The Study of Adaptive Learning Sequence in the Knowledge Space based on Formal Concept Analysis

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ABSTRACT

Through the practical teaching experiences and investigations, we discover that there exists a gap between learners' current performances and learning objectives. In this thesis, we focus on the problem that individual's learning needs are not satisfied and explore the solutions to these instruction problems. We provide an instructional design aimed at improving individual learning. For achieving the objective, we carry out the following two designs:

1. Providing multiple learning sequences of a specific knowledge domain.
2. Providing individual remedial learning sequences according to the learners' knowledge states.

We integrate Formal Concept Analysis with Knowledge Space Theory in one unified framework and provide a knowledge landscape with multiple learning paths for students to navigate according to their own preferences, knowledge states and learning objectives.

KEYWORD: Formal Concept Analysis, Knowledge Space Theory, Knowledge Representation.

1. INTRODUCTION

There exists a gap between learners' current performances and learning objectives. Students' results of examinations do not achieve the set goals. J.K. Burton and P. F. Merrill called the situation mentioned above normative need [17]. The problems between instruction and learning are complex. The possible reasons are as follows.

1. There are deficiencies in instruction designs and in execution.
2. Individual's learning needs are not satisfied.
3. Students lack the correct learning motivation, teachers are short of general capability in the specific field, and instruction management is in disorder etc. [16] [17].

It is essential to make learning materials of a knowledge domain a landscape for students to be navigated in multiple ways rather than a line with one start and one end. The reasons are as follows.

1. Individuals may be different in cognitive structures; they have different learning needs [13].
2. Optimal content structure and optimal learning sequence for individuals will make learning more effective [1].
3. By learning in multiple contexts, students may build highly interconnected knowledge structures that permit greater flexibility that knowledge can be used [15].

For constructing a visual landscape of domain knowledge, Formal Concept Analysis is a methodology of data analysis and knowledge representation. It has been applied to a variety of applications, like linguistic applications, restructuring help systems, Document Retrieval for Email Search and Discovery [6] [8] [14]. The applications above emphasize that the concept lattice of FCA serves as a means for navigating collections of objects using a visual lattice metaphor rather than a tree. It provides a multiple search paths in the lattice. Moreover, it classifies an object according to multiple orthogonal criteria (attributes or scales).

The main objective of this thesis is to provide an instructional design aimed at improving individual learning. For achieving the main objective, we carry out the following two designs.

1. Providing multiple learning sequences in a

specific knowledge domain.

By analyzing the precedence, contribution, and prerequisite between learning units, we may construct a knowledge landscape with multiple learning sequences. For further considerations of teachers' didactic preferences and student's learning practice, we provide two levels (one for instruction and the other for tests) learning context. The first learning sequence is controlled by teachers in order to class presentation. Through this teacher-oriented sequence, student may get an initial view of the domain knowledge. The second or more sequences may be taken by students. They may guide themselves through the knowledge landscape according to their own preferences, competences, etc.

2. Providing individual remedial learning sequences.

By analyzing the results of examinations taken by a group of students, we construct a hierarchical diagram of concepts. We may recognize the prerequisite relations between the concepts that students fail from the diagram. According to the approach, students may learn remedially in an effective way instead of reviewing the whole learning materials or just studying the isolated concepts repeatedly.

2. RELATED WORK

In this section, we introduce knowledge representation, knowledge space theory, formal concept analysis and some related works.

2.1 Knowledge Representation

A knowledge representation is (i) a medium of human expression, (ii) a set of ontological commitments, (iii) a surrogate, (iv) a fragmentary theory of intelligent reasoning and (v) a medium for pragmatically efficient computation [7]. There are several knowledge representation formalisms like Description Logics, Conceptual Graphs, Concept Maps, Formal Concept Analysis, etc. The differences of these formalisms may be described through Figure 2.1.

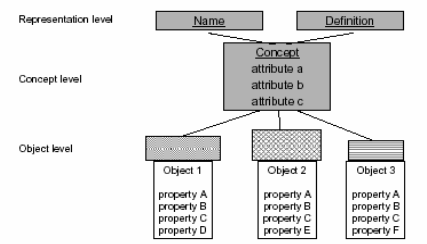

Figure 2.1: Object level, concept level, and representation level according to ISO 704

The standard ISO 704 in Figure 2.1 distinguishes three levels: object level, concept level, and representation level. On the object level, there is no immediate relationship between objects and names. This relationship is rather provided by concepts. On the concept level, the objects under discussion constitute the extension of the concept, while their shared properties the intension of the concept. On the representation level, a concept is specified by a definition and is referred to by a name. FCA is on the concept level, while other knowledge representation formalisms mainly focus on the representation level [18].

2.2 Knowledge Space Theory

In knowledge space theory, a domain of knowledge is a collection of items Q, i.e. problems or questions in a given field of knowledge. The knowledge state of a student is given by the subset K of all problems in Q that the student masters. A knowledge structure for Q is a collection of knowledge states K, and it contains the empty set and the set Q. The subsets K are elements of the collection K [11] [12].

Due to prerequisite relationships between items in Q, there exists surmise relations in the knowledge space. Formally, a surmise relation is a binary relation on the set Q, which will be denoted by $\prec$. For example, the expression $a \prec b$ means that whenever problem a is solved correctly then we can surmise a correct solution to problem b. In other words, the mastery of problem a implies the mastery of b. Surmise relations are partial orders on Q, and they can be illustrated through Hasse diagrams [12].

