_1\rangle$. Then, $\langle a_1, b, a_3, a_2, w, a_4, P, a_1\rangle$ forms a hamiltonian cycle of $J(G)$ with length $|V(G)|+2$. (See Figure 5.) Consequently, this theorem is proved. $\square$

Theorem 8 *Let G be an equitable bipartite graph with at least four nodes. If G is edge-bipancyclic, then $J(G)$ is also edge-bipancyclic.*

Proof. Let e be any edge in $J(G)$. To prove that $J(G)$ is edge-bipancyclic, we need to find cycles containing edge e of every even length from 4 to $|V(G)|+2$.

Case 1: $e \in E(G)$.

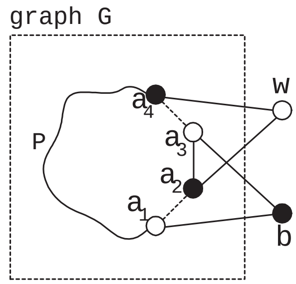

Figure 5: Pancyclicity of $J(G)$.

Let $e = (x_i, y_j)$ for some i, j. By the assumption that G is edge-bipancyclic, $J(G)$ has cycles containing edge (x_i, y_j) of every even length from 4 to $|V(G)|$. Let y, x_i, y_j, x be four consecutive nodes in the hamiltonian cycle of G containing edge (x_i, y_j). By definition, edges (w, y), (w, y_j), (b, x), and (b, x_i) belong to $E(J(G))$. Then, $\langle y, w, y_j, x_i, b, x, P_1, y\rangle$ forms a hamiltonian cycle of $J(G)$ with length $|V(G)| + 2$. (See Figure 6(a).)

Case 2: $e \in E(J(G)) - E(G)$.

Without loss of generality, we may consider only the case that $e = (b, x_i)$ for any i. Consequently, we will find a cycle containing edge (b, x_i) of every even length from 4 to $|V(G)| + 2$ in $J(G)$. Since G is bipancyclic, there are cycles of every even length from 4 to $|V(G)|$ in G. Let x_i, a_2, a_3, a_4 be four consecutive nodes in this cycle; and the color of x_i,a_3 be white, and a_2,a_4 be black. Thus, we may label this cycle as $\langle x_i, a_2, a_3, a_4, P_2, x_i\rangle$. Therefore, $\langle x_i, b, a_3, a_2, w, a_4, P_2, x_i\rangle$ form cycles containing edge (b, x_i) of every even length from 6 to $|V(G)|+2$ of $J(G)$. (See Figure 6(b).) In addition, $\langle x_i, b, a_3, a_2, x_i\rangle$ forms a cycle containing edge (b, x_i) of length 4 of $J(G)$. (See Figure 6(c).) This theorem is complete. $\square$

In the following two theorems, we establish that X is also a recursively construction scheme for bipancyclic and edge-bipancyclic graphs.

Theorem 9 *Let G be an equitable bipartite graph with at least four nodes. If G is bipan-*

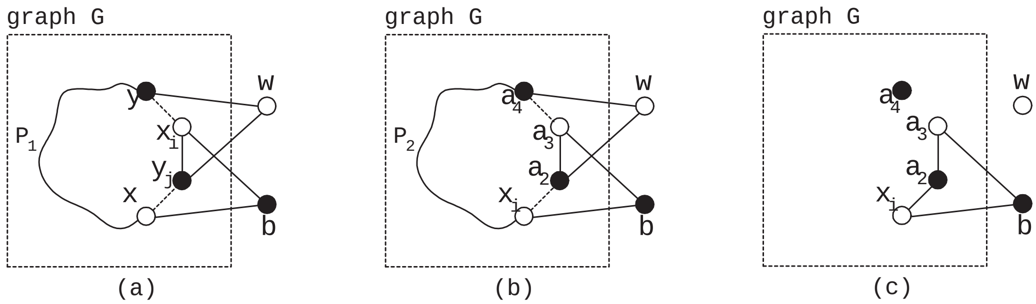

Figure 6: Edge-bipancyclicity of $J(G)$.

cyclic, then $X(G)$ is also bipancyclic.

Proof. By the assumption that G is bipancyclic, to prove that $X(G)$ is also bipancyclic, we need to find a cycle of length $|V(G)|+2$ in $X(G)$. Since G is equitable and bipancyclic, there will be a hamiltonian cycle in G. Let $V(G)$ be the union of white nodes $\{x_1, x_2, \cdots, x_n\}$ and black nodes $\{y_1, y_2, \cdots, y_n\}$. Let x_n and a be two consecutive nodes on this hamiltonian cycle. Thus, we may label this cycle as $\langle x_n, a, P, x_n\rangle$. Therefore, $\langle x_n, b, w, a, P, x_n\rangle$ forms a hamiltonian cycle of length $|V(G)|+2$ of $X(G)$. (See Figure 7.) This completes the proof. □

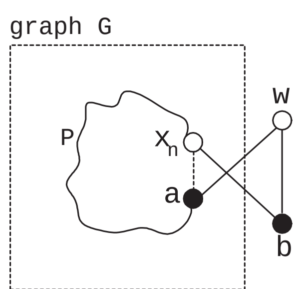

Figure 7: Pancyclicity of $X(G)$.

