

# Image Coding Using Adaptive Wavelet Zerotree

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## Abstract

Wavelet image decompositions generate a tree structured components that provide a hierarchical data structure for representing images. A new class of recently proposed image compression algorithms has focused on new ways for exploiting dependencies between this hierarchy of wavelet coefficients using *zerotree* data structures. This paper deals with a particular type of adaptive subband image coding where we focus on the image coder's ability to adjust its coding length of each coefficient of high frequency subbands. Our model is inspired by a variation on the embedded zerotree wavelet (EZW). Our subband image coder based on the proposed adaptive quantization idea exhibits excellent rate-distortion performance. For popular test images, it is comparable or superior to most of the state-of-the-art coders in the literature.

*key words:* image compression, discrete wavelet transform, EZW, *P-zerotree*.

## I. Introduction

Hierarchical decomposition to image coding has been a popular and promising framework for highly efficient image compression [1], motivated by their ability to offer space-frequency resolution tradeoffs that are well matched to the characteristics of real images. The interest in wavelet/subband coding techniques was renewed thanks in part to the recent work by [2] that introduced a method to generate embedded bit streams at a reasonable algorithmic cost while still outperforming the stand block-based (e.g., DCT-based) algorithms. In fact, many wavelet/subband image coders were submitted to the ongoing JPEG 2000 standardization effort, and a wavelet-based coder will be adopted for the standards.

Wavelet/subband coding techniques provide excellent space/frequency energy compaction so that within each subband energy tends to be clustered spatially. The space/frequency localization of subband image data takes advantage of efficient data structures for spatial decorrelation, e.g., zerotrees in [2], [3], [10] hierarchical trees in [4]. These techniques utilize the correlation across and within subbands and

energy compaction properties [5]. In addition, wavelet/subband coding techniques usually consist the use of quantization of subband coefficients, and optional entropy coding of the quantizer's indexes—to change from one image to another, or even locally within an image.

In this paper, we concentrate on incorporating adaptive quantization techniques in the subband image-coding framework where: 1) we develop a modeling structure, modified zerotree, called *P-zerotree*, for coding the location information, 2) instead of conventional quantization of the subband, in which a specific fixed bit rate is typically allocated for each subband, the proposed new adaptive quantization techniques explicitly compare the ratio of significant coefficients inside each subband with those of its scale and quantize only such coefficients.

## II. Modified Embedded Zerotree Wavelet Coding

Shapiro [2] developed an algorithm that exploits a relation between subbands in image compression. Zerotree wavelet coding is proven a technique for coding wavelet transform coefficients [1-8], [10]. The advantages of zerotree wavelet coding include embedded bitstream structure, scalability and superior compression performance. In the algorithm, zerotree has been combined with bit plane coding and demonstrates the effectiveness of wavelet based coding. The algorithm is based on the zerotree that efficiently represent many insignificant coefficients. The compression algorithm has three key procedures: 1) wavelet decomposition, 2) exploiting the self-similarity inherent in the wavelet transform to predict the location of significant information for symbol generation, 3) entropy coding. In this section, first we briefly review Shapiro's EZW [2]. Then, we describe the proposed technique, which provides higher compression efficiency.

### A. Wavelet transform of image

The wavelet transform is identical to a hierarchical subband system, where the subbands

are logarithmically spaced in frequency. The basic idea in the DWT for a two dimensional image is as follows. An image is first decomposed into four parts of high, middle, and low frequencies, i.e., LL1, HL1, LH1, HH1 subbands, by cascading horizontal and vertical two channel critically subsampled filter banks. The subbands labeled HL1, LH1, and HH1 represent the finest scale wavelet coefficients. To obtain the next coarser scale of wavelet coefficients, the subband LL1 is further decomposed and critically subsampled. This process is continued an arbitrary number of times, which is determined by the application at hand. Fig. 1 shows an image is decomposed into ten subbands for three levels (scales). Each level has various band-information such as low-low frequency band, low-high frequency band, high-low frequency band, and high-high frequency band.