Knowledge structures of surmise relations satisfy following properties. They are under union and intersection closure, i.e. for any two knowledge states S and S', their union ($S \cup S'$) and their intersection ($S \cap S'$) are also knowledge states. If a knowledge structure is closed under union but not under intersection, it is defined to be a knowledge space (Doignon and Falmagne, 1985). According to the surmise relations of items, we may construct a corresponding knowledge space which is a lattice-like diagram. Therefore, by applying knowledge space theory to tutoring, we obtain the concept of multiple learning paths [3][10]. For determining a knowledge space, we may query expert, analyze students' data and systematically construct the problems of contents.

Table 2.1: Example of a knowledge domain *Q*={a, b, c, d, e} consisting of five basic computer concept problems

Symbol	Problem
a	下列何者為非？ 1. 暫存器是 CPU 內部的記憶體，存取時間比主記憶體短 2. 硬碟大小為 10GB，這表示其容量為 10^{11} bytes 3. 所有的算術運算都在算術/邏輯運算單元完成 4. 具 64 位元資料匯流排及 32 位元位址匯流排的 CPU，其定址最大線性記憶體空間為 4GB
b	下列何者為非？ 1. 在 XP 中複製磁片，需要先將磁片格式化 2. 在 XP 中可以透過「顯示內容」交談視窗改變桌面背景 3. 按右鍵選新增資料夾可以自訂一個屬於自己的資料夾 4. 作業系統使用一段時間後會產生一些不必要的檔案，那是可以刪除的
c	下列何者為非？ 1. TCP/IP 中 TCP 負責將資料正確的送達接收端，IP 負責設定資料封包的 IP 位址及選擇最佳傳輸路徑 2. 以 9600BPS 的傳送速度傳送 6000 個 BIG-5 中文字碼，需要 20 秒 3. IP 位址是由四組 8 位元的數字組成，其中每組數值均介於 0~255 之間 4. 上網瀏覽網頁時，對方網站資料已下載儲存在自己的電腦中
d	下列何者為非？ 1. 1024*768 像素全彩的影像儲存於電腦中，佔用 2.25MB 的儲存空間 2. 127 筆已排序完成的資料，使用二分搜尋法去搜尋某一資料時再多需要 10 次即可找到該筆資料 3. 副程式通常是應用在程式中有某部份需經常用到者，將其寫成副程式 4. 物件導向語言是指具有封裝、繼承、多型三種特性的程式語言
e	下列何者為非？ 1. 使用分封交換技術傳輸資料必須先將資料分割成許多特定大小的封包，接收端在收到資料後不需要花費時間重組資料封包的順序 2. hero@haven.ntctc.edu.tw 由以上的電子信箱中，其中的 haven 是指提供服務的主機名稱 3. 結構化程式設計，是一種由上而下且模組化的設計方法，擁有單一入口、單一出口明確的設計概念 4. 在解析度為 800*600 模式下顯示全彩色那顯示卡記憶體至少需要 4MB

Table 2.1 presents an example of a knowledge domain *Q*={a, b, c, d, e} consisting of five basic computer concept problems, and Figure 2.2 presents a surmise relation defined on the knowledge domain *Q* of Table 2.1.

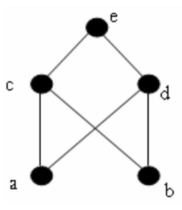

Figure 2.2: Surmise relation on the knowledge domain Q of Table 2.1

The knowledge structure *K* consisting of the knowledge states induced by the surmise relation of Figure 2.2. It is given

K = { ∮ ,{a},{b},{a,b},{a,b,c},{a,b,d},*Q*}.

A lattice-like diagram of the knowledge structure is induces because of set inclusion. We may construct the lattice by Formal Concept Analysis which will be mentioned in the next section. Figure 2.3 provides an illustration of the resulting lattice.

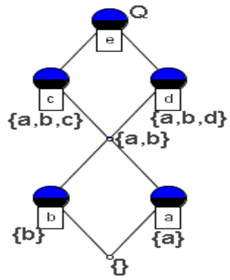

Figure 2.3: Knowledge structure induced by the surmise relation of Figure 2.2

The sequences of upwards directed line from knowledge state ∮ to the set *Q* of full mastery may be interpreted as possible learning paths. We may easily verify in Figure 2.3 that the sequence ∮ ,{a},{a,b},{a,b,d},*Q* of knowledge states forms a possible learning path.

2.3 Formal Concept Analysis

Formal Concept Analysis is a mathematization of the philosophical understanding of concept and a method to visualize data and its inherent structures, implications and dependencies. It is mainly a human-centered method to structure and analyze data. We start from the definition of "concept", "context", "formal concept" and "concept lattice". The description of a concept is based on sets of objects, attributes and a relation form them. For example, the concept "car" can be described by some attributes, objects and an incidence relation between the attributes and the objects which is showed in Figure 2.4 [19].

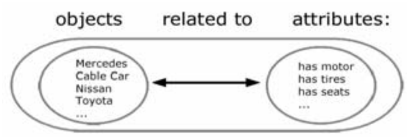

Figure 2.4: Description of the concept "car"

Formally, a concept is constituted by two parts: one is a set of objects and the other is a set of attributes. All objects belonging to this concept have all the attributes of *B*, and all attributes belonging to this concept are shared by all objects of *A*. *A* is called the concept's extension and *B* is called the concept's intension. The formal context is a universe that subsumes the sets of concepts and their relations as showed below in Figure 2.5. We can derive formal concepts, deduce implications base on the context.

