Image Compression Using Grey-Based Neural Networks In the Wavelet Transform Domain

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Abstract

The wavelet transform has recently emerged as a powerful tool for image compression. In this paper, the Grey theory is applied to a two-layer modified Competitive Learning Network (GCLN) to generate optimal solutions for VQ. In accordance with the degree of similarity measures between training vectors and codevectors, the grey relational analysis is used to measure the relationship degree among them. The input image is first decomposed into four subbands in the 1-level wavelet transform. Then the corresponding transformed coefficients are trained using GCLN to form individual codebooks for each subband. The compression performances using the proposed approach are compared with GCLN and the conventional vector quantization LBG method. Experimental results show that promising performance can be obtained using the GCLN with wavelet decomposition.

Key words: Image compression, Competitive learning network, Grey theory, Wavelet transform.

1. Introduction

In recent years, wavelet coding [1-2] plays an important role in image compression. The wavelet transform is identical to a hierarchical subband system. Basically, the wavelet transform decomposes an image into a set of sub-image blocks that are more stationary and hence provide better coding performance. A higher compression ratio can be achieved by the exploitation of quantizers adapted to the statistics of the sub-image blocks.

A number of vector quantization algorithms for data compression have proposed over the years [3-5]. The purpose of vector quantizaton is to create a codebook such that the average distortion between training vectors and their corresponding codevectors in the codebook is minimized. Codebook design can be considered as a clutering process in which each training vector is classified to a specific class. The clustering process updates the codebook iteratively such that the average distortion between training vectors and codevectors in the codebook becomes smaller and smaller.

Neural networks with competitive learning have been demonstrated capable of performing vector quantization [6-8]. In addition to the neural network-based techniques, the grey relational theory proposed in 1982 [9-12] has also been demonstrated to address the codebook design in this paper. In Grey-based Competitive

Learning Network (GCLN), the learning rule and stopping criterion of the original competitive learning neural network are modified to address the codebook design using the grey relational strategy. The problem of VQ is regarded as a minimization process of an object function. This object function is defined as the average distortion between the training vectors in a divided image to the cluster centers represented by the codevectors in the codebook. The modified competitive learning network is simpler than that of the conventional competitive leaning network, and is constructed as a two-layer fully interconnected array with the input neurons representing the training vectors and output neurons representing the codevectors in the codebook.

In this paper, the input image is first decomposed into four subbands LL1, LH1, HL1, and HH1 in the 1-level wavelet transform. Then the corresponding transformed coefficients are trained using GCLN to form individual codebooks for each subband. Computer simulations show that the GCLN used with VQ is promising for image compression.

2. Competitive Learning Network

A competitive learning network is an unsupervised network which selects a winner based on similarity measure over the feature space. A proper neuron state is updated if and only if it wins the competition among all neurons. Many schemes for competitive learning networks have been proposed [13-14].

In the simple competitive learning network, the single output layer consists of cluster centers, each of which is fully connected to the inputs via interconnection strength. In conventional competitive learning only one output unit is active at a time and the objective function is given by

$$J_c = \frac{1}{2}\sum_{j=1}^{c}\sum_{i=1}^{n} u_{i,j}\left\|\mathbf{x}_i - \boldsymbol{\omega}_j\right\|^2 \qquad (1)$$

where n and c are the number of training vectors and the number of clusters respectively. $u_{i,j}=1$ if $\mathbf{x}_i$ belongs to cluster j and $u_{i,j}=0$ for all other clusters. The neuron that wins the competition is called the winner-take-all neuron. Thus $u_{i,j}$ indicates whether the input sample $\mathbf{x}_i$ activates neuron j to be a winner. $u_{i,j}$ is given by

$$u_{i,j} = \begin{cases} 1 & \text{if } \left\|\mathbf{x}_i - \boldsymbol{\omega}_j\right\| \le \left\|\mathbf{x}_i - \boldsymbol{\omega}_k\right\|, \quad \text{for all } k \; ; \\ 0 & \text{otherwise.} \end{cases} \qquad (2)$$

The incremental $\Delta\boldsymbol{\omega}_j$ is given by [13-14]

$$\left\langle \Delta\boldsymbol{\omega}_j \right\rangle = -\eta \frac{\partial J_c}{\partial \boldsymbol{\omega}_j} = \eta \sum_{i=1}^{n}\left(\mathbf{x}_i - \boldsymbol{\omega}_j\right) u_{i,j}, \quad j = 1,2,\cdots,c. \qquad (3.a)$$

where η is the learning-rate parameter. Although Eq. (3.a) is written as a sum over all samples, practically it is usually used incrementally, i.e.