Furthermore, from these DWT coefficients, the original image can be reconstructed. This reconstruction process is called the inverse DWT (IDWT). Let  $I[m, n]$  represent an image. The DWT and IDWT for  $I[m, n]$  can be similarly defined by implementing the DWT and IDWT for each dimension  $m$  and  $n$  separately:  $DWT_n$  [ $DWT_m I[m, n]$ ]. Fig. 2 shows the original and DWT decomposition of  $512 \times 512$  Lena image. The coarsest subband is a low-pass approximation of the original image, and the other subbands are finer-scale refinements.

A parent-child relationship can be defined between wavelet coefficients at different scales corresponding to the same location. Excepting the highest frequency subbands, i.e., HL1, LH1, and HH1, every coefficient at a given scale can be related to a set of coefficients at the next finer scale of similar orientation. The coefficient at the coarse scale is called the parent, and all coefficients corresponding to the same spatial location at the next finer scale of similar orientation are called children. For a given parent, the set of all coefficients at all finer scales of similar orientation corresponding to the same location are called descendants. A wavelet tree that descending from a coefficient in the subband LH3 is shown in Fig. 3.

**Definition 1:** A wavelet coefficient  $x_n(i, j) \in D$  is a parent of  $x_{n-1}(p, q)$ , where  $D$  is a subband labeled HL<sub>n</sub>, LH<sub>n</sub>, HH<sub>n</sub>,  $p = i*2-1$ ,  $q = j*2-1$ ,  $j*2$ ,  $n > 1$ ,  $i > 1$ , and  $j > 1$ .

**Definition 2:** If a wavelet coefficient  $x_n(i, j)$  at the coarsest scale and its descendants  $x_{n-k}(p, q)$  satisfy  $|x_n(i, j)| < T$ ,  $|x_{n-k}(p, q)| < T$  for a given threshold  $T$ , then they are called wavelet zerotrees.

**Definition 3:** If a wavelet coefficient  $x_n(i, j)$  at the coarsest scale satisfy  $|x_n(i, j)| > T$  for a given

threshold  $T$ , then  $x_n(i, j)$  is called a significant coefficient [2].

The zerotree is defined as follows. Given an amplitude threshold  $T$ , if a wavelet coefficient  $|x(i, j)|$  satisfies  $|x(i, j)| < T$ , then the  $|x(i, j)|$  is said to be insignificant over a given threshold  $T$ . If a coefficient and all of its descendants are insignificant over  $T$ , then we call the set of these wavelet coefficients zerotree for the threshold  $T$ . An element of a zerotree for threshold  $T$  is a *zerotree root* if it is not the descendant of a previously found zerotree root for the threshold  $T$ .

The zerotree is based on the hypothesis that if a wavelet coefficient at a coarse scale is insignificant with respect to a given threshold  $T$ , then all wavelet coefficients of the same orientation in the same spatial location at finer scales are likely to be insignificant with respect to  $T$ . In [2], Shapiro represented the EZW algorithm for image compression using the *zerotree* of wavelet coefficients.

### B. Embedded Zerotree Wavelet Coding [2]

EZW coding is based on discrete wavelet transform (DWT). The DWT decomposed the input image into several varying resolutions of subbands. EZW scans wavelet coefficients subband by subband. Parents are scanned before any of their children, but only after all neighboring parents have been scanned. There are two types of passes performed: 1) a dominant pass, 2) and a subordinate pass [2]. The dominant pass finds significant coefficients to a given threshold, and the subordinate pass refines all significant coefficients found in all previous dominant passes. Four symbols are used to tell a dominant pass to a decoder. Two symbols are used for a significant according to its sign—POS (*positive significant*) or NEG (*negative significant*). A ZTR (*zerotree root*) symbol is used to signify a coefficient below the threshold  $T$ , with all its children in the zerotree data structure are also insignificant. An IZ (*isolated zero*) symbol signs a coefficient is insignificant but has at least one significant child. The use of ZTR and IZ symbols is to inform locations of significant coefficients as efficiently as possible.

After a dominant pass, EZW further encodes coefficients using a successive approximation quantization (SAQ) scheme. Coding is done bit-plane by bit-plane. The successive approximation approach to quantization of the wavelet coefficient leads to the embedded nature of an EZW coded bitstream [8]. These two passes are entropy-coded with an adaptive arithmetic coder [6].

