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**Forgery Attack on Xia-You's Group Signature**

**Abstract**

Recently, Xia and You proposed an ID-based group signature scheme with strong separability. This article shows a universal forgery attack on the scheme.

*Index term:* Cryptography, group signature.

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# Comments: Forgery Attack on Xia-You's Group Signature

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Recently, Xia and You proposed an ID-based group signature scheme with strong separability. This article shows a universal forgery attack on the scheme.

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## 1. Introduction

Chaum and Heijst first proposed the concept and a scheme of group signature [1] by which a group member can represent his group to sign a signature without revealing his identity. However, the group manager can open the signer's identity, in case of a later dispute. Since then, several group signature schemes [2-5] have been proposed to either improve the performance or improve the functionalities. However, some schemes [4-5] were found insecure later.

A secure group signature scheme should satisfy the following properties:

*Unforgeability:* Only group members are able to sign messages on behalf of the group.

*Anonymity:* Given a valid group signature of some message, it is computationally hard for anyone except the group manager to identify the actual signer.

*Unlinkability:* Deciding whether two different valid group signatures were signed by the same member is computationally hard.

*Exculpability:* Neither a group member nor the group manager can sign on behalf of other group members.

*Traceability:* The group manager is always able to open a valid signature and identify the actual signer.

An ID-based cryptosystem [7] has the advantage of eliminating the cost of

maintaining the public key directory and verifying a user's public key, since a user's identity is his public key. The schemes [4-5] are ID-based group signature schemes. Recently, Xia and You [6] proposed a new ID-based group signature scheme with strong separability, where the role of a group manager is further divided into two parts: a membership manager and a revocation manager. A membership manager maintains the membership of a group, and a revocation manager can alone open the identity of a valid group signature, without the co-operation of the membership manager. This separation of the privilege has important applications in E-commerce [6]. This article shows a universal forgery attack on Xia-You's group signature scheme.

## 2. Review of Xia-You's group signature scheme

### Initialization:

Initially, the Trusted Authority (TA) randomly chooses two primes  $p_1$  and  $p_2$  of about 100 decimal digitals, and lets  $m = p_1 \cdot p_2$ , where  $p_1 \equiv \pm 1 \pmod{8}$  and  $p_2 \equiv \pm 3 \pmod{8}$  so that the Jacobi symbol  $(2/m)$  equals  $-1$  [8]. TA publishes  $m$  and a generator  $g$ , where  $g < \min(p_1, p_2)$ . A signer  $U_i$ 's  $ID_i$  is defined in Equation (1) to ensure each  $ID_i$  has a discrete logarithm modulo a composite number  $m$ , where  $D_i$  is the signer's public identity information [6]. TA now computes  $x_i$  as  $U_i$ 's secret key, such that  $g^{x_i} \equiv_m ID_i$ .

$$ID_i = \begin{cases} D_i & \text{if } (D_i/m)=1 \\ 2D_i & \text{if } (D_i/m)=-1, \text{ where } (D_i/m) \text{ is the Jacobi symbol.} \end{cases} \quad (1)$$

Now the membership manager computes  $n = p_3 \cdot p_4$ , where  $p_3$  and  $p_4$  are two large primes such that  $p_3 - 1$  and  $p_4 - 1$  are not smooth and  $n > m$ . He publishes  $(e, n)$  and  $h()$  as the group public parameters, where  $h()$  is a secure one-way hash function and  $e$  is the RSA's public key satisfying  $e \cdot d = 1 \pmod{\phi(n)}$ . When a signer  $U_i$  wants to

join the group, the membership manager computes  $z_i \equiv_n ID_i^d$  as  $U_i$ 's secret membership key.

Next, the revocation manager randomly chooses two integer  $x \in Z_m$  and  $h \in Z_m^*$ , computes  $y = h^x \bmod m$  satisfying  $y \in Z_m^*$ , and publishes  $h$  and  $y$  as the group's public parameters, in addition to  $(e, n, h)$ .

