Workshop: Computer Networks

A Genetic Algorithm for Multiple Multicast on WDM Ring Network

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Abstract

The *Optimal multiple multicasts problem* (*OMMP*) on *wavelength division multiplexing* (*WDM*) ring networks without wavelength conversion is considered in this paper. When the physical network and the set of multicast requests are given, OMMP is the problem that selects a suitable path or (paths) and wavelength (or wavelengths) among the many possible choices for each multicast request such that not any paths using the same wavelength pass through the same link. This problem can be proven to be *NP-hard* problem. In the paper, a formulation of OMMP is given and a *genetic algorithm* (*GA*) is proposed to solve it. Experimental results indicate that GA is robust for this problem.

Keyword: genetic algorithm, multiple multicast, integer programming, WDM ring, single-hop.

1 Introduction

All-optical network based on *wavelength division multiplexing* (*WDM*) using the wavelength routing technique is considered as a very promising approach for the realization of future large bandwidth networks [12][13]. WDM technology is used to accommodate several wavelength channels on a fiber. This technology could enhance the line capacity of networks. Moreover, since the large bandwidth network requires not only transmission line capacity enhancement but also cross-connect node processing capability enhancement, WDM should be used in combination with wavelength routing [8][9].

In wavelength routing, data signals are carried on an unique wavelength from a source node to a destination node passing through nodes where the signals are optically routed and switched without regeneration in the electrical domain. When a physical network is given and connections among the nodes in the network are required, we must establish an optical path (*light-path*) with a dedicated wavelength for each required connection. The *routing and wavelength assignment* (*RWA*) problem is to select suitable paths and wavelengths among the many possible choices for the required connections. To avoid collision, no two paths using the same wavelength pass through the same link[8]. By practical limitations on the transmission technology, the number of available wavelengths on a fiber is restricted. So, a good solution to RWA problem is important to increase the efficiency of WDM networks. Ring networks are the predominant topology for current MAN/interoffice networks, and are expected to be the first topology to be used for WDM networks in real world. Wavelength conversion is the ability to convert the data on one wavelength to another wavelength. Moreover, optical wavelength converters are still laboratory curiosities[11]. In ring networks, RWA without wavelength conversion is known to be *NP-hard* [3]. Recently, several studies for RWA on ring networks have been performed [4][7][8][10][12][13], researchers have concentrated on the development of heuristic algorithms. A genetic algorithm for solving RWA problem on WDM ring has been proposed in [2].

In order to solve various application problems on a WDM network, mechanisms must be developed to handle not only point-to-point communication but also group communication involving transporting information in the network. A typical group communication is multicast that transports information from one source to a set of destination nodes. A more general version of group communication is multiple multicast that contains multiple groups of multicast, each with its own source node and destination set [11]. Multiple multicast covers all existing types of communications. In [6], H. Shen *et al* studied the multiple multicast problem in *multi-hop* WDM networks, they presented three heuristic efficient algorithms to construct an optimal/sub-optimal multicast tree for each multicast and minimize the network congestion on wavelengths. To my knowledge, there is no other work reported on the problem of multiple multicast in WDM networks.

In this paper, each WDM node is assumed to be *multicast incapable* and has some functions as follows[10]:

- *drop only*: When the locally attached router is a destination, and there is no need to forward a copy to any downstream node.
- *continue only*: When the locally attached router is not a destination and there is a down-stream destination.
- *drop and continue*: When the locally attached router is a destination and there is a down-stream destination.

Assume that each node has the function that it is possible to send multiple copies to the same output using different wavelengths and along clockwise and counter-clockwise paths. That is, each node has multiple transmitters, and hence can transmit to as many children as needed when constructing a multicast path. Similarly, a source can transmit to its children on different wavelengths using different transmitters.

In this paper, the optimal multiple multicast problem (OMMP) on WDM ring networks without wavelength conversion and with static traffic is considered, since finding an optimal solution to this problem is NP-hard [3], an exact search for optimal solutions is impractical due to exponential growth in execution time. Moreover, traditional heuristic methods and greedy approaches should trap in local optima. *Genetic algorithms* (*GA*) have been trusted as a class of general-purpose search strategies that strike a reasonable balance between exploration and exploitation. Genetic algorithm proposed by

John Holland [5] has been constructed as robust stochastic search algorithms for various optimization problems. GA searches by exploiting information sampled from different regions of the solution space. The combination of crossover and mutations helps GA escape from local optima. These properties of GA provide a good global search methodology for the OMMP problem. In this paper, first, an formulation of OMMP is proposed, then a genetic algorithm for finding the minimal number of required wavelengths on ring network without wavelength conversion is presented.

This paper is organized as follows. In Section 2, the formulation of the OMMP problem is described. The background of Genetic Algorithm is discussed in Section 3. In Sections 4, the proposed genetic algorithm for finding the optimal solution of the OMMP problem is described. Experimental results and conclusions are given in Sections 5 and 6.

2 Problem Formulation

Consider a ring network $G(V, E)$, which has n nodes, indexed from 1 to n in the clockwise direction. In general and most practical cases, each edge in G is bidirectional so that messages can be transmitted in either direction. Let V be the set of nodes, $V= \{1, 2, ..., n\}$; E be the set of undirected links, $E = \{(1, 2), (2, 3), ..., (n-1, n), (n, 1)\}$; and for each link, there are W wavelengths associated with it.

There are r groups of multicast $M_i = (s_i,\ D_i)$, where $D_i = \{d_i^1, d_i^2, ..., d_i^{k_i}\}$, s_i is the source and $d_i^1, d_i^2, ..., d_i^{k_i}$ are the destinations of multicast request M_i, for integers i and k_i, $1 \leq i \leq r$, $1 \leq k_i \leq n$. Assume $d_i^1 < d_i^2 < ... < d_i^{k_i}$ and $s_i = d_i^l$, for some l, $1 \leq l \leq k_i$. Let M_i alone (without considering the existence of other groups) can be realized by a multicast tree MT_i, and all groups of multicast can be realized by multicast forest $MF = \cup_{i=1}^{r} MT_i$. Clearly, in the general case, many edges of MF will fail into the same edge of G and use the same wavelength at that edge, thus causing congestion when broadcasting these r sources simultaneously and violating the wavelength continuous constraint of WDM network.

At each node, assume that no wavelength conversion will occur and no light splitting will available. But a separate transmitter is required for transmitting optical signals for each outgoing link. The *optimal multiple multicast problem (OMMP)* is then defined as follows. Given graph $G(V, E)$ and a set of multicast requests $\{M_i = (s_i, D_i) | i = 1, 2, ..., r\}$, find a multicast forest MF of requests and the wavelength assignment of each multicast tree MT_i (i=1, 2, ..., r) such that the number of used wavelengths is minimized. The optimal multiple multicast problem (OMMP) can be proved to be *NP-hard* because a special case of it is equivalent to the RWA problem.