### C. Adaptive Zerotree Entropy (AZE) Coding

The proposed coding method, AZE coding, is an efficient technique for coding wavelet transform coefficient. Like EZW, this new AZE exploits the self-similarity inherent in the wavelet transform of images to predict the location of information across wavelet scales. AZE introduced a data structure called a *P-zerotree*, also built on the parent-child relationship. The *P-zerotree* structure takes advantage of the principle that if a wavelet coefficient  $x$  at a coarse scale is insignificant with respect to a given threshold  $T$ , then all wavelet coefficients of the same orientation at the same spatial location at finer wavelet scales are also likely to be insignificant with respect to  $T$  [2], [8]. In such situation,  $x$  is a *P-zerotree* root, and will be quantized to zero. AZE coding organizes quantized wavelet coefficients into wavelet trees and then uses *adaptive entropy coding* to reduce the number of bits required representing those trees. Subjectively, AZE differs from EZW in four major ways.

- 1) *P-zerotree model*: The *P-zerotree* model consists of a tree structure that crosses multiple resolutions of the wavelet-decomposed image. A *P-zerotree* structure data group is formed using units that have the same location inside each subband along the horizontal, vertical, and diagonal directions. Thus, each tree node has  $2 \times 2$  children, and each child has its own  $2 \times 2$  children, and so on until the highest frequency subbands are reached. If all of the elements inside a tree structure data group are tested to be insignificant, then the data group is labeled as the *P-zerotree*. The definition of *P-zerotree* is as follows:

$P_k$  is a *P-zerotree* root for a given threshold  $T$ , if

$$\bigwedge_{k \geq i \geq 2} (P_i < T), \text{ or}$$

$$\bigwedge_{k=2 \geq i \geq 1} (P_i < T),$$

where  $P_k$  is a parent of  $P_{k-1}$ ,  $k \geq 2$ .

*P-zerotree* model is based on one assumption: The symbol ZTR is used to signify a coefficient  $x_k$ ,  $k > 2$ , at a  $k$ -scale coarser subband below  $T$ , with all its children in the zerotree data structure, except the finest scale, also below  $T$ . The potential reason is that if  $x_k, x_{k-1}, \dots, x_2$  are all below  $T$ , the descendant  $x_1$  of  $x_k$  has a higher possibility below  $T$ .

- 2) Quantization at the coarsest subband is performed distinctly from the *P-zerotree* growing process, thereby making it possible to reduce image distortion.
- 3) EZW takes advantage of successive-approximation quantization

(SAQ) to perform the embedded coding. The SAQ sequentially applies a sequence of thresholds  $T_0, \dots, T_{n-1}$  to determine significance, where the thresholds are chosen so that  $T_i = T_i / 2$ . While in the proposed AZE coding, as will be seen in next section, the coding length of significant coefficients at a subband is adaptively dependent on the weight through significant coefficients at such scale.

- 4) Being the variation of coefficients among different scale, significant coefficients at different scale are further encoded independently for enhancing the image quality.

### III. Proposed Framework

Fig. 4 shows a block diagram of the encoding/decoding structure of the proposed framework. The corresponding image coding algorithms consist of two major stages: 1) a pyramid wavelet decomposition/reconstruction stage, and 2) a quantization/coding stage. The pyramid decomposition stage achieves efficient spatial decorrelation using *P-zerotree* data type, and usually concentrates most of the subjectively important information in the coarsest frequency subband.

In this section, a simple example will be used to illustrate the order of operations used in the AZE algorithm. Consider the simple 3-scale wavelet transform of an  $8 \times 8$  image. The array of value is shown in Fig. 5. The AZE coding steps are as follows:

#### A. *P-zerotree* scanning

- 1) We can choose our threshold  $T$ , to evaluate whether a coefficient is significant or insignificant. Let  $T = 32$ .
- 2) The threshold  $T$  is taken to generate *P-zerotree*. AZE coding reserves all coefficients at the coarsest subband,  $HL3$  in Fig. 5, to be significant.
- 3) AZE scans wavelet coefficients subband by subband. Parents are scanned before any of their children. In Fig. 5, coefficients at subband  $HL3$ ,  $-34$ , is first scanned, and its amplitude is greater than  $T$ , coefficient  $-34$  is significant. The rest of coefficients at subband  $HL3$  are then scanned and evaluated. After that, coefficient at  $LH3$ , i.e.,  $-31$  is evaluated to be smaller than  $T$ , and each child at  $LH2$  is also small than  $T$ . The target coefficient  $-31$  is replaced with '0' to signify *P-zerotree* root of its children at subband  $HL2$  and  $HL1$ . The scan result is

listed in Fig. 6.