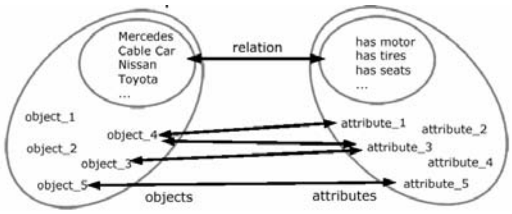

Figure 2.5: A context: the universe of concepts

Formally, a formal context (*G*, *M*, *I*) is a group of objects *G*, attributes *M* and a relation *I*. A context table is a way to specify the incidence relation between objects and attributes. Figure 2.6 presents a formal context, the cell marked "x" means the object has the attribute. Transposing the matrix, changing objects and attributes, creates the dual structure – the same diagram, but flipped top down.

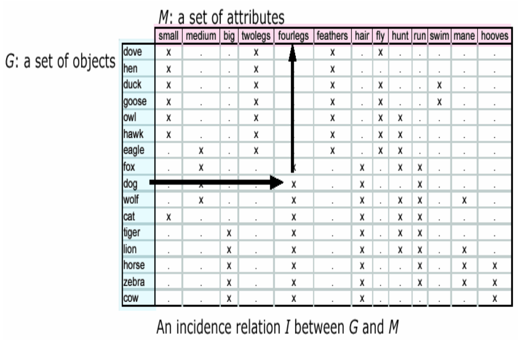

Figure 2.6: A formal context

For a set of object *A*, *A′* is defined as: *A′*= (all attributes in *M* shared by the objects of *A*). For a set of attributes *B*, *B′* is defined as: *B′* = (all objects in *G* that have all attributes of *B*). The pairs of sets (*A*, *B*) of objects and attributes that fulfill the conditions *A′*=*B* and *B′*=*A* are called formal concepts. For example, referring to the above formal context, we can pick any set *A* of objects *G*, e.g. *A* = {duck} to derive the attributes *A′*= {small, two legs, feathers, fly, swim} and to derive (*A′*)′= {small, two legs, feathers, fly, swim}′= {duck, goose}. Then (*A″*,*A′*) = ({duck, goose}, {small, two legs, feathers, fly, swim}) is a formal concept [19].

Conceptual scales are used to group related attributes together. A diagram based on a subset of attributes of a formal context is called a conceptual scale. The process of creating single-valued contexts from a many-valued data set is called conceptual scaling which mostly relies on the human interpretation. Conceptual scaling can also be applied to one-valued contexts in order to reduce the complexity of the visualization. For example, Table 2.2 presents a context. We may split it into two scales. The fist scale is the group of topic numbers, and the second is the style of learning objects [19]. Figures 2.7 and 2.8 present the corresponding concept lattices of Table 2.2.

Table 2.2: A context table with two scales

A	B	C	D	E	F	G	H
	Topic1	Topic2	Topic3	Topic4	Topic5	Instruction	Problem
LO1	X					X	
LO2		X				X	
LO3	X	X				X	
LO4		X	X			X	
LO5	X	X	X			X	
LO6		X	X		X	X	
LO7	X	X	X	X		X	
LO8	X	X	X		X	X	
LO9	X	X	X	X	X	X	
LO10	X	X	X				X
LO11	X	X	X	X	X		X

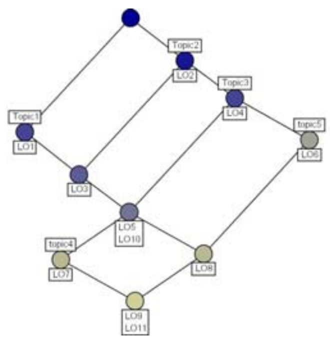

Figure 2.7: A lattice constructed by the first scale (Topic1-Topic5) of Table 2.2

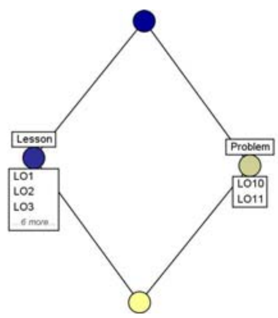

Figure 2.8: A lattice constructed by the second scale (Instruction, Problem) of Table 2.2

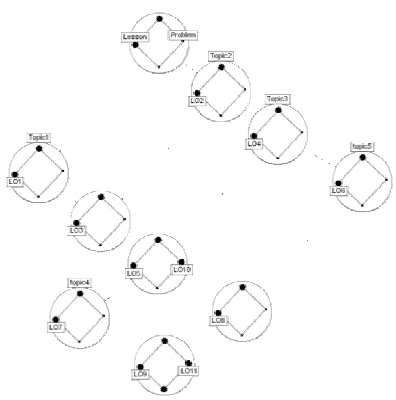

Figure 2.9: A nested lattice with two scales of Table 2.2

Knowledge space theory and FCA both are based on set inclusion principle. Surmise relations of knowledge space theory and implications of FCA are equivalent. According to surmise relations of knowledge space theory, we may construct a subsumption hierarchical diagram, a concept lattice, through FCA. Browsing an ontology based on knowledge space theory to be supported by visualization techniques of Formal Concept Analysis.