Let MT_i be the multicast tree for the multicast request $M_i = (s_i, D_i)$ with source s_i and destinations set $D_i = \{d_i^l | i = 1, 2, ..., k_i\}$. Since $d_i^1 < d_i^2 < ... < d_i^{k_i}$ and $s_i = d_i^l$, for some l, $1 \leq l \leq k_i$. Let $P^c(s_i, d_i^q)$ be a path from source node s_i to destination d_i^q on ring G in clockwise direction, $q \in \{1, 2, ..., k_i\}$, i=1, 2, ..., r. Let $P^r(s_i, d_i^q)$ be a path from source node s_i to destination d_i^q on ring G in counter-clockwise

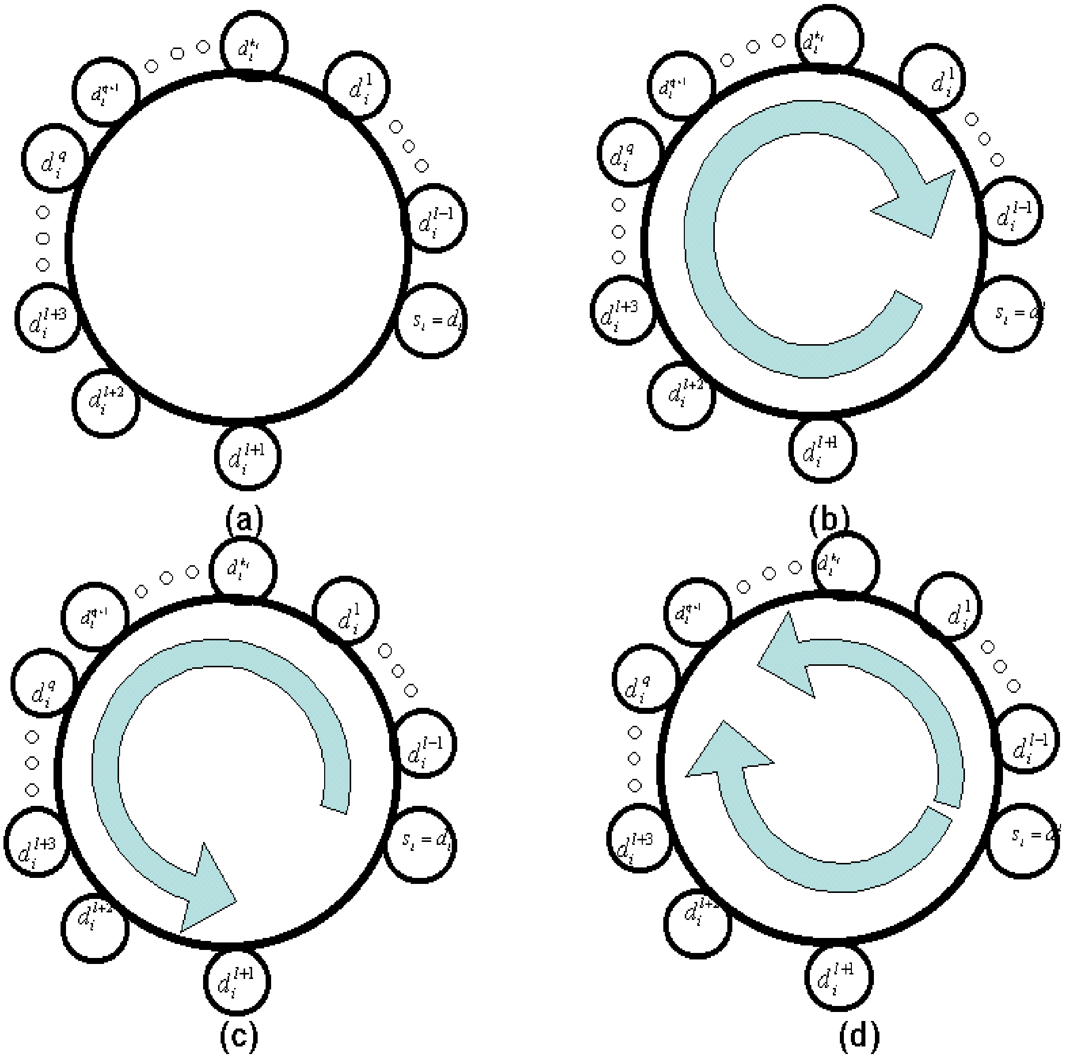


Figure 1: (a) Multicast M_i= (s_i, D_i); (b) $P^c(s_i, d_i^{l-1})$; (c)$P^r(s_i, d_i^{l+1})$; (d) $P^c(s_i, d_i^q)$ and $P^r(s_i, d_i^{q+1})$.

direction, $k \in \{1, 2, ..., k_i\}$, i=1, 2, ..., r. That is,

$$P^c(s_i, d_i^q) = \{ \begin{array}{ll} s_i = d_i^l \rightarrow d_i^{l+1} \rightarrow d_i^{l+2} \rightarrow ... \rightarrow d_i^q & if q > l \\ s_i = d_i^l \rightarrow d_i^{l+1} \rightarrow d_i^{l+2} \rightarrow ... \rightarrow d_i^{k_i} \rightarrow d_i^1 \rightarrow d_i^2 \rightarrow ... \rightarrow d_i^q & otherwise \end{array}$$

and

$$P^r(s_i, d_i^q) = \{ \begin{array}{ll} s_i = d_i^l \rightarrow d_i^{l-1} \rightarrow d_i^{l-2} \rightarrow ... \rightarrow d_i^k & if q < l \\ s_i = d_i^l \rightarrow d_i^{l-1} \rightarrow d_i^{l-2} \rightarrow ... \rightarrow d_i^1 \rightarrow d_i^{k_i} \rightarrow d_i^{k_i-1} \rightarrow ... \rightarrow d_i^q & otherwise \end{array}.$$

Thus, each MT_i can be established by following methods:

(1) a clockwise-direction path: $P^c(s_i, d_i^{l-1})$ (as in Fig. 1-(b))

(2) a counter-clockwise-direction path: $P^r(s_i, d_i^{l+1})$ (as in Fig. 1-(c)), or

(3) two paths: clockwise-direction path $P^c(s_i, d_i^q)$ and clockwise-direction path $P^r(s_i, d_i^{q+1})$ for some $q \in D_i$ (as in Fig. 1-(d)).

Let y_w=1, if wavelength $w \in \{1, 2, ..., W\}$ is used; y_w=0, otherwise. From the discussion above, it is easily to find that each multicast M_i = (s_i, D_i) can be routed by using at most two paths $P^c(s_i, d_i^q)$ and $P^r(s_i, d_i^{q+1})$ for some $q \in D_i$. Let M be the set of selected pairs of nodes, M=$\{(s_1, d_1^{q_1})$,

$(s_1, d_1^{q_1+1}), (s_2, d_2^{q_2}), (s_2, d_2^{q_2+1}), ..., (s_r, d_r^{q_r}), (s_r, d_r^{q_1+1})\}$. Thus, the goal is to assign wavelengths to paths in M so that the number of used wavelengths can be minimized.