- 4) Note that coefficient at HH3 and its children at HH2 are all small than  $T$ , thus coefficient at HH3 is replaced with '0' to be  $P$ -zerotree root. It indicates that its children at HH2 and HH1 are all replaced with '0' to signify insignificant.
- 5) After all of coefficients at 3-scale bands are scanned complete. AZE will scan non-zero coefficient part of 2-scale bands.
- 6) The magnitude 49 at subband HL2 is significant with respect to  $T$ .
- 7) The magnitude 10 at HL2 and all of its children at HL1 are lower than  $T$ , thus, magnitude 10 is replaced with '0' to signify  $P$ -zerotree root, and its children are replaced with '0' to signify insignificant with respect  $T$ .
- 8) The magnitudes (7, 13, 3, 4) at subband HL1 remains to be significant because its parent, i.e., 49, at HL2 is significant.
- 9) The scan results are listed in Fig. 6.

#### B. Construct significant/insignificant coefficient map

- 1) We can construct significant/insignificant map from the results of Fig. 6. The map is generated with the sequence of subbands at 3-scale, 2-scale, and 1-scale, i.e., HL3, LH3, HH3, HL2, LH2, ....
- 2) In Fig. 7, A significant coefficient is encoded using the symbol "01" (*negative significant*) or "10" (*positive significant*). Only  $P$ -zerotree root is encoded using the symbol "00".

#### C. Adaptive entropy coding

- 1) Next, adaptive coding is applied to significant coefficients. Significant coefficients at each scale are quantized to  $n$  intervals, where

$$B = \text{Round}((NC_l / TC_l) / (1/3)) + 3,$$

$$l = 1, 2, 3$$

$$n = 2^B - 1,$$

where  $B$  denotes representation bits for each significant coefficient,  $NC_l$  represents the number of significant coefficients at the  $l_{th}$ -scale image data, and  $TC_l$  represents the number of coefficients at the  $l_{th}$ -scale image data. The relationship between  $NC_l / TC_l$  and coding length for each coefficient is shown in Fig. 8.

- 2) Each significant coefficient  $Coef_i$  at the  $l_{th}$ -scale resolution is then encoded to  $Coef'_i$  as follows.

$$D_l = (MAX\_Coef_l - MIN\_Coef_l) / n_l,$$

$$Coef'_i = (Coef_i - MIN\_Coef_l) / D_l, l = 1, 2, 3$$

where  $MAX\_Coef_l / MIN\_Coef_l B$  denotes

the largest/smallest significant coefficient at the  $l$ -scale resolution. It is clearly that the value of  $Coef'_i$  is at the range of 0 and  $n_l$ .

## IV. Experimental Results

We have implemented an image coder using the adaptive  $P$ -zerotree entropy coding to generate actual compressed bit streams. We use in our experiments popular  $512 \times 512$  gray scale image. Our subband image coder employs a four-scale  $p$ -zerotree decomposition except the coarsest subband. We apply our adaptive entropy quantization to the significant coefficients to shrink down the variances among different scale subbands.

Fig. 9 includes the compressed Lena image at various rates. The bit rate is based on AZE coding. We compare the peak signal-to-noise ratio (PSNR)

$$\text{PSNR} = 10 \log_{10} \frac{255^2}{MSE},$$

where  $MSE$  is the mean square error for each compressed image. We can see that the reconstructed images offer good quality at a very low rate of 0.125 b/pixel.

Fig. 10 shows the results PSNR versus bit rate performance of proposed coder for Lena. We also summarize the PSNR results of AZE in Table I, for Lena.

## V. Conclusion

We have proposed a novel subband image coder that produces a fully embedded bit stream. By modeling the image subbands except the lowest one using the parametric  $P$ -zerotree decomposition model, we were able to provide a conceptual framework to develop the image coder as well as simple implementation.

In this work, adaptive entropy quantization is exploited to balance the coefficients' variation of each subband among different scale. We showed that a symbol stream is coded with less entropy using the  $P$ -zerotree relation. We used a modified zerotree data structure for subband decomposition and adaptive entropy coding but still achieved a very good image coder. The experimental results show that our AZE is competitive with other image coder in the literature. Experimentally, our AZE subband coder has 0.27~0.85 dB better performance than the EZW's.

