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RECURSIVE SELF-CORRECTION ALGORITHM AND APPLICATION TO DISCRETE SIGNAL PROCESSING

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ABSTRACT

A recursive self-correction algorithm is introduced to improve the calculation accuracy of discrete signal processing under failing the fulfilling of the required application conditions. This algorithm is applied to the discrete Fourier transform (DFT) and averaging algorithms of discrete periodic signals. The analytical and simulation results show the accuracy improvement for processing discrete periodic signals sampled non-synchronously. Exact DFT analysis and averaging for sinusoid signals are realized by the recursive self-correction algorithm under fulfilling the sampling theorem. For non-sinusoid periodic signals the processing accuracy is much higher than that of other algorithms. The principle can also be used for the accuracy improvement of some approximation algorithms.

1. INTRODUCTION

The well-known data processing algorithms, such as DFT and averaging algorithms, etc., are based on the application condition: $f_s= Nf$, where f_s and f are the sampling frequency and the signal frequency respectively, and N is the sampling number per signal period. If this condition is not fulfilled, see (7), remarkable calculation errors are caused to influence the data processing accuracy [1, 2, 3]. Furthermore the calculation accuracy of the most approximation algorithms is also dependent upon the calculation conditions. The failing of the fulfilling of the required conditions causes additional calculation error.

In this paper a general recursive self-correction algorithm is derived from the recursive Self-Correction DFT algorithm proposed in [2, 3, 4, 5] to solve the problem mentioned above. A recursive self-correction principle is applied to the accuracy improvement of the discrete signal processing and approximation algorithms under failing the required conditions. The calculation error is minimized by the recursive self-correction algorithm. This algorithm is applied to the discrete Fourier Transform and the averaging of discrete periodic signals.

In the following sections we introduce firstly the recursive self-correction algorithm and then give the recursive self-correction DFT and averaging algorithms of the discrete periodic signals as examples. The simulation results will show the advantages of these algorithms compared with other algorithms, such as standard and interpolated DFT [1, 6, 7, 8, 9] and averaging algorithms.

2. RECURSIVE SELF-CORRECTION ALGORITHM

In the recursive self-correction algorithm we assume that the input data and output data are $x_0(k)$ and $y_i(n)$ respectively, where the number of the data volume k and n are normally unequal. These data are functions in the time or frequency domain. Reference data $y_{ri}(n)$ for the self-correction are assigned by using the output data. The reconstruction data $x_i(k)$ are calculated by the corresponding signal model, for example, inverse DFT and regression etc. The index i denotes the iteration times of the recursive algorithm if $i \geq 1$. These data definitions are considered in the following algorithms.

2.1 Recursive Self-Correction Algorithm

The recursive self-correction algorithm is based on the self-calibration by using *internal reference data*, which are calculated by the *standard algorithm* with the use of the original input data $x_0(k)$ or the reconstruction data $x_i(k)$. This algorithm (Fig. 1) consists of four main parts, that is, 1) initialization, 2) self-correction unit, 3) error minimization unit, and 4) output, etc.

In the initialization the original output data $y_0(n)$ are calculated by the corresponding standard algorithm (S-A) by using the input data $x_0(k)$, where the index 0 denotes the original data. The original output data $y_0(n)$ serve for the initialization of the reference data $y_{r1}(n)$ for the first iteration and for the error correction of the output data $y_i(n)$ of the i-th iteration. In the case of not fulfilling of the application conditions the output data $y_0(n)$ deviate from the exact data so that a self-calibration method is needed to determine the deviation data and compensate them. The following steps contribute to the self-correction unit.

In the self-correction unit (Fig. 2) reference data $y_{ri}(n)$ are assigned by the output data $y_{i-1}(n)$:

$$y_{ri}(n) = y_{i-1}(n) \quad (1)$$

where $i=1$ denotes the assignment by the original output data $y_0(n)$ at the first iteration.