Let P_k^c be the path for connection $c_z = (s_z, d_z) \in M$ having clockwise-direction and P_k^r be the path for connection $c_z = (s_z, d_z) \in M$ having counter-clockwise direction. The set of possible paths for all c_z in M is denoted by $P = \cup_{c_z \in M}\{P_k^r, P_k^c\}$. Then, the set of connections whose clockwise and counter-clockwise direction paths pass edge $e \in E$ are denoted by M_e^c and M_e^r[8].

In WDM network, given the multiple multicast request, the problem is to select paths on G for all required multicast request and determine the wavelength assignments of paths such that no two paths using the same wavelength pass through the same link. The objective is to minimize the number of required wavelengths.

With the above notation, the OMMP problem can be formulated as the follows:

$$\textit{Objective}: \ \mathtt{min} \sum_{\mathtt{w} \in \mathtt{W}} \mathtt{y_w} \tag{1}$$

s.t.

$$\begin{aligned} \sum_{w \in W} x_{wc}^k &\leq 1, \ \mathtt{for\ all} \ e \in P_k^c & (2) \\ \sum_{w \in W} x_{wr}^k &\leq 1, \ \mathtt{for\ all} \ e \in P_k^r & (3) \end{aligned}$$

$$\sum_{w \in M_e^c} x_{wc}^k + \sum_{w \in M_e^r} x_{wr}^k \leq y_w, \ \mathtt{for\ all\ w \in W, e \in E} \tag{4}$$

$$\begin{aligned} x_{wc}^k, x_{wr}^k &\in \{0,1\}, \ \mathtt{for\ all\ c_k \in M} & (5) \\ y_w &\in \{0,1\}, \ \mathtt{for\ all\ w \in W} & (6) \end{aligned}$$

The binary decision variable $x_{wc}^k = 1$, if wavelength w is selected to establish clockwise-direction path P_k^c; $x_{wc}^k = 0$, otherwise. Similar, the binary decision variable $x_{wr}^k = 1$, if wavelength w is selected to establish counterclockwise-direction path P_k^r; $x_{wr}^k = 0$, otherwise. The binary decision variable $y_w = 1$, if wavelength is used; $y_w = 0$, otherwise. The objective is to minimize the number of wavelengths used. Constraints (2) and (3) mean that every required multicasts must be established. Constraints (4) ensure that the paths using wavelength can be selected only when wavelength is used and at most one path using wavelength passes edge. The number of available wavelengths is assumed to be large enough to establish all the required connections.

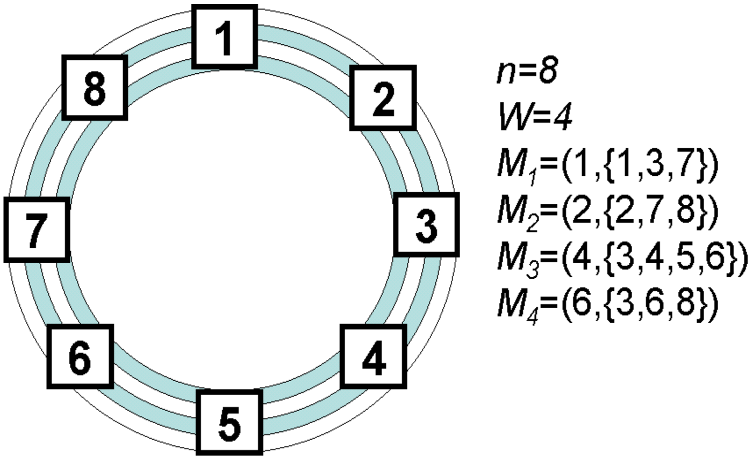


Figure 2: Example of multiple multicast.

Example 1. Consider the graph shown in Fig. 2. There are eight nodes and links in ring network G; each fiber between two nodes can provides 4 wavelengths, i.e., W=4. r=4 groups of multicast are requested to be established, they are M_1=(1,{1,3,7}), M_2=(2,{2,7,8}), M_3=(4,{3,4,5,6}), M_4=(6,{3,6,8}). A possible assignment of Fig. 2 is shown in Fig. 3. Observation form Fig. 3 shown that four wavelengths are used to assign these multicast requests.

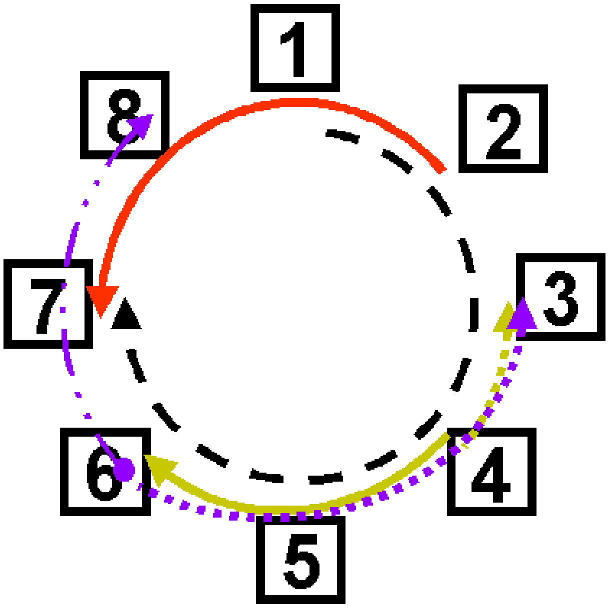


Figure 3: Possible assignment of the Example 1.

3 Concept of GA

The search space in GA is composed of possible solutions to the problem. A solution in the search space is represented by a sequence of 0s and 1s. This solution string is referred as a chromosome in the search space. Each chromosome has an associated objective function called the *fitness*. A good chromosome is the one that has a high/low fitness value, depending upon the nature of the problem (maximization/minimization). The strength of a chromosome is represented by its *fitness value*. Fitness values indicate which chromosomes are to be carried to the next generation. A set of chromosomes and associated fitness values is called the *population*. This population at a given stage of GA is referred to as a *generation*. The general GA proceeds as follows:

```
Genetic Algorithm()
    Begin
     Initialize population;
     while (not terminal condition) do
       Begin
         choose parents from population; /* Selection */
         construct offspring by combining parents; /* Crossover */
         optimize (offspring); /* Mutation */
         if suited (offspring) then
         replace worst fit (population) with better offspring;
         /*Survival of the fittest */
      End;
    End.
```

There are three main processes in the while loop for GA:

(1) The process of selecting good strings from the current generation to be carried to the next generation. This process is called *selection/reproduction*.