#### ACKNOWLEDGMENT

The authors thank Mr. Y.-H. Huang for providing partial program codes for the

experiments and many fruitful discussions.

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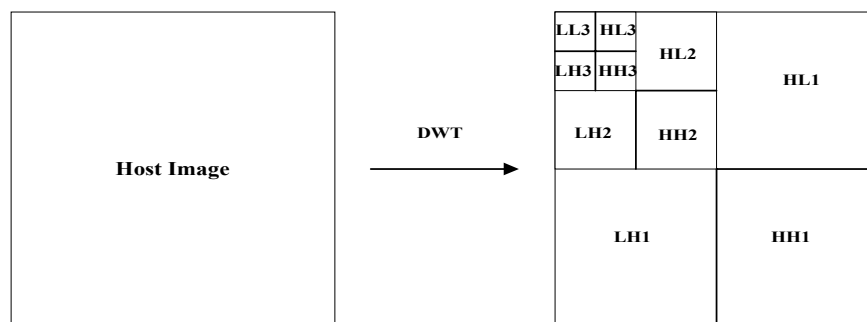


Fig. 1. DWT decomposition of an image: Note that the lowest frequency subband is the top left, and the highest frequency subband is at the bottom right.

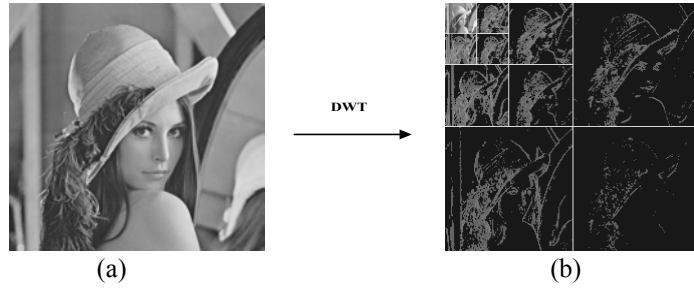


Fig. 2. (a) The original and (b) DWT decomposition of  $512 \times 512$  Lena image.

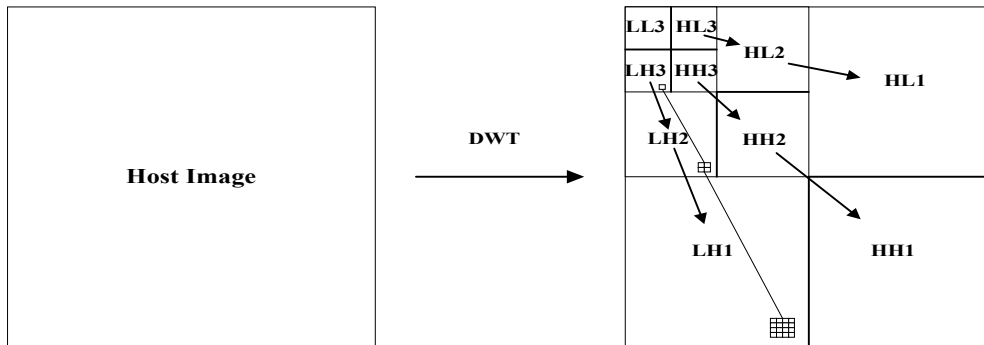


Fig. 3. DWT decomposition of an image: Note that the arrow points from the subband of the parents to the subband of the children. The lowest frequency subband is the top left, and the highest frequency subband is at the bottom right. Also shown is a wavelet tree consisting of all the descendants of a single coefficient in the subband LH3. The coefficient in LH3 is a zerotree root if it is insignificant and all of its descendants are insignificant.