These reference data $y_{ri}(n)$ are transformed into reference reconstruction data $x_{ri}(k)$ according to the signal modeling. If reference output data $y_i^*(n)$ are calculated by the same standard algorithm (S-A) with the use of the reference reconstruction data $x_{ri}(k)$, the calculation deviation $\Delta y_i(n)$ of the standard algorithm can be derived from the difference between the reference output data $y_i^*(n)$ and the reference data $y_{ri}(n)$:

$$\Delta y_i(n) = y_i^*(n) - y_{ri}(n) \tag{2}$$

These data denote the calculation error of the standard algorithm. Therefore an error correction of the original output data $y_0(n)$ can be realized by the following operation:

$$y_i(n) = y_0(n) - \Delta y_i(n) \tag{3}$$

After the *i-th* error correction the output data $y_i(n)$ is more approximately to the exact output data than the original output data $y_0(n)$. The corrected output data are assigned as reference data for the following iteration if the calculation accuracy does not satisfy the requirements.

The reason for using the recursive algorithm is that the reference reconstruction data $x_{r1}(k)$ of the first iteration is unequal to the input data $x_0(k)$. In this case the deviation data $\Delta y_1(n)$ calculated by the standard algorithm at the first iteration is not the same as the calculation deviation of the original output data $y_0(n)$. Therefore, the reference reconstruction data $x_{ri}(k)$ should be approximated step by step to the original input data $x_0(k)$, and the deviation $\Delta y_i(n)$ is approached to the error of the original output data $y_0(n)$ by using a recursive self-correction algorithm.

This approximation, however, is nonlinear in some cases [2]. Thus an *Error-Minimization Unit* is introduced in this self-correction algorithm in order to optimize the calculation accuracy, that is, the optimal iteration times i_o.

In this unit the output data $y_i(n)$ are inversely transformed into the reconstruction data $x_i(k)$ by the corresponding signal model. The error between the reconstruction data $x_i(k)$ and the original input $x_0(k)$ can be determined by

$$e_i(k) = x_i(k) - x_o(k) \tag{4}$$

We define the *error band* E_i of these error data as the error evaluation of the *i-th* iteration:

$$E_i = \max\{e_i(k)\} - \min\{e_i(k)\} \tag{5}$$

At each iteration the error band E_i is checked. The iteration times are defined as the optimal iteration times i_o if the error is minimized. The iteration process is not ended until the evaluation error E_i is minimized by the self-correction algorithm. By means of this algorithm we can finally obtain the optimal output data $y_{io}(n)$.

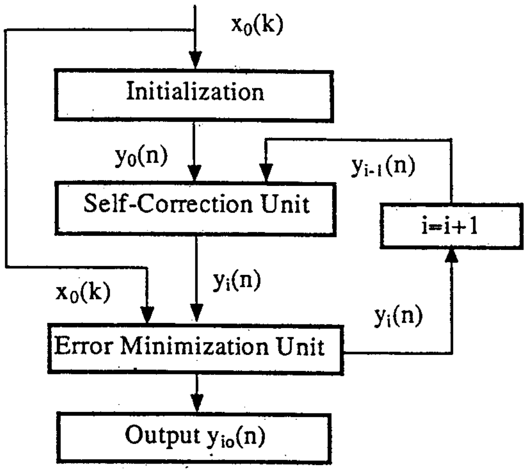


Fig. 1 Recursive Self-Correction Algorithm

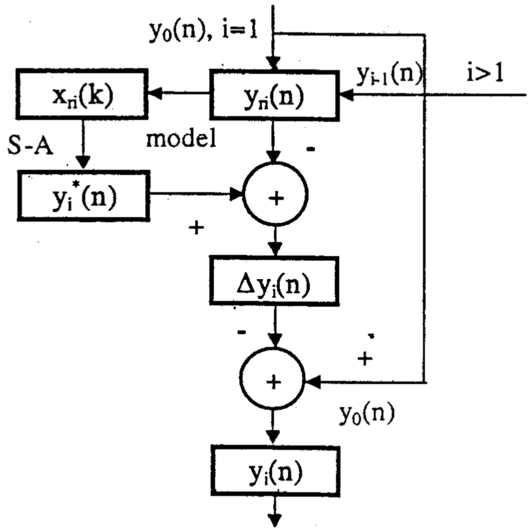


Fig. 2 Self-correction unit

In the following this recursive self-correction algorithm will be illustrated by the example algorithms for the averaging and DFT of discrete periodic signals.