$$\Delta\boldsymbol{\omega}_j = \eta\left(\mathbf{x}_i - \boldsymbol{\omega}_j\right) u_{i,j}, \quad j = 1,2,\cdots,c. \qquad (3.b)$$

The updating rule is given by

$$\boldsymbol{\omega}_j(t+1) = \boldsymbol{\omega}_j(t) + \Delta\boldsymbol{\omega}_j(t). \qquad (4)$$

The Modified Competitive Learning Network (MCLN) has the same architecture as the conventional competitive learning network. It is an unsupervised competitive learning network using the modified competitive learning rule and stopping criterion. Similar to the standard competitive learning rule in Eqs. (3.b) and (4), the least squared error solution can be obtained by [13]

$$\boldsymbol{\omega}_j(t+1)=\begin{cases}\boldsymbol{\omega}_j(t)+\eta(\mathbf{x}_i-\boldsymbol{\omega}_j) & \text{if } \|\mathbf{x}_i-\boldsymbol{\omega}_j\|\le\|\mathbf{x}_i-\boldsymbol{\omega}_k\|, \quad \text{for all } k \; ; \\ \boldsymbol{\omega}_j(t) & \text{otherwise}\end{cases}$$

(5)

The MCLN algorithm modifies only output neurons without updating the interconnection strengths. Instead of updating the interconnection strengths using the winner-take-all scheme in the conventional competitive learning network and for the purpose of simplifying the hardware architecture, the MCLN only modifies the output states (cluster centroids).

3. Grey-Based Competitive Learning Network for VQ

Suppose an image is divided into n blocks (vectors of pixels) and each block occupies $\ell\times\ell$ pixels. A vector quantizer is a technique that maps the Euclidean $\ell\times\ell$ -dimensional space $\mathbf{R}^{\ell\times\ell}$ into a set $\{\boldsymbol{\omega}_j, j=1,2,...,c\}$ of points in $\mathbf{R}^{\ell\times\ell}$, called a codebook. It looks for a codebook such that each training vector is approximated as close as possible by one of the code vectors in the codebook. A codebook is optimal if the average distortion is at the minimum value. The average distortion $E[d(\mathbf{x}_i,\boldsymbol{\omega}_j)]$ between an input sequence of training vectors $\{\mathbf{x}_i, i=1,2,...,n\}$ and its corresponding output sequence of code vectors $\{\boldsymbol{\omega}_j, j=1,2,...,c\}$ is defined as

$$D=E[d(\mathbf{x}_i,\boldsymbol{\omega}_j)]=\frac{1}{n}\sum_{i=1}^{n}d(\mathbf{x}_i,\boldsymbol{\omega}_j) \qquad (6)$$

The grey system is usually divided into several topics such as grey theory, grey mathematics, grey prediction, grey generating space, grey decision, and grey relational analysis. Grey relational theory demonstrates the measurement of similarity between training vectors and codevectors based on the grey relational space. Let $\mathbf{x}_i$ be a training vector and $\boldsymbol{\omega}_j$ be the codevector *j*, then the grey relational coefficient is defined as

$$\gamma\left(\mathbf{x}_i,\boldsymbol{\omega}_j\right)=\frac{\Delta_{\min.}+\xi\Delta_{\max.}}{\Delta_{ij}+\xi\Delta_{\max.}} \qquad (7)$$

where

$$\Delta_{\min.}=\min\left|\mathbf{x}_i-\boldsymbol{\omega}_j\right|$$

$$\Delta_{\max.}=\max\left|\mathbf{x}_i-\boldsymbol{\omega}_j\right|$$

$$\Delta_{ij}=\left|\mathbf{x}_i-\boldsymbol{\omega}_j\right|$$

and $0<\xi<1$ is the distinguished coefficient. The grey relational grade is given by

$$\gamma_{i,j}=\frac{1}{\ell\times\ell}\sum_{m=1}^{\ell\times\ell}\gamma\left(\mathbf{x}_i,\boldsymbol{\omega}_j\right) \qquad (8)$$

where *m* is the dimension of the training vector $\mathbf{x}_i$ and the codevector $\boldsymbol{\omega}_j$.

In this paper, the grey theory is applied to a two-layer MCLN in order to generate optimal solution for VQ. The modified competitive learning rule is modified as

$$\boldsymbol{\omega}_j(t+1)=\begin{cases}\boldsymbol{\omega}_j(t)+\eta(\mathbf{x}_i-\boldsymbol{\omega}_j) & if \quad \gamma_{i,j}\geq\gamma_{i,k}, \quad for\ all\ k\ ; \\ \boldsymbol{\omega}_j(t) & otherwise\end{cases}$$

(9)

The steps of codebook design using the grey-based competitive learning network are given as follows.