(2) The process of shuffling two randomly selected strings to generate new offspring is called *crossover*. Sometimes, one or more bits of a chromosome are complemented to generate a new offspring. This process of complementation is called *mutation*.

(3) The process of replacing the worst performing chromosomes based on the fitness value.

The population size is finite in each generation of GA, which implies that only relatively fit chromosomes in generation (i) will be carried to the next generation $(i+1)$. The power of GA comes from the fact that the algorithm terminates rapidly to an optimal or near optimal solution. The iterative process terminates when the solution reaches the optimum value. The three genetic operators, namely, selection, crossover and mutation, are discussed in the next section.

3.1 Selection / Reproduction

Since the population size in each generation is limited, only a finite number of good chromosomes will be copied in the *mating pool* depending on the fitness value. Chromosomes with higher fitness values contribute more copies to the mating pool than do those with lower fitness values. This can be achieved by assigning proportionately a higher probability of copying a chromosome that has a higher fitness value[1]. Selection/reproduction uses the fitness values of the chromosome obtained after evaluating the objective function. It uses a biased roulette wheel[1] to select chromosomes, which are to be taken in the mating pool. It ensures that highly fit chromosomes (with high fitness value) will have a higher number of offspring in the mating pool. Each chromosome (i) in the current generation is allotted a roulette wheel slot sized in proportion (p_i) to its fitness value. This proportion p_i can be defined as follows. Let Of_i be the actual fitness value of a chromosome (i) in generation (j) of g chromosomes, $Sum_j = \sum_{i=1}^{g} Of_i$ be the sum of the fitness values of all the chromosomes in generation j, and let $p_i = Of_i / Sum_j$.

When the roulette wheel is spun, there is a greater chance that a better chromosome will be copied into the mating pool because a good chromosome occupies a larger area on the roulette wheel.

3.2 Crossover

This phase involves two steps: first, from the mating pool, two chromosomes are selected at random for mating, and second, crossover site c is selected uniformly at random in the interval $[1, n]$. Two new chromosomes, called *offspring*, are then obtained by swapping all the characters between positions $c + 1$ and n. This can be shown using two chromosomes, say P and Q. each of length $n = 6$ bit positions

chromosome P: 111|000;

chromosome Q: 000|111.

Let the crossover site be 3. Two substrings between 4 and 6 are swapped, and two substrings between 1 and 3 remain unchanged; then, the two offspring can be obtained as follows:

chromosome R: 111|111;

chromosome S: 000|000.

3.3 Mutation

Combining the reproduction and crossover operations may sometimes result in losing potentially useful information in the chromosome. To overcome this problem, mutation is introduced. It is implemented by complementing a bit (0 to 1 and vice versa) at random. This ensures that good chromosomes will not be permanently lost.

4 Genetic Algorithm (GA) for WDM multiple multicast

In this section, the details of GA developed to solve the optimal multiple multicast problem (OMMP) on WDM ring network are given. The development of GA requires: (1) a chromosomal coding scheme, (2) a fitness function definition, (3) genetic crossover operators, (4) mutation operators, (5) a replacement strategy, and (6) termination rules.

4.1 Encoding

Since the problem involves representing paths of multiple multicasts and wavelengths, a coding scheme that uses integral numbers is employed. A chromosome (one-dimension arrays with size $4\times r$) is introduced to represent the routing paths and assignments of paths which denoted *routing chromosome* R. Another chromosome is used to represent the usage of the wavelength, denoted as *wavelength chromosome* Y. The structure of routing chromosme R is divided into two parts, the first part denoted as MG (multicast gene), $MG = \{mg_1^1, mg_1^2, mg_2^1, mg_2^2, ..., mg_r^1, mg_r^2\}$, where the element $mg_i^k \in D_i$, for $i = 1, 2, ..., r; k = 1, 2$. If $mg_i^1 = s_i$ $(1 \leq i \leq r)$, the i^{th} multicast request is routed through the counter-clockwise-direction path $P^r(s_i, d_i^{j+1})$; if $mg_i^2 = s_i$ $(1 \leq i \leq r)$, the i^{th} multicast request is routed through the clock-direction path $P^c(s_i, d_i^{j-1})$; otherwise, the i^{th} multicast request is routed through the clockwise-direction path $P^c(s_i, d_i^q)$ and clockwise-direction path $P^r(s_i, d_i^{q+1})$ for some $d_i^q \in D_i$. The second part of the routing chromosome R denoted as AG (assignment gene); $AG = \{ag_1^1, ag_1^2, ag_2^1, ag_2^1, ..., ag_r^1, ag_r^2\}$, where both elements ag_q^1 and ag_q^2, q=1, 2, ..., r; are fell in $\{1, 2, ..., W\}$ and used to represent the wavelength assignments of paths $P^c(s_i, d_i^q)$ and $P^r(s_i, d_i^{q+1})$, respectively. The structure of the routing chromosome R is shown in Fig. 4-(a) and the routing chromosome of the possible assignment of the Example 1 in Fig. 2 is shown in Fig. 4-(b).

A one-dimension array Y=$\{y_w | w = 1, 2, ..., W\}$ is used to represent the wavelength chromosome, where $y_w = 1$, if the w^{th} wavelength is used; $y_w = 0$, otherwise. For example, the chromosome of the possible routing paths and wavelength assignment of the Example 1 in Fig. 2 is shown in Fig. 5. It is worth noting that if there exist one element ag_i^k in AG such that $ag_i^k = w$ and $ag_i^k \neq s_i$; then $y_w = 1$, for i=1, 2, ..., r; k=1, 2; w = 1, 2, ..., W. For example, the relationships of routing and wavelength chromosomes of the possible assignment of the Example 1 in Fig. 2 is shown in Fig. 5.

4.2 Fitness function definition

Generally, genetic algorithms use fitness functions to map objectives to costs and achieve the goal of finding optimally results. According to the content of the routing chromosome, the value of wavelength chromosome can be determined. Since the goal is to minimize the number of required wavelengths, the fitness function is $\sum_{w=1}^{W} y_w$. A fitness function value is associated with each wavelength chromosome,