3. System Framework

3.1 Structuring Contents and Creating Learning Sequences

An important task in the development of an adaptive tutoring system is the determination of structures for instructions and problems serving as a basis for the adaptively. In this section, we combine FCA and ontology notions to structure learning domain.

Table 3.1: Contents of Basic Computer Concept

單元主題	內容綱要
一、電腦科技與職業生活	（一）在個人方面的應用 （二）在家庭方面的應用 （三）在學校方面的應用 （四）在社會方面的應用 （五）在職業生活方面的應用
二電腦硬體知識	（一）電腦的發展簡史 （二）電腦的架構與連接 （三）電腦的操作與保養 （四）電腦的需求評估 （五）其他相關知識
三、電腦作業系統	（一）作業系統的功能 （二）作業系統的類型 （三）作業系統實例 （四）其他相關知識
四、應用軟體實作	（一）文書處理 （二）電子試算表 （三）簡報 （四）電腦繪圖 （五）電腦音樂 （六）其他相關知識
五、電腦網路的基本知識	（一）資料瀏覽與查詢方法 （二）簡易網頁製作方法 （三）資訊智慧財產權的意義 （四）資訊安全與保護 （五）其他相關知識
六、演算法與程式語言	（一）演算法的簡介與實例 （二）演算法的表示與設計 （三）程式語言的類型與組成 （四）結構化程式實例 （五）其他相關知識
七、電腦科技的相關應用	（一）網路與通訊 （二）語音處理 （三）影像處理 （四）虛擬實境 （五）人工智慧 （六）其他相關知識

The topics from unit one to unit seven in Table 3.1 may be regarded as ontology concepts as well as FCA attributes. In standard Formal Concept Analysis, the set of attributes does not carry any structure. By considering this set of the topics as a set of ontology concepts, we may model relations and dependencies between attributes [5].

For structuring contents through merging multiple ontologies via FCA, we should specify the formal contexts according to different ontologies respectively. The objects are the (sub) learning objectives, and the attributes are the ontology concepts (topics). If a topic is a part of a learning objective, we make an incidence relation between them. For simplification, we use alphabet symbols to substitute for chapter topics in the following descriptions. The contrast table is shown in Table 3.2.

Table 3.2: The contrast table of symbols and topics

Symbol	Topic	
a.	第一章.	電腦簡介
b.	第二章.	電腦硬體
c.	第三章.	數字系統和資料表示法
d.	第四章.	電腦軟體
e.	第五章.	作業系統
f.	第六章.	應用軟體實作
g.	第七章.	VB 程式語言(一)
h.	第八章.	VB 程式語言(二)
i.	第九章.	電腦網路與通訊
j.	第十章.	電腦科技的相關知識與應用

According to the precedence relation, the context and the corresponding lattice diagram are shown in Figure 3.1.

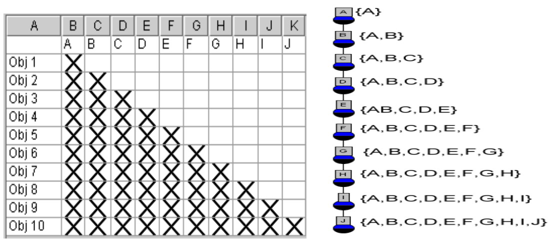

Figure 3.1: The context and lattice considering the precedence relation of topics

Figure 3.1 represents a single learning path. Each node is a sub learning objective. A hierarchy of the topics displayed in Figure 3.2 is a forest with duplicate nodes. The steps of attribute exploration by Concept Explorer are as follows.
Step 1: confirm or rejecting implication.
Step 2: provide counterexample.

The resultant lattice diagram is shown in Figure 3.3. It may provide multiple learning paths and guide learners through a domain with constrains of the relations.

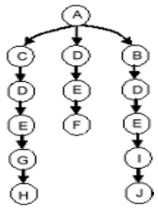

Figure 3.2: The hierarchical diagram of the topics in Table 3.2

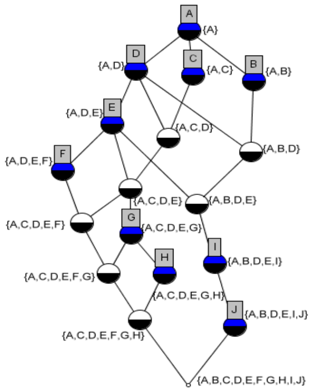

Figure 3.3: The context and lattice considering prerequisite relation between topics

3.2 Construct relations between problems and queries

We determine the contribution relations of topics through analyzing problems and querying experts. Figure 3.4 shows the contribution relations of topics. We stand alone this contribution relation ontology for the reason that it is useful to design multi-level questions according to Bloom's classification [4]. For superior students, it offers a synthetic way to review the teaching contents and therefore enhance students' problem solving abilities [9].

Benjamin Bloom created taxonomy for categorizing level of questions [4]. The levels of questions are knowledge, comprehension, application, analysis, synthesis and evaluation. We may design questions of level referring to the hierarchical property of the lattice.