Step 1: Initialize the codevectors $\boldsymbol{\omega}_j\,(2\leq j\leq c)$, learning rate η, maximum error (*ME*), total error (*TE*), and a threshold value ε.

Step 2: Input a training vector $\mathbf{x}_i$ and find the winner's codevector based on the maximum grey relational grade.

Step 3: Apply Eq. (9) to update the winner's codevector and set $TE=TE+\Delta_{ij}$.

Step 4: Repeat Steps 2 and 3 for all input samples, then if $(ME-TE)/ME<\varepsilon$, go to step 5; otherwise replace *ME* content from *TE*, and go to Step 2.

Step 5: Complete the codebook design.

4. Wavelet Transform and GCLN

The wavelet transform is a signal decomposition technique [16]. The mother function of wavelets, $\Psi(x)$, can be any function if it satisfies the following condition

$$\int_{-\infty}^{\infty}\left|\Psi(x)\right|^2 dx<\infty \tag{10}$$

Basically, most of the mother functions are derived from the scaling function $\Phi(x)$ which is any function satisfying the scaling equation

$$\Phi(x)=\sum_{i\in Z}c_i\Phi(2x-i) \tag{11}$$

For any integer *j*, we define vector space V^j as follows

$$V^j=span\{\Phi_i^j(x)\} \tag{12}$$

$$\Phi_i^j(x)=\Phi(2^j x-i), \qquad i\in Z \tag{13}$$

If V^j satisfies the following four multiresolution analysis conditions

Condition 1: $f(x)\in V^j \Leftrightarrow f(2^{-j}x)\in V^0, \forall j\in Z$

Condition 2: $\bigcap_{i\in Z}V^i=\{0\}$

Condition 3: $\bigcup_{i\in Z}V^i=L^2(R)$

Condition 4: $\cdots\subset V^{-2}\subset V^{-1}\subset V^0\subset V^1\subset V^2\subset\cdots$

it is easy to show that the mother function of wavelets is given by

$$\Psi(x)=\sum_{i\in Z}(-1)^i c_{1-i}\Phi(2x-i) \tag{14}$$

One of the most important characteristics of the wavelet transform is multiresolution. The wavelet transform converts the pixel values of images to wavelet domain, without lose any information in the spatial domain. A wavelet decomposition of Lena image using the 1-level Haar basis wavelet transform is shown in Fig. 1.

Vector quantization has been demonstrated to be an efficient method for image compression. The motivation for the proposed GCLN with wavelet decomposition scheme is based upon the fact that the lower resolution wavelet coefficients hold more information and higher resolution wavelet coefficients hold less information. The basic idea is described as follows. The 1-level wavelet transform is first performed on the input image to generate the corresponding transformed coefficient. Then, the coefficients of each scale are trained separately to obtain an individual codebook. For example, as shown in Fig. 2, the coefficients of LL1 and LH1 are divided into the blocks of size 2×2 and 4×4 from which two codebooks of size = 512, and 64 were built respectively. The coefficients of HL1 and HH1 are totally discarded since they rarely hold information, and are reconstructed by searching the best-matched codevectors from the codebook of LH1. The GCLN is eventually employed as vector quantization technique of image compression and thus reducing bit rate and training time.

5. Experimental Results

Codebook design is a primary problem in image compression based on vector quantization. In this paper, the quality of the images reconstructed from the designed codebooks was compared with that from the LBG and GCLN methods. The training vectors were extracted from 256×256 with 8-bit gray level real images, each of which is divided into 4×4 blocks to generate 4096 non-overlapping 16-D training vectors. Three codebooks of size = 64, 128, and 256 were built by these training vectors. The resulting images were evaluated subjectively by the mean squared error (MSE) and peak signal to noise ratio (PSNR) defined for images of size N×N as

$$PSNR = 10\log_{10}\frac{255^2}{MSE} \qquad (15)$$

and

$$MSE = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(x_{ij} - \hat{x}_{ij}\right)^2 \qquad (16)$$