Theorem 10 *Let G be an equitable bipartite graph with at least four nodes. If G is edge-bipancyclic, then $X(G)$ is also edge-bipancyclic.*

Proof. Let $V(G)$ be the union of white node set $\{x_1, x_2, \cdots, x_n\}$ and black node set $\{y_1, y_2, \cdots, y_n\}$. Let e be any edge in $X(G)$. We shall prove that $X(G)$ is edge-bipancyclic by finding cycles containing edge e of every even length from 4 to $|V(G)| + 2$ in $X(G)$.

Case 1: $e \in E(G)$.

Because G is edge-bipancyclic, we can find cycles of every even length from 4 to $|V(G)|$ in G. Let $\langle y_i, x_n, y_j, P_1, y_i\rangle$ be a hamiltonian cycle of G containing edge e where y_i, y_j are two nodes adjacent to x_n. Of course, at least one of (x_n, y_i) and (y_j, x_n) is not edge e, say $(x_n, y_i) \neq e$. By definition, edges (w, b), (w, y_i), and (b, x_n) belong to $E(X(G))$. Thus, $\langle b, x_n, y_j, P_1, y_i, w, b\rangle$ forms a cycle containing edge e of length $|V(G)| + 2$ in $X(G)$. (See Figure 8(a).)

Case 2: $e = (w, b)$.

Since G is edge-bipancyclic, we can find cycles of every even length from 4 to $|V(G)|$ in G. We may label this cycle of G as $\langle x_n, a, P_2, x_n\rangle$ where a is a neighbor of x_n. Thus, $\langle x_n, b, w, a, P_2, x_n\rangle$ is a cycle containing edge (w, b), so there are cycles containing edge (w, b) of every even length from 6 to $|V(G)| + 2$, and $\langle x_n, b, w, a, x_n\rangle$ is a cycle containing edge (w, b) of length 4. (See Figure 8(b).)

Case 3: $e = (w, y_n)$ or $e = (b, x_n)$.

The case holds for $e = (b, x_n)$ as shown in Case 2. For $e = (w, y_n)$, it can be proved similarly.

Case 4: $e \in \bigcup_{(x_n, y_i) \in E(G)} (w, y_i)$ or $e \in \bigcup_{(y_n, x_i) \in E(G)} (b, x_i)$.

Without loss of generality, we may consider only $e = (w, y_i)$ for some i such that $(x_n, y_i) \in E(G)$. Of course, $(x_n, y_i) \in E(G)$. Since G is edge-bipancyclic, there exist cycles containing edge (x_n, y_i) of every even length from 4 to $|V(G)|$ in G. Let $\langle x_n, y_i, P_3, x_n\rangle$ be one such cycle. Then, $\langle x_n, b, w, y_i, P_3, x_n\rangle$ is a cycle containing edge (w, y_i) with two more edges. Thus, there are cycles containing edge (w, y_i) of every even length from 6 to $|V(G)|+2$ in $X(G)$; and $\langle x_n, b, w, y_i, x_n \rangle$ is a cycle containing edge (w, y_i) of length 4. (See Figure 8(c).) Therefore, the proof of this theorem is complete. $\square$

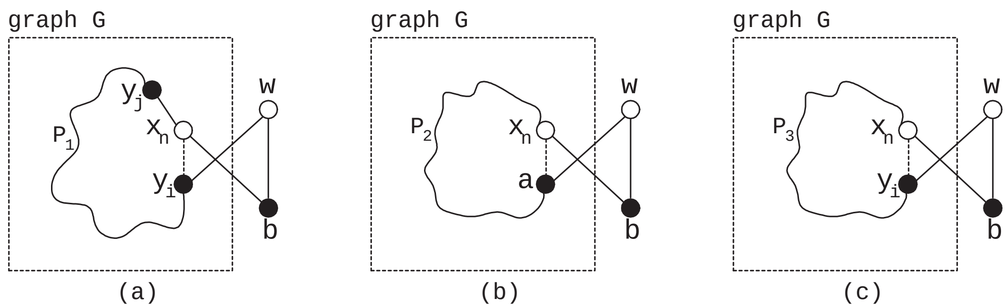

Figure 8: Edge-bipancyclicity of $X(G)$.

With a similar argument, we have the following two results.

Theorem 11 *Let G be an equitable bipartite graph with at least four nodes. If G is bipancyclic, then $J'(G)$ is also bipancyclic.*

Theorem 12 *Let G be an equitable bipartite graph with at least four nodes. If G is edge-bipancyclic, then $J'(G)$ is also edge-bipancyclic.*

4 Concluding Remarks

In this paper, we extend the result presented in [7] and we show that operation J performing on *hamiltonian laceable* graphs is not only *hamiltonian laceable*, but also *strongly hamiltonian laceable*. Furthermore, we show that by adding one more specific edge to $J(G)$, the resulting graph $J'(G)$ becomes both *strongly hamiltonian laceable* and *hyper-hamiltonian laceable*. We observe that $J(G)$ and $J'(G)$ are two graphs by adding as many as $O(|V(G)|)$ edges to G. Thus, we have another operation, X, adding considerably less number of edges to G to recursively construct *hamiltonian laceable* graphs.

It is noticed that Hamiltonian laceability is an important topic which deals with embedding a hamiltonian path in a given graph. On the other hand, *bipancyclicity* and *edge-bipancyclicity* for bipartite graphs are important issues in interconnection networks. We show that the three operations, J, J', and X, performing on *bipancyclic* and *edge-bipancyclic* graphs can yield *bipancyclicity* and *edge-bipancyclicity*, respectively.

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