A	B	C	D	E	F	G	H	I	J	K	L
	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11
A									X	X	
B	X			X	X	X			X	X	
C	X							X		X	
D		X	X						X		
E		X	X								
F			X								X
G							X				
H							X				
I					X	X		X			
J				X		X					

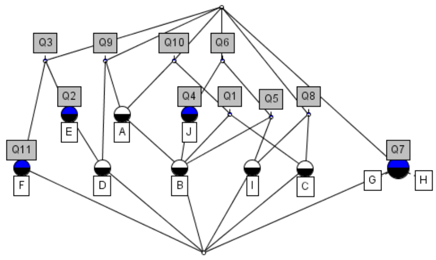

Figure 3.4: The formal context and lattice based on contribution relations of topics

3.3 Conceptual Scaling and Teaching Didactic

The tutoring system may be a combined structure of lessons and problems of tests. For the purpose, we extend the partial ordering of topics by adding two attributes, "instruction" and "problem" as shown in Table 2.2. Figures 3.5 and 3.6 show the conceptual scaling diagrams in a nested view.

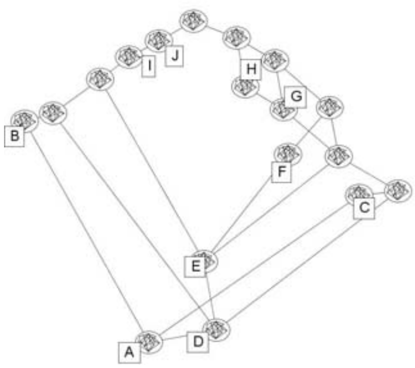

Figure 3.5: The nested lattice of Figures 3.3 and 3.4

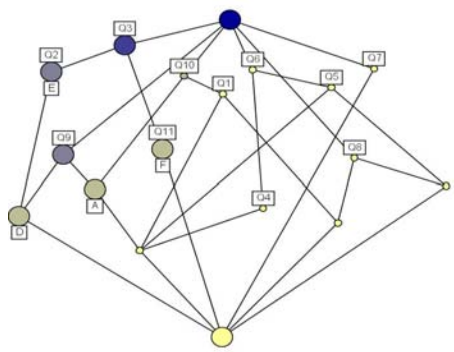

Figure 3.6: The inner scale of Figure 3.5

In Figure 3.5, we may utilize the nested lattice in a didactic view. Students navigate the knowledge domain with constrained paths in the outer scale (topics), and zoom into the inner scale (instruction & problem) which provides different types of learning objects.

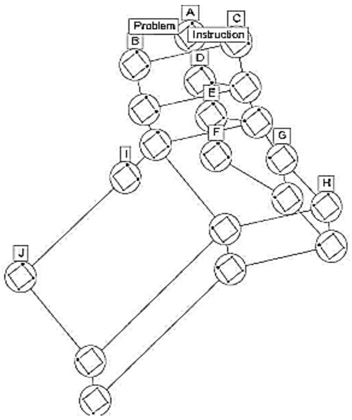

Figure 3.7: The nested lattice 1 of teaching didactic

In Figure 3.5, students may choose to take quizzes or learn instructions from the outer scale (instruction & problem) firstly then they zoom into the inner scale to navigate the knowledge domain.

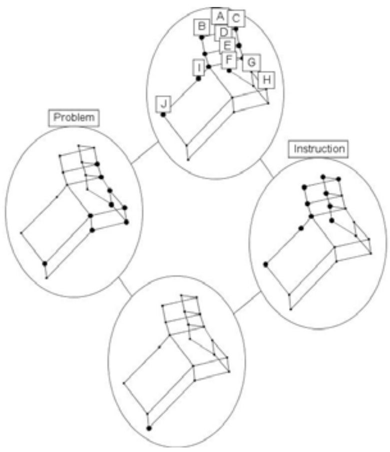

Figure 3.8: The nested lattice 2 of teaching didactic

3.3 An Application of FCA in Remedial Learning

3.3.1 Constructing a Concept Lattice in a Pedagogic View

Remedial learning process emphasizes the specific concepts which students fail to achieve learning goals. But it is not mean to teach/learn isolated concept one by one. It is effective to teach/learn the related concepts simultaneously in a remedial lesson.

We obtain students' knowledge states through examinations than determine the prerequisite relations of concepts by means of comparing students' results of examinations. We specify a formal context according to the relations between the concepts and design remedial learning sequences through the constructed formal concept lattice. We explore the prerequisite relations between concepts by the following three directions:

1. Top-down oriented direction: the relations of concepts are determined by instructors.
2. Bottom-up oriented direction: the relations of concepts are determined by analyzing students' data and utilizing FCA as a tool.
3. Both 1 and 2.

In this section, we concern the bottom-up oriented direction with help of FCA first. The resultant ontology may be modified by instructors if necessary. The steps are as follows:

Step 1: Create a matrix, which will be introduced in Section 3.3.2, to record the relations between students and the units which he/she fails.

Step 2: Transform the matrix in step1 to a formal context, the object are units and the attributes are students. A unit is related to a student if he/she fails the unit.

Step 3: Compute a lattice according to the formal context in Step2.

Step 4: Use the hierarchical and clustering information in the lattice to produce the relations between units。The following are the procedures describing how to use the information in the lattice.

(1) How to find the students who fail: Find the object concept having unit$_i$ (the concept labeled unit$_i$) as extent and follow the lines up to the attribute concepts. The labels of the attribute concepts are the students who fail unit$_{i.}$

(2) How to find the students who fail unit$_i$ and unit$_j$: Find the object concepts having unit$_i$ and unit$_j$ (the concept labeled unit$_i$ and the one labeled unit$_j$) as extent and follow the lines up to the attribute concepts where there join.