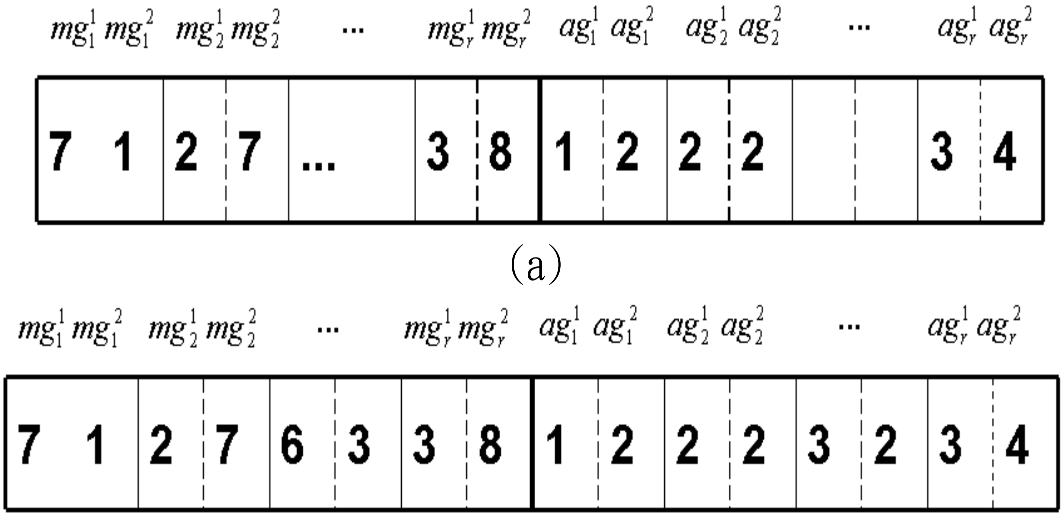
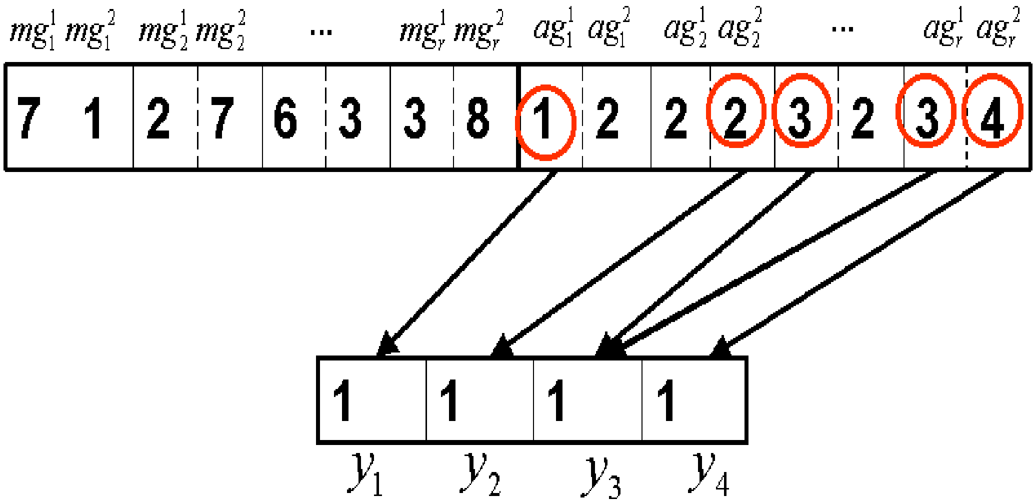


Figure 4: (a) Structure of routing chromosome R. (b) Routing chromosome of the Example 1.



$$y_j = \begin{cases} 1 & \textit{if}\ (\text{there is one element } \mathrm{ag}_\mathrm{i}^\mathrm{k}, \text{ such } \mathrm{ag}_\mathrm{i}^\mathrm{k} = j) \text{ and } (mg_i^k \neq s_i); \\ & \text{for } \mathrm{i} = 1,2,\ldots,\mathrm{r}; \mathrm{k} = 1,2) \\ 0 & \qquad\qquad\qquad \textit{otherwise} \end{cases}$$

Figure 5: Routing chromosome of the Example 1.

which is the same as the measure mentioned above. The objective function is

$$minimize \sum_{w=1}^{W} y_w. \tag{7}$$

Note that if constraints (2), (3) and (4) in Section 2 are considered with this objective function, then a complete problem formulation is found. While breeding chromosomes, the genetic algorithm does not require the chromosome to reflect a feasible solution. Thus, a penalty is needed to attach to the fitness function in the event that the solution is not constraint-satisfied. For a pair of routing paths p_i^k=$P^c(s_i, d_i^{l_i})$ (if k=1 or $p_i^k = P^r(s_i, d_i^{l_i})$ if k=2) and $p_j^{k'}$=$P(s_{i'}, d_{i'}^{l_{i'}})$ (if (k=1 or $p_j^{k'} = P^r(s_j, d_j^{l'_i})$ if k = 2) in routing chromosome R, let $conflict(p_i^k, p_j^{k'}) = 1$, if p_i^k and $p_j^{k'}$ are assigned to the same wavelength and the routing paths of these paths are overlapped. Then, the formulation above can be rewrote in an unconstrained form:

$$minimize \quad cost = \sum_{w=1}^{W} y_w + \alpha \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{2} \sum_{k'=1}^{2} conflict(p_i^k, p_j^{k'}), \tag{8}$$

where α is the penalty weight. It is worth noting that the value of α should be greater than W. Since the best-fit chromosomes should have a probability of being selected as parents that is proportional to their fitness, they need to be expressed in a maximization form. This is done by subtracting the objective from a large number C_{max}. Hence, the fitness function becomes:

$$maximum \quad C_{max} - \left(\sum_{w=1}^{W} y_w + \alpha \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{2} \sum_{k'=1}^{2} conflict(p_i^k, p_j^{k'}) \right), \tag{9}$$

where C_{max} denotes the maximum value observed, so far, of the cost function in the population. Let C_{cost} be the value of the cost function for the chromosome; C_{max} can be calculated by the following iterative equation:

$$C_{max} = max\{C_{max}, C_{cost}\}, \tag{10}$$

where C_{max} is initialized to zero.

For the computation of penalty cost, consider a routing chromosome R, if two routing paths are assigned to the same wavelength and the routing paths are overlapped then conflict occurred. For each pair of routing paths, if conflict occurred then a penalty should be attached into the cost function. Then give that

$$conflict(p_i^k, p_j^{k'}) = \{ \begin{matrix} 1 & if\ (ag_i^k = ag_j^{k'})\ and\ (\text{paths } p_i^k \text{ and } p_j^{k'}\, are \text{ overlapped}) \\ 0 & otherwise. \end{matrix}$$

Clearly, whether two given paths are overlapped or not can be determined in constant time on ring network. Thus, the time spent for computing penalty cost is $O(r^2)$.

4.3 Genetic crossover operator

Five types of crossover operators were used to develop this algorithm: (1) *single point crossover* (*SPC*), (2) *Single point wavelength crossover* (SPWC), (3) *Single point routing path crossover* (SPRPC), (4) *Single assigning wavelength exchanging operator* (SAWEO), and (5) *Wavelength exchanging operator* (WEO). These operators only operated on the routing chromosome.

- *Single point crossover* (SPC): Randomly select a crossover point from 1 to r. Two routing chromosomes are crossovered as following example.

  Parent P_1 is

  7 1 2 7 6 3 3 8 ‖ 1 2 2 2 3 2 3 4 and

  parent P_2 is

  1 7 7 2 3 4 6 3 ‖ 2 1 2 2 1 2 1 4 .