2.2 Recursive Self-Correction Averaging (SC-AVE)

Fig. 3 shows the recursive self-correction algorithm for the averaging of discrete periodic signals. This algorithm is based on the *standard averaging (S-AVE) algorithm*, which is realized by

$$y(n) = \frac{1}{M}\sum_{j=1}^{M} x[n + N(j-1)] \tag{6}$$

where $x(k)$ is discrete periodic signal in the time domain, $y(n)$ is the averaging data in one period, and N and M denotes the number of the output data per signal period and the averaging times respectively. The condition for the self-correction algorithm is written by

$$f_s = (N + \alpha) f \qquad (7)$$

where f and f_s are the frequency of the periodic signal and the sampling frequency of the input data, and α is defined as a deviation factor with normally $|\alpha|<0.5$. An exact calculation of the averaging output data is realizable only at $\alpha=0$ by using (6) [10]. The calculation error for $\alpha\neq0$ must be reduced by the recursive self-correction algorithm.

The original averaging data $y_0(n)$ are calculated by the S-AVE algorithm with the use of the input data $x_0(k)$ in the initialization and assigned as reference data $y_{r1}(n)$ at the first iteration in the self-correction unit. The reference data $y_{r1}(n)$ are transformed to reference reconstruction data $x_{r1}(k)$ by signal model. The signal model is realized by the recursive self-correction DFT algorithm [2, 3, 4, 5] for sinusoid signals and by a linear regression algorithm for triangle and ramp signals.

After calculating the reference averaging data $y_1^*(n)$ by the S-AVE algorithm with the use of the reference reconstruction data $x_{r1}(k)$, the calculation errors of the S-AVE algorithm can be determined by (2). The error correction of the original averaging data is made by (3). The corrected averaging data $y_1(n)$ are transformed into reconstruction data $x_1(k)$. The error evaluation E_1 can be calculated by (5) according to the deviation data $e_1(k)$ between the reconstruction data $x_1(k)$ and the input data $x_0(k)$. It is checked whether the error evaluation parameter E_1 is minimized or not. The corrected output data $y_1(n)$ are assigned as reference data $y_{r2}(n)$ for the following iteration if the error is not minimized. The iteration calculation is not ended until the error evaluation E_i is minimized for a given maximal iteration number. Thus an error minimization is realizable by using the recursive self-correction algorithm.

The error minimization procedure of this averaging algorithm is normally linear convergent for sinusoid signals so that the optimal accuracy is easily realizable under the convergence condition.