### Signing phase:

To sign a message  $M$ ,  $U_i$  randomly chooses five integers  $\mathbf{a}, \mathbf{b}, \mathbf{q}, \mathbf{w} \in Z_m$  and  $\mathbf{d} \in Z_n$ , computes  $A \equiv_n (y^{\mathbf{a}} \cdot z_i)$ ,  $B = y^{\mathbf{w}} \cdot ID_i$ ,  $\hat{B} \equiv_m B$ ,  $C \equiv_m h^{\mathbf{w}}$ ,  $v \equiv_n (A^e / B)$ ,  $t_1 \equiv_n y^{\mathbf{d}}$ ,  $t_2 \equiv_m (y^{\mathbf{b}} \cdot g^{\mathbf{q}})$ ,  $t_3 \equiv_m h^{\mathbf{b}}$ ,  $\mathbf{e} = \mathbf{a} \cdot e - \mathbf{w}$ ,  $E = \mathbf{d} - D \cdot \mathbf{e}$ ,  $F = \mathbf{b} - D \cdot \mathbf{w}$ ,  $G = \mathbf{q} - D \cdot x_i$ , and  $D = h(y \| g \| h \| A \| B \| \hat{B} \| C \| v \| t_1 \| t_2 \| t_3 \| M)$ . Finally, he sends  $(A, B, C, D, E, F, G)$  as a signature for message  $M$ .

### Verification phase:

A verifier can verify the signature  $(A, B, C, D, E, F, G, M)$  as follows. He computes  $\hat{B}' \equiv_m B$ ,  $v' \equiv_n (A^e / B)$ ,  $t_1' \equiv_n v'^D \cdot y^E$ ,  $t_2' \equiv_m \hat{B}'^D \cdot y^F \cdot g^G$ ,  $t_3' \equiv_m C^D \cdot h^F$ , and  $D' = h(y \| g \| h \| A \| B \| \hat{B}' \| C \| v' \| t_1' \| t_2' \| t_3' \| M)$ . The verifier will accept the signature if  $D'$  equals  $D$ .

## 3. Universal forgery attack

This section will show a universal forgery attack on Xia-You's scheme, where an attacker can easily forge a valid group signature for any message. Let  $M'$  be a message on which the attacker is about to forge a signature. The attacker randomly chooses five integers  $k_1$  and  $G \in Z_m$ , and  $k_2, k_3$ , and  $k_4 \in Z_n$ . He then computes  $C \equiv_m h^{k_1}$ ,  $A \equiv_n y^{k_2}$ ,  $B = y^{k_1}$ ,  $v \equiv_n \frac{A^e}{B}$ ,  $t_1 \equiv_n y^{k_3}$ ,  $t_2 \equiv_m y^{k_1} \cdot g^G$ ,  $t_3 \equiv_m h^{k_4}$ , and  $D = h(y \| g \| h \| A \| B \| \hat{B} \| C \| v \| t_1 \| t_2 \| t_3 \| M')$ . He finally sends  $(A, B, C, D, E, F, G)$  as a group signature on message  $M'$ .

A verifier will accept the signature. It can be easily checked that  $D'$  equals  $D$ , since the verifier will compute  $v' \equiv_n \frac{A^e}{B} \equiv_n y^{e \cdot k_2 - k_1} \equiv_n v$ ,  $\hat{B}' \equiv_m B$ ,  $t_1' \equiv_n v'^D \cdot y^E \equiv_n y^{k_3} \equiv_n t_1$ ,  $t_2' \equiv_m \hat{B}'^D \cdot y^F \cdot g^G \equiv_m y^{k_3} \cdot g^G \equiv_m t_2$ ,  $t_3' \equiv_m C^D \cdot h^F \equiv_m h^{k_3} \equiv_m t_3$ .

#### 4. Conclusions

In this article, we have shown that an attacker can easily forge a signature for any message on Xia-You's group signature scheme.

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