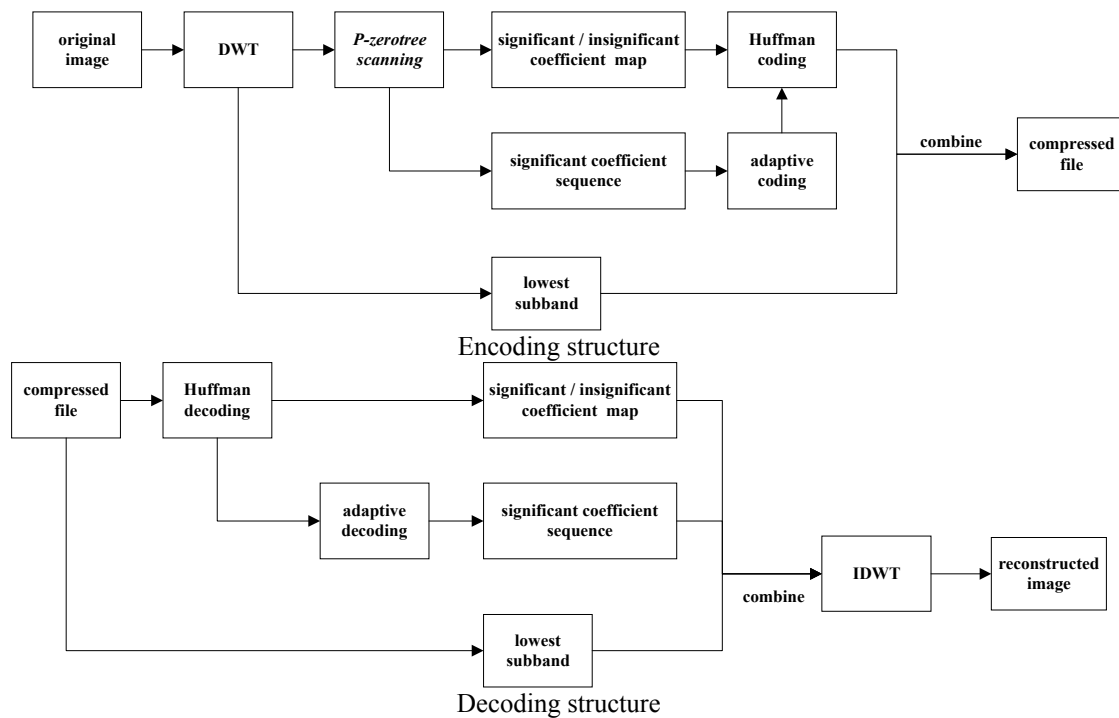


Fig. 4. Encoding/decoding structure of proposed framework.

63	-34	49	10	7	13	-12	7
-31	23	14	-13	3	4	6	-1
15	14	3	-12	5	-7	3	9
-9	-7	-14	8	4	-2	3	2
-5	9	-1	47	4	6	-2	2
3	0	-3	2	3	-2	0	4
2	-3	6	-4	3	6	3	6
5	11	5	6	0	3	-4	4

Fig. 5. Example of 3-scale wavelet transform of an  $8 \times 8$  image.

63	-34	49	0	7	13	0	0
0	0	0	0	3	4	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Fig. 6. Scanned results.

01	00	00	10	00	00	00	10	10	10	10
-34			49				7	13	3	4
HL <sub>3</sub>	LH <sub>3</sub>	HH <sub>3</sub>		HL <sub>2</sub>					HL <sub>1</sub>	

Fig. 7. Significant/insignificant coefficients map.

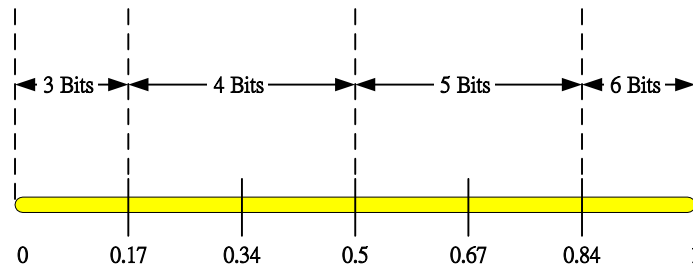


Fig. 8. The relationship between  $NC_l / TC_l$  and coding length for each coefficient at the  $l_{th}$ -scale image data.