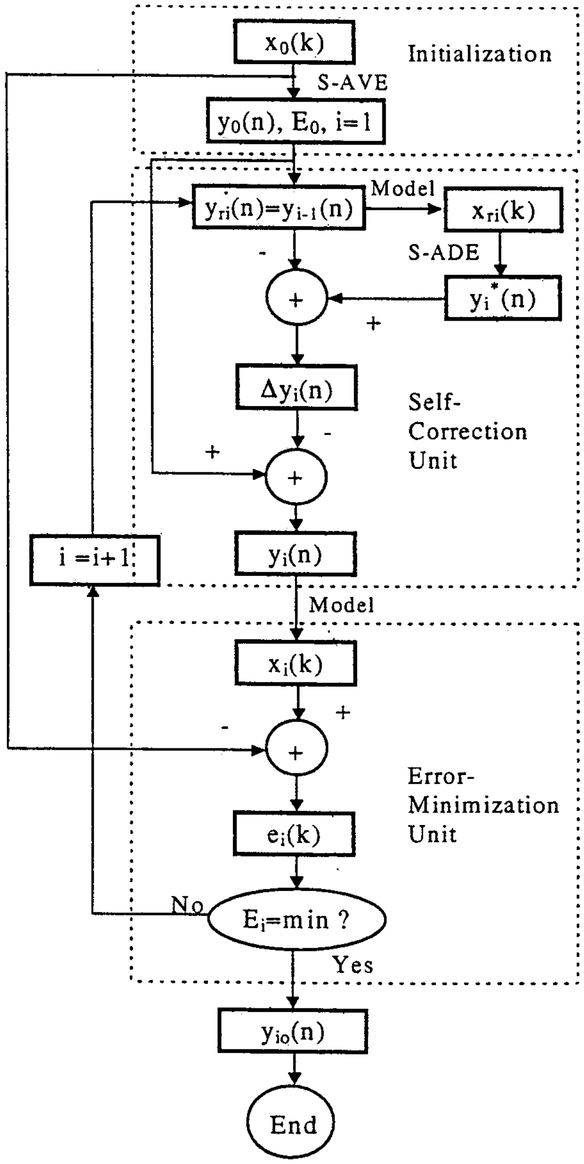


Fig. 3 Recursive self-correction algorithm for averaging of discrete signals

2.3 Recursive Self-Correction DFT (SC-DFT)

Fig. 4 shows the recursive self-correction algorithm for the discrete Fourier Transform. The standard DFT (S-DFT) algorithm can be written by the following periodic complex functions [2]:

$$S(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{-j2\pi \frac{kn}{N}} \qquad (8)$$

and

$$x(k) = \sum_{n=0}^{P} S(n) e^{j2\pi \frac{kn}{N}} \qquad (9)$$

where $x(k)$ denotes the input data or reconstruction data in the time domain, $S(n)$ is a corresponding frequency function and connotes the spectrum of the *n-th* harmonic, N is the sampling number per signal period and P is the highest order of the harmonics of the frequency function. According to the sampling theorem the condition: $P<N/2$ should be fulfilled. Equation (8) is known as the *Discrete Fourier Transform* (DFT) and (9) the *Inverse Discrete Fourier Transform* (IDFT).

The frequency function $S(n)$ calculated by the S-DFT under the condition (7) with $\alpha \neq 0$ are not exact [1, 2, 3, 4, 5]. Thus the recursive self-correction algorithm has to be used to improve the calculation accuracy in this case.

In this algorithm (Fig. 4) the input data $x_0(k)$ and the reconstruction data $x_i(k)$ calculated by the IDFT are time functions while the output data $S_i(n)$ and reference data $S_{ri}(n)$ are frequency functions. The principle here is the same as the recursive self-correction averaging algorithm, see also [2, 3, 4, 5]. The approximation procedure of the recursive self-correction algorithm, however, is normally nonlinear convergent [2]. Therefore a special algorithm should be used for the error minimization. The convergence condition here depends on the error minimization algorithm.

In the following section we will take a closer look at the comparison between different algorithms.

3. SIMULATION RESULTS

In order to make a comparison between the recursive self-correction algorithm and the standard algorithm as well as interpolation-algorithm [1, 6, 7, 8, 9], we generate two typical signal wave-forms: sinusoid and triangle signals for the simulation. The *relative error* E_r is used for the evaluation and is defined as [2]

$$E_r = \frac{E_{io}}{x_{\max} - x_{\min}} \qquad (10)$$

where E_{io} denotes the minimal error band of the reconstruction data $x_i(k)$ determined by (5), and x_{max} and x_{min} denote the maximum and minimum values of the input data $x_0(k)$. Analytical results prove that the relative error is dependent on 1) the iteration number, 2) the deviation factor α, and 3) the frequency ratio $R=f_s/f$.

Fig. 5 shows the approximation procedure of the recursive self-correction averaging algorithm for sinusoid signals.

The averaging is carried out under the conditions: $N=21$, $\alpha=-0.5$ and $M=7$. The relative error is defined as the absolute reconstruction error $e_i(k)$ (4) divided by the maximum x_{max} of the input data. From Fig. 5 we find that the original error (64.9%) of the standard averaging (S-AVE) is reduced by the self-correction averaging (SC-AVE) with the increase of the iteration number. After the 20th iteration SC-AVE(20) the relative error is reduced to less than 0.01%. A calculation error of less than 0.0001% is realizable by using a recursive self-correction alogrithm with 30 iterations.

Fig. 6 shows the simulation results of three averaging algorithms using a sinusoid signal. From Fig. 6(a) we find that the relative errors of the S-AVE and I-AVE algorithm increase with the absolute deviation factor $|\alpha|$. The error of the I-AVE algorithm is less than that of the S-AVE. The relative error of the SC-AVE, however, is reduced to be negligible and nearly independent on the deviation factor. Fig. 6(b) shows the relation between the relative error and the frequency ratio $R_f=f_s/f$. Similarly, the relative errors of the S-AVE and I-AVE algorithms