Step 5: Obtain a matrix, which will be introduced in Section 3.3.2, to represent the prerequisite relations between units according to the data resulting from step 4 and the criterion mentioned in Section 3.3.2.

Step 6: Compute a lattice, the objects and attributes both are units, according to the matrix resulting from Step 5.

Step 7: Design the remedial learning paths for individuals by means of the lattice resulting from Step 6.

The processes of producing a remedial learning environment are described in detail in Section 3.3.3.

3.3.2 Criterion to Determine the Prerequisite Relation of Any Two Learning Units

A criterion is presented to determine the prerequisite relation of any two units. Let *m* be the number of the students. Let U_i (i = 1–*n*) denote unit i and *n* be the number of the unit. The number of the students who fail U_i is x_i and the number of the students who fail U_j is x_i.

The number of the students who fail U_i as well as U_j is x_{ij}. If the both conditions below are satisfied, we may consider one unit is the prerequisite of the other one.

(1) $x_{ij} / m \geq r_1$

(2) $\max(\frac{x_{ij}}{x_i}, \frac{x_{ij}}{x_j}) \geq r_2$

Where r_1 and r_2 are prescribed values related with the degree of difficulty for the problem and the level of ability for the student. Accordingly, if $\max(\frac{x_{ij}}{x_i}, \frac{x_{ij}}{x_j}) = \frac{x_{ij}}{x_i}$ and $x_{ij} / m \geq r_1$, then unit *i* is the prerequisite of unit *j*.

3.3.3 Procedures to Determine a Matrix M for Constructing a Formal Concept Lattice

In this section, we create a matrix $M = [m_{ij}]_{nxn}$ in order to construct a formal concept lattice representing the hierarchy and clustering of unit.

Let $s_j (j = 1 - m)$ denote the *j*-th student. We define a matrix $R = [r_{ij}]_{nxm}$ that *i*-th row represents U_i, and the *j*-th column represents s_j. If s_j fails U_i, let the value of the element r_{ij} equal to 1, otherwise equal to 0. Based on the results of examinations, we obtain matrix R.

In addition, we define a matrix $N = [n_{ij}]_{nxn}$, the row *i* and the column *j* represent U_i and U_j respectively. The element n_{ij} represents $\frac{x_{ij}}{x_j}$. According to matrix R, we obtain x_{ij} and matrix N. Furthermore, we define a matrix $M = [m_{ij}]_{nxn}$ that both column and row represent units. Let the value of the element m_{ij} be equal to 1 if the unit *j* is the prerequisite of unit *i*. The following procedures are used to determine matrix M.

First, if any two units *i* and *j* (*i*≠*j*) are independent, then let the corresponding two element m_{ij} and m_{ji} to be 0. In addition, let m_{ii} equal to 0. Secondly, according to the criterion mentioned in section 3.3.1, determine prerequisite relation of any two units. Lastly, we obtain matrix M, and identify a formal context according to the matrix, then compute a formal concept lattice for remedial learning [2].

4. Example

The steps of determine the prerequisite relations between units are described following. First, we record the students' results of the examination and transform them to the form of a context shown below in Figure 3.8. In the context, the objects are units and the attributes are students. A unit is related to a student if he/she fails the unit.

A	B	C	D	E	F	G	H	I	J	K
	S01	S02	S03	S04	S05	S06	S07	S08	S09	S10
U1							X			X
U2		X			X			X		
U3					X			X		
U4	X	X			X			X		
U5				X			X	X		
U6		X		X	X			X		X
U7	X		X							
U8						X			X	
U9	X	X		X	X		X	X		
U10		X	X	X			X		X	

Figure 4.1: The context indicating the incidence relations between students and units (Cross mark means that some student fails some unit)

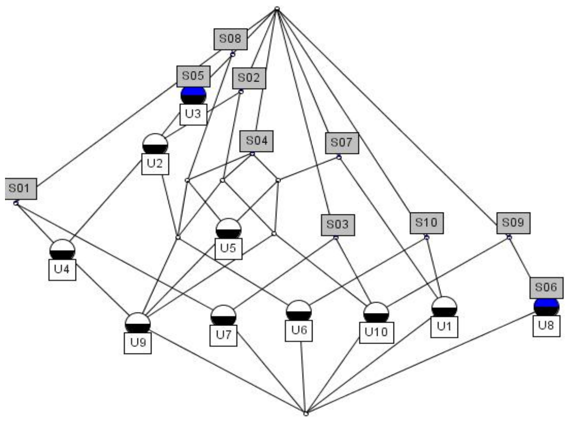

Figure 4.2: The formal concept lattice of Figure 4.1

Firstly, the sub-lattice in Figure 4.3 shows the object concept u10 which has 5 attributes. We may explain the sub-lattice that the students who fail unit u10 are s02, s03, s04, s07 and s09. Secondly, the sub-lattice in Figure 4.4 shows the attribute concept s07 with 4 objects. We may explain the sub-lattice that the student s07 fails units: u1, u5, u9 and u10. Lastly, the sub-lattice in Figure 4.5 shows the formal concept ({u9, u10}, {s02, s04, s07}). We may explain the sub-lattice that the students who fail u9 and u10 are s02, s04 and s07.