  Assume crossover point is 2, after operating, children C_1 and C_2 are as follows:

  C_1: 7 1 2 7 **3 4 6 3** ‖ 1 2 2 2 **1 2 1 4**

  C_2: **1 7 7 2** 6 3 3 8 ‖ **2 1 2 2** 3 2 3 4 .

- *Single point wavelength crossover* (SPWC): Randomly select a crossover point from 1 to r. Two routing chromosomes are crossovering as following example, it it worth notice that only the AG part of the routing chromosome is applied.

  Parent P_1 is

  7 1 2 7 6 3 3 8 ‖ 1 2 2 2 3 2 3 4 and

  parent P_2 is

  1 7 7 2 3 4 6 3 ‖ 2 1 2 2 1 2 1 4 .

  Assume crossover point is 2, after operating, children C_1 and C_2 are as follows:

  C_1: 7 1 2 7 6 3 3 8 ‖ 1 2 2 2 **1 2 1 4,** and

  C_2: **1 7 7 2 3 4 6 3 ‖ 2 1 2 2** 3 2 3 4 .

- *Single point routing path crossover* (SPRPC): Randomly select a crossover point from 1 to r. Two routing chromosomes are crossovering as following example, it it worth notice that only the MG part of the routing chromosome is applied.

  Parent P_1 is

  7 1 2 7 6 3 3 8 ‖ 1 2 2 2 3 2 3 4 and

parent P_2 is

$$\begin{array}{|cccccccc||cccccccc|}\hline \mathbf{1} & \mathbf{7} & \mathbf{7} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{6} & \mathbf{3} & \mathbf{2} & \mathbf{1} & \mathbf{2} & \mathbf{2} & \mathbf{1} & \mathbf{2} & \mathbf{1} & \mathbf{4} \\ \hline \end{array}.$$

Assume crossover point is 2, after operating, children C_1 and C_2 are as follows:

$$C_1: \begin{array}{|cccccccc||cccccccc|}\hline 7 & 1 & 2 & 7 & \mathbf{6} & \mathbf{3} & \mathbf{3} & \mathbf{8} & 1 & 2 & 2 & 2 & 1 & 2 & 1 & 4, \\ \hline \end{array} \text{ and}$$

$$C_2: \begin{array}{|cccccccc||cccccccc|}\hline \mathbf{1} & \mathbf{7} & \mathbf{7} & \mathbf{2} & 3 & 4 & 6 & 3 & \mathbf{2} & \mathbf{1} & \mathbf{2} & \mathbf{2} & \mathbf{3} & \mathbf{2} & \mathbf{3} & \mathbf{4} \\ \hline \end{array}.$$

- *Single assigning wavelength exchanging operator* (SAWEO): First, randomly select one chromosome for operating from previous generation. Then randomly select i from 1 to r, the values of ag_i^1 and ag_i^2 in these routing chromosome are exchanged. Assume

  parent P_1 is

  $$\begin{array}{|cccccccc||cccccccc|}\hline 7 & 1 & 2 & 7 & 6 & 3 & 3 & 8 & 1 & 2 & 2 & 2 & 3 & 2 & 3 & 4 \\ \hline \end{array},$$

  i=4, and routing paths ag_4^k are selected.

  After operating, parent P_1 is changed to

  $$\begin{array}{|cccccccc||cccccccc|}\hline 7 & 1 & 2 & 7 & 6 & 3 & 3 & 8 & 1 & 2 & 2 & 2 & 3 & 2 & \mathbf{4} & \mathbf{3} \\ \hline \end{array}.$$

- *Wavelength exchanging operator* (WEO): Randomly select one chromosome for operation from previous generation. Then, randomly select two wavelength assignment ag_i^k and $ag_j^{k'}$, $1 \leq i, j \leq r$; k, k'=1,2 and the assigned wavelengths of these two paths are exchanged. Assume

  parent P_1 is

  $$\begin{array}{|cccccccc||cccccccc|}\hline 7 & 1 & 2 & 7 & 6 & 3 & 3 & 8 & 1 & 2 & 2 & 2 & 3 & 2 & 3 & 4 \\ \hline \end{array},$$

  and assignments ag_2^1 and ag_4^2 are selected.

  After operating,

  parent P_1 is changed to

  $$\begin{array}{|cccccccc||cccccccc|}\hline 7 & 1 & 2 & 7 & 6 & 3 & 3 & 8 & 1 & 2 & \mathbf{4} & 2 & 3 & 2 & 3 & \mathbf{2} \\ \hline \end{array}.$$

4.4 Mutations

Five types of mutations are used to develop the genetic algorithm: (1) *Single routing path mutation* (SRPM), (2) *Multiple routing paths mutation* (MRPM), (3) *Single wavelength assignment mutation* (SWAM), (4) *Multiple wavelength assignment mutation* (MWAM), and (5) *Multicast assignment mutation* (MAM).

- *Single routing path mutation* (SRPM): Randomly select a multicast M_i in routing chromosome, the single routing path mutation (SRPM) changes the destination of the clockwise routing path

$P^c(s_i, mg_i^1)$ to a random integral number d_i^q, which is in $D_i = \{\ d_i^1,\ d_i^2,\ ...,\ d_i^{k_i}$. Then the value of the gene mg_i^2 is changed to d_i^{q+1}.

Assume parent P_1 is

$$\begin{array}{|cccccccc||cccccccc|}\hline 7 & 1 & 2 & 7 & 6 & 3 & 3 & 8 & 1 & 2 & 2 & 2 & 3 & 2 & 3 & 4 \\ \hline\end{array}\ ,$$

routing path mg_3^1 is selected, $D_3 = \{3, 4, 5, 6\}$, and assume new destination $d_3^q = 5$.

After operating,

parent P_1 is changed to

$$\begin{array}{|cccccccc||cccccccc|}\hline 7 & 1 & 2 & 7 & \mathbf{5} & \mathbf{6} & 3 & 8 & 1 & 2 & 4 & 2 & 3 & 2 & 3 & 2 \\ \hline\end{array}\ .$$

- *Multiple routing paths mutation* (MRPM): Randomly select z $(1 \leq z \leq |M|)$ multicasts in routing chromosome R, the MRPM changes each destination of the clockwise routing path $P^c(s_i, mg_i^1)$ to a random integral number d_i^q, which is in D_i. Then the value of the gene mg_i^2 is changed to d_i^{q+1}.

- *Single wavelength assignment mutation* (SWAM): Randomly select a path, the single wavelength assignment mutation (SWAM) changes the assignment wavelength of the path to a random integral number w, which is in $\{1,\ 2,\ ...,\ W\}$.