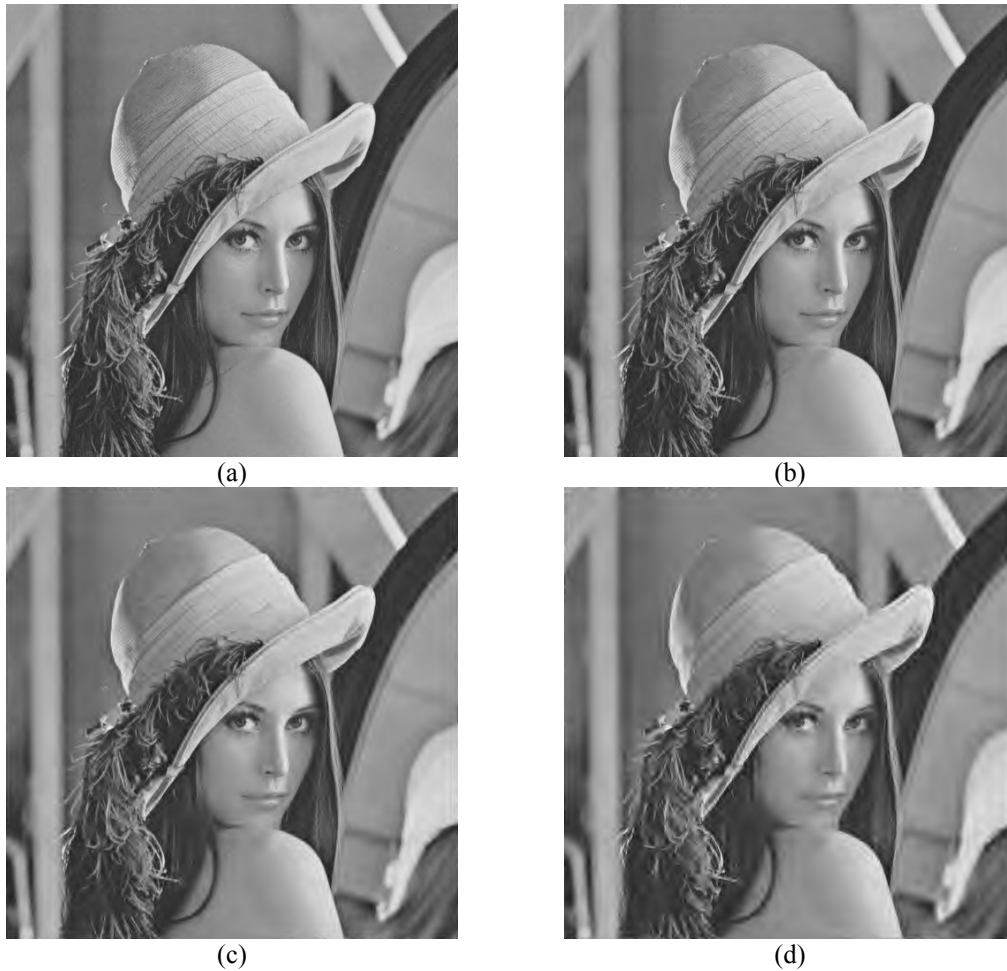


Fig. 9. Compressed Lena images using AZE. (a) Rate = 1.0 b/pixel (PSNR = 40.12). (b) Rate = 0.5 b/pixel (PSNR = 36.95). (c) Rate = 0.25 b/pixel (PSNR=34.02). (d) Rate = 0.125/pixel (PSNR = 30.50).

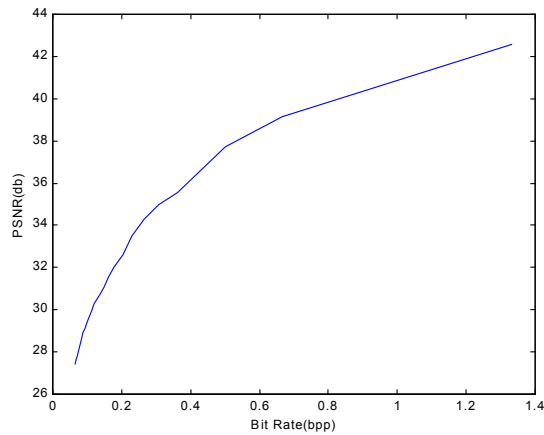


Fig. 10. PSNR versus bit rate performance for proposed coder, for Lena.

TABLE I  
CODING RESULT FOR  $512 \times 512$  LENA SHOWING PSNR (JPEG, EZW AND AZE)

Compress Ratio	Bit Rate(bpp)	JPEG	EZW	AZE
8	1.0	38.81	39.55	40.12
16	0.5	35.15	36.28	36.95
32	0.25	29.86	33.17	34.02
64	0.125	24.63	30.23	30.50