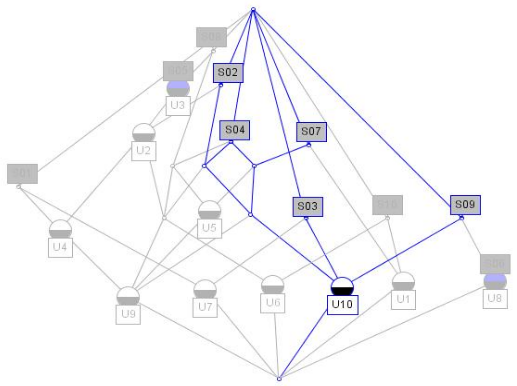

Figure 4.3: The sub-lattice of object concept u10

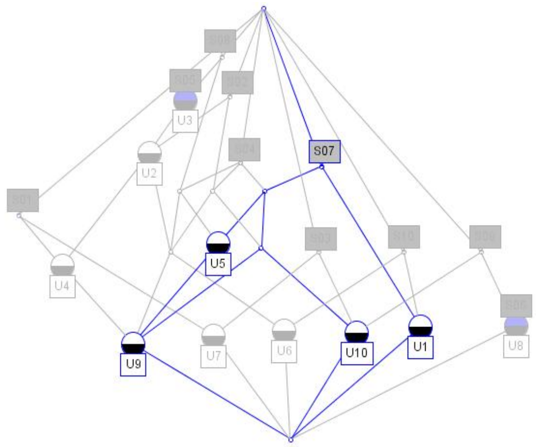

Figure 4.4: The sub-lattice of attribute concept s07

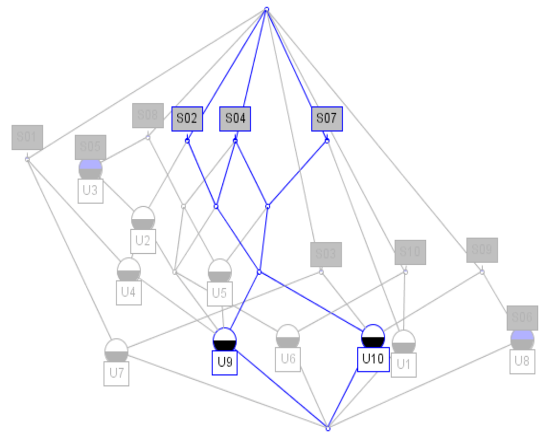

Figure 4.5: The sub-lattice of formal concept ({u9, u10}, {s02, s04, s07})

We imply the procedures described in Section 3.3.3, apply the first criterion described in Section 3.3.2, and set the prescribed values: r1=0.2, r2=0.4 for this example. Consequently, we obtain Matrix N_{nxn} shown in Table 4.1. Figure 4.6 is a directed line diagram of N_{nxn} which represents the weighted prerequisite relations between units.

Table 4.1: The relations between units

	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10
U1	0	0	0	0	0	0	0	0	0	0
U2	0	0	1	0.75	0	0.6	0	0	0.5	0
U3	0	0.7	0	0.5	0	0.4	0	0	0	0
U4	0	1	1	0	0	0.6	0	0	0.6	0
U5	0	0	0	0	0	0.4	0	0	0.5	0.4
U6	0	1	1	0.75	0.7	0	0	0	0.6	0.4
U7	0	0	0	0	0	0	0	0	0	0
U8	0	0	0	0	0	0	0	0	0	0
U9	0	1	1	1	1	0.8	0	0	0	0.6
U10	0	0	0	0	0.7	0.4	0	0	0.5	0

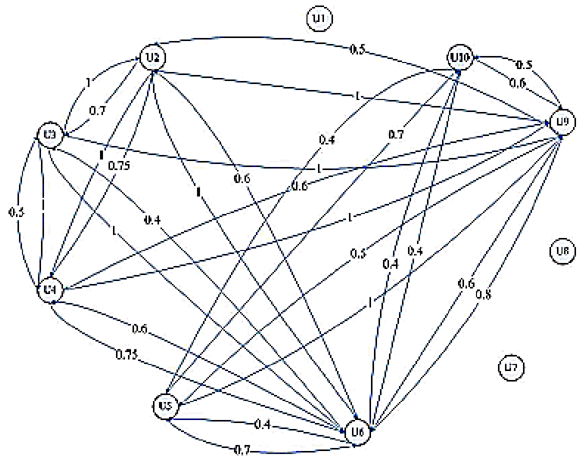

Figure 4.6: The directed line diagram representing the relations between units

With N_{nxn} in Table 4.1, we may obtain the matrix M_{nxn} in Figure 4.7.

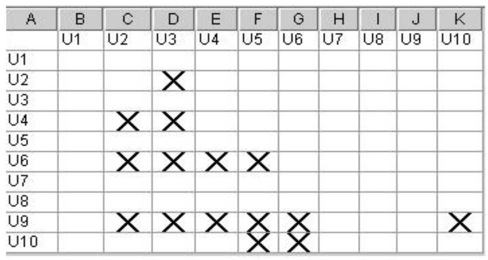

Figure 4.7: The formal context transformed from Table 4.1, the objects and attributes both are units

Figure 4.8 represents the lattice computed from the formal context in Figure 4.7. It represents the prerequisite relations between units and provides multiple remedial learning paths. We illustrate the steps of finding a remedial learning path for a specific student in Figures 4.9-4.12.