  Assume

  parent P_1 is

$$\begin{array}{|cccccccc||cccccccc|}\hline 7 & 1 & 2 & 7 & 6 & 3 & 3 & 8 & 1 & 2 & 2 & 2 & 3 & 2 & 3 & 4 \\ \hline\end{array}\ ,$$

  ag_2^1 is selected and $w = 4$.

  After mutation,

  parent P_1 is changed to

$$\begin{array}{|cccccccc||cccccccc|}\hline 7 & 1 & 2 & 7 & 6 & 3 & 3 & 8 & 1 & 2 & \mathbf{4} & 2 & 3 & 2 & 3 & 4 \\ \hline\end{array}\ .$$

- *Multiple wavelength assignment mutation* (MWAM): Randomly select z $(1 \leq z \leq |M|)$ paths, the MWAM changes the assignment wavelengths of these paths to random integral integers, which are in $\{1,\ 2,\ ...,W\}$.

  Assume

  parent P_1 is

$$\begin{array}{|cccccccc||cccccccc|}\hline 7 & 1 & 2 & 7 & 6 & 3 & 3 & 8 & 1 & 2 & 2 & 2 & 3 & 2 & 3 & 4 \\ \hline\end{array}\ ,$$

  z=4, ag_2^1, ag_3^1, ag_3^2, ag_4^2 are selected and the corresponding wavelengths are $\{4,\ 2,\ 3,\ 2\}$.

  After mutation,

parent P_1 is changed to

$$\begin{array}{|cccccccc||cccccccc|}\hline 7 & 1 & 2 & 7 & 6 & 3 & 3 & 8 & 1 & 2 & \mathbf{4} & 2 & \mathbf{2} & \mathbf{3} & 3 & \mathbf{2} \\ \hline \end{array}.$$

- *Multicast assignment mutation* (MAM): Randomly select a multicast, the MAM changes the assignment wavelengths of the clockwise path and the counter-clockwise path to the same random integral number w, which is in $\{1, 2, ..., W\}$.

  Assume

  parent P_1 is

$$\begin{array}{|cccccccc||cccccccc|}\hline 7 & 1 & 2 & 7 & 6 & 3 & 3 & 8 & 1 & 2 & 2 & 2 & 3 & 2 & 3 & 4 \\ \hline \end{array},$$

  ag_3^1 and ag_3^2 are selected, and w=4.

  After mutation,

  parent P_1 is changed to

$$\begin{array}{|cccccccc||cccccccc|}\hline 7 & 1 & 2 & 7 & 6 & 3 & 3 & 8 & 1 & 2 & 2 & 2 & \mathbf{4} & \mathbf{4} & 3 & 4 \\ \hline \end{array}.$$

4.5 Replacement strategy

This subsection discusses a method used for creating new generation after crossover and mutation is carried out on the chromosomes of previous generation. There are several replacement strategies proposed in the literature, a good discussion can be found in [1] . The most common strategies are to probabilistically replace the poorest performing chromosomes the previous generation. The *elitist strategy* appends the best performing chromosome of a previous generation to the current population and thereby ensures that the chromosome with the best objective function value always survives to the next generation.

Initially, assume $N_{population}$ be the number chromosomes to be generated and N_{parent} connection chromosomes are randomly constructed. In the process of selection, $N_{population}/2$ pairs of connections are randomly selected for crossover to generated the new generation of chromosomes. After crossover, chromosomes are sorted according to the fitness function in increasing order.

The algorithm developed here combines both the concepts maintained above. Each offspring generated after crossover is added to the new generation if it has a better objective function value than both of its parents. If the objective function value of an offspring is better than only one of the parents, then we select a chromosome randomly from the better parent and the offspring. If the offspring is worse than both parents then any one of the parents is selected at random for the next generation. This ensures that the best chromosome is carried to the next generation, while the worst is not carried to the succeeding generations.

4.6 Termination rules

Execution of GA can be terminated when the number of generations exceeds an upper bound ($N_{generation}$) specified by the user.

5 Experimental Results

In order to evaluate the performance of the proposed genetic algorithm, the algorithm has been implemented and applied it to solve problems that were randomly generated. The results of these experiments are reported below. In all the experiments, the implementation was conducted in C, and all the experiments were run on a personal computer (PC) with a Pentium III 1GHZ CPU and 512MB RAM.

The algorithm has been test on ring network with 20 nodes. For the test, a set of multicast requests is generated in all nodes in the networks. Assume W=30, r=20, and the maximal number of destinations of each multicast request is set to be constant MAX_D=5.

In the following, the effects of the various parameters of the genetic algorithm are discussed. The parameters of GA are: crossover probability P_c=1.0, mutation probability P_m=0.3, population size $N_{population}$ =1000, generation size $N_{generation}$=2000, and α= W.

(1) Effect of crossover probabilities: In Fig. 6, the effect of the crossover probability on the evolution of the required wavelengths in GA is presented. In this experiment, assume P_c is in {0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95, 1.00 } It can be found that the algorithm leads to both fast convergence and to the global optimal when the crossover probability is large (0.75, 0.90, 1.00).

(2) Effect of mutation probabilities: In Fig. 7, the effect of the mutation probability on the evolution of the required wavelengths in GA is presented. In this experiment, assume P_c is in {0.00, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 1.00 } It can be found that the algorithm leads to both fast convergence and to the global optimal when the mutatio probability is large (0.70, 0.80, 0.90).

(3) Effect of population size: In Fig. 8, the effect of the different number of chromosome in populations ($N_{population}$), where $N_{population}$ is in {200, 400, 600, 800, 1000, 1200, 1400, 1600, 1800, 2000}. It can be easily to find that $N_{population}$ =1600 or 2000 get the best results; $N_{population}$= 400 is the worst.

6 Conclusions

In this paper, the optimal multiple multicast problem (OMMP) on WDM ring networks without wavelength conversion and with static traffic is investigated and formulated. Since finding the optimal solution of this problem can be found to be NP-hard, a genetic algorithm is proposed to solve this problem. Simulation results show that the proposed genetic algorithm is robust for this problem.