where x_{ij} and $\hat{x}_{ij}$ are the pixel gray levels from the original and reconstructed images, and 255 is the peak gray level. Table 1 shows the PSNR and MSE of the “F16”, “Girl” and “Lena” images reconstructed from three codebooks of size 64, 128, and 256 designed by the LBG and the GCLN methods without wavelet decomposition. The comparison between GCLN without decomposition (named GCLN) and GCLN with wavelet decomposition (named by WT+GCLN) is shown in table 2. In table 2, the compression ratio (CR) is [CR_LL1 + (CR_LH1 × 3)] / 4 = $\left(\frac{2\times 2\times 8}{9} + \frac{4\times 4\times 8}{6}\times 3\right)\div 4 = 16.89$ for WT+GCLN, and $\frac{4\times 4\times 8}{6} = 21.33$, $\frac{4\times 4\times 8}{7} = 18.29$, $\frac{4\times 4\times 8}{8} = 16$ for GCLN respectively. From the experimental results, the reconstructed images obtained from the GCLN are superior to those obtained from the LBG algorithm, and those from the WT+GCLN are significantly better than those from the GCLN algorithm.

6. Conclusions

In this paper, a two-layer modified competitive learning neural network based on gray relational theory for VQ in wavelet transform domain has been presented. Instead of updating the interconnection strengths using the winner-take-all scheme in the conventional competitive learning network, the GCLN algorithm only modifies output neurons and omits the updating of the interconnection strengths. Experimental results show that the GCLN and WT+GCLN methods produce reconstructed images better than those reconstructed by the LBG method.

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Fig. 1 1-level wavelet transformed Lena image (Haar basis)

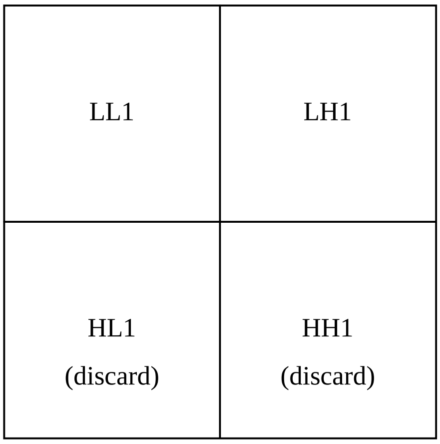

Fig. 2 1-level wavelet transformed four subbands

Table 1. PSNR and MSE of the images reconstructed from codebooks of various size designed by the LBG and proposed GCLN algorithms.

Images/Algorithms \ Codebook Sizes		64 PSNR	64 MSE	128 PSNR	128 MSE	256 PSNR	256 MSE
F16	LBG	24.11	222.31	25.29	192.30	26.34	142.94
	GCLN	24.75	217.68	25.35	189.69	26.64	140.90
Girl	LBG	27.68	109.62	28.51	91.60	29.69	77.09
	GCLN	28.79	85.98	29.91	66.44	31.05	51.02
Lena	LBG	25.26	146.89	26.37	127.01	27.06	106.71
	GCLN	26.09	160.01	27.23	123.11	28.84	84.89

Table 2. PSNR and MSE of the images reconstructed from various compression ratios (CR) designed by the GCLN and WT+GCLN methods.

Algorithms/CR \ Images		F16 PSNR	F16 MSE	Girl PSNR	Girl MSE	Lena PSNR	Lena MSE
GCLN	CR=21.33	24.75	217.68	28.79	85.98	26.09	160.01
WT+GCLN	CR=16.89	26.64	140.90	31.31	48.10	28.91	83.51
GCLN	CR=18.29	25.35	189.69	29.91	66.44	27.23	123.11
WT+GCLN	CR=16.89	26.64	140.90	31.31	48.10	28.91	83.51
GCLN	CR=16	26.64	140.90	31.05	51.02	28.84	84.89
WT+GCLN	CR=16.89	26.82	135.27	31.31	48.10	28.91	83.51

Image Compression Using Grey-Based Neural Networks In the Wavelet Transform Domain

應用灰色理論的神經網路於小波轉換域之影像壓縮技術

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摘要

小波轉換合併影像壓縮技術近來已成為有力的工具。本篇論文中，灰色理論被應用到一個兩層的修正競爭式學習網路上。其目的在於建立一編碼簿使得介於訓練向量與編碼簿中之編碼向量的灰關聯度最大。輸入影像首先用小波轉換將其分解成四個次頻帶，然後其對應的小波轉換係數再應用灰關聯度的修正競爭式學習網路對個別次頻帶訓練編碼簿。根據實驗結果顯示，基於灰色理論最大關聯準則之修正競爭式學習網路於小波轉換域上所產生的影像壓縮編碼簿具有良好的效能。

關鍵字：影像壓縮 (Image compression)，競爭學習網路 (Competitive learning network)，灰色理論 (Grey theory)，小波轉換 (Wavelet transform)。