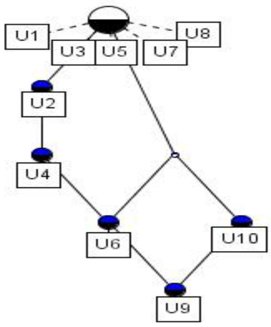

Figure 4.8: The lattice computed from the context in Figure 4.7

By the lattice in Figure 4.9, we may recognize each student's learning circumstances easily. For example, student s03 fails u07 and u10. For visualizing the detail prerequisite relations between 07 and u10, we may utilize the inner scale.

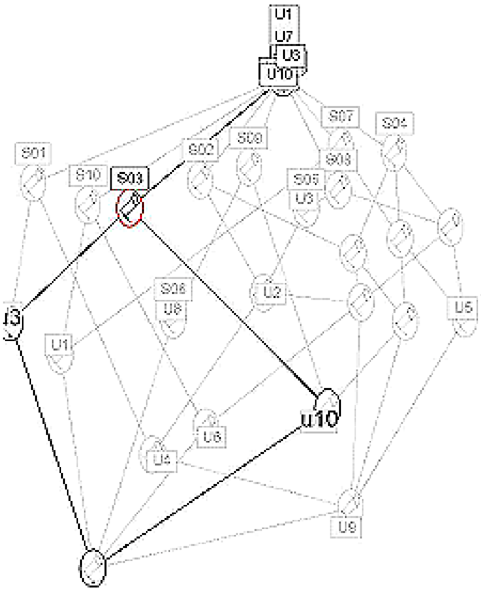

Figure 4.9: The nested lattice diagram. In the outer scale, attributes are students and objects are units

For the lattice in Figure 4.9 is a nested diagram, we zoom into the inner scale as shown in Figure 4.10, and recognize the prerequisite relation between the units which units student s03 fails. In Figure 4.11, we may recognize that unit u7 has no prerequisite. Therefore, the first remedial learning unit is u7. With respect to unit u10, it has prerequisites u5 and u10 in order as shown in Figure 4.12. Accordingly, the remedial learning sequence for student s03 is (u7, u5, u6, u10).

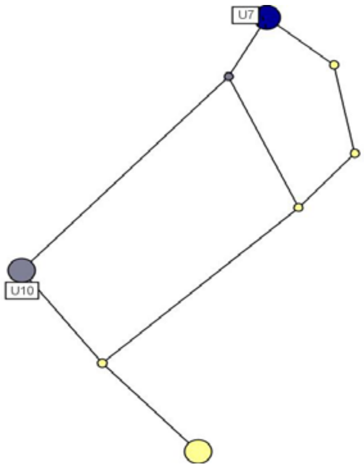

Figure 4.10: The lattice which is identical with Figure 4.8, and emphasizes the units: u7 and u10 that student s03 fails

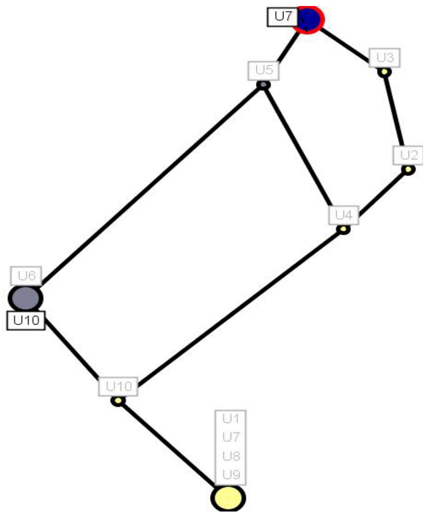

Figure 4.11: The lattice which is identical with Figure 4.8, and emphasizes the unit u7 that has no prerequisite

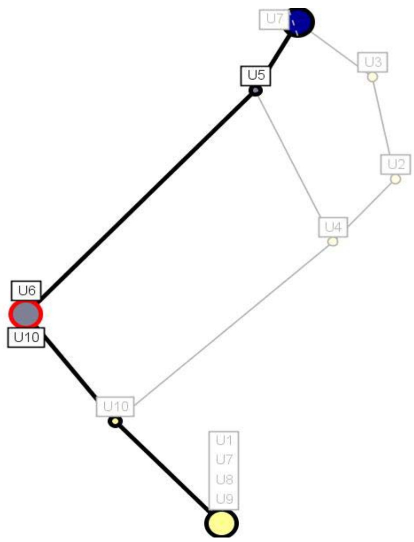

Figure 4.12: The lattice which is identical with Figure 4.8, and emphasizes the unit u10 that has two prerequisites: u5 and u6 in order.

5. CONCLUSION

In this thesis, we have integrated Formal Concept Analysis and Ontology Engineering in one unified framework. Since we have some understanding of the domain, we use top-down (ontological) approach to do classifications first. FCA can help refine build ontologies in bottom-up process. Through the two directions, we provide a knowledge landscape with multiple learning paths for students to navigate according to their own preferences, knowledge states and learning objectives. In the view of establishing effective remedial learning sequences, we analyze the concepts which students fail and construct a remedial concept hierarchy via FCA.

Furthermore, we utilize conceptual scaling and nested scaling of FCA to manipulate multiple classifications of learning materials and didactic preferences. Multiple classifications of learning materials may be regarded as multiple learning contexts. Learning in multiple contexts reflects the way that knowledge is learned and used in different views. The pedagogical features will benefit the ability of solving complex problems and complete tasks. Besides, we provide the inner sequences by increasing formal concept's depth. Students may zoom into a nested formal concept for detailed or deeply description of the concept if necessary.

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