In the proposed method, routing and wavelength chromosomes are used to represent the routing

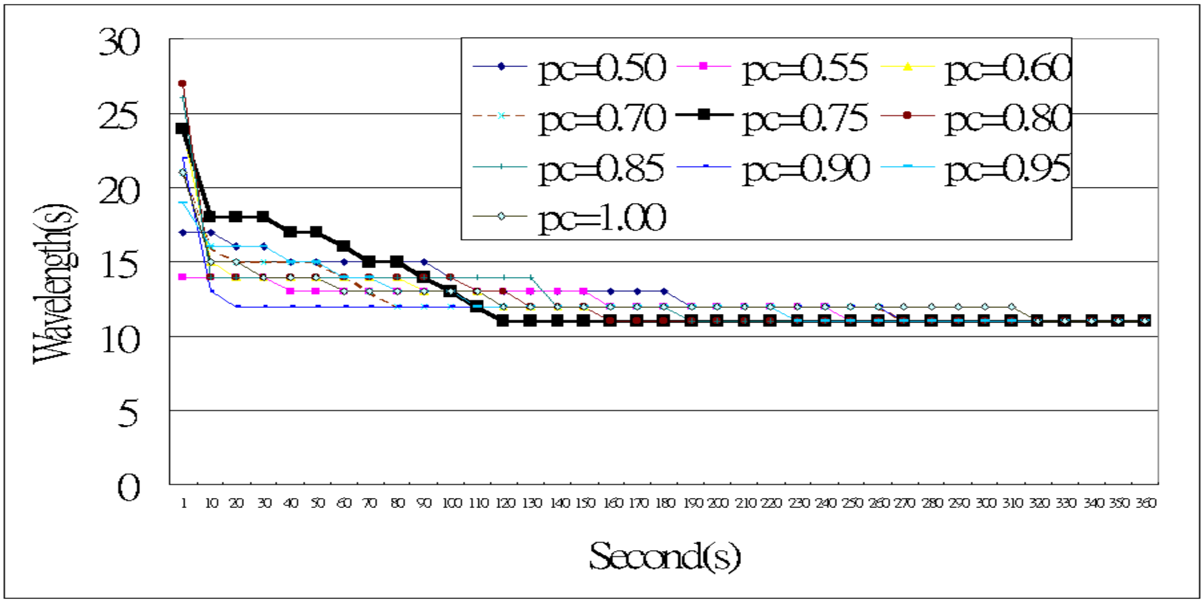


Figure 6: Experiment of different values of crossover probabilities.

paths of multicasts and the usage of wavelengths, respectively. In the GA method, three general genetic operators - selection, crossover, and mutation - are employed. Five types of operators (SPC, SPWC, SPRPC, SAWEO, and WEO) and five types of mutations (SRPM, MRPM, SWAM, MWAM and MAM) are employed in the proposed genetic algorithm. Experimental results indicate that GA runs efficiently.

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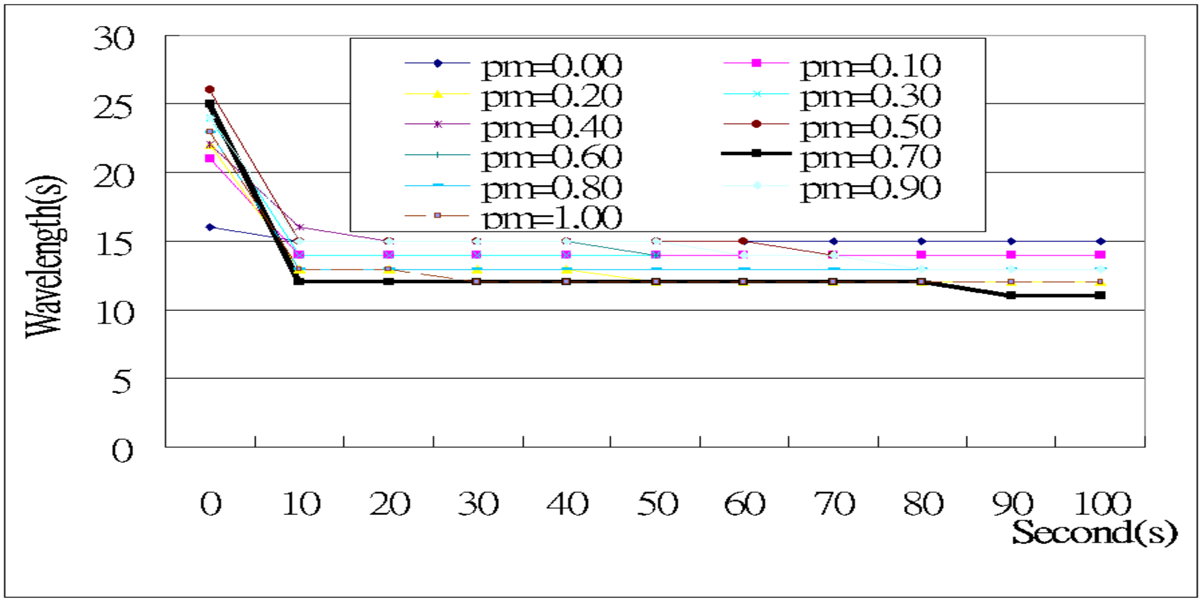

Figure 7: Experiment of different values of mutation probabilities.

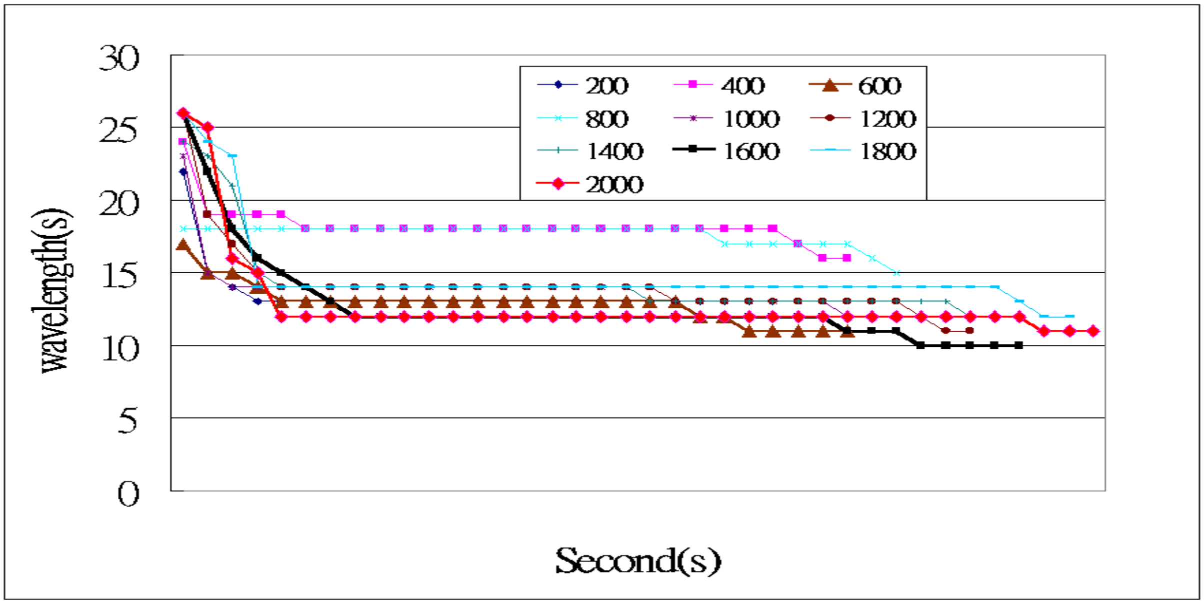

Figure 8: Experiment of different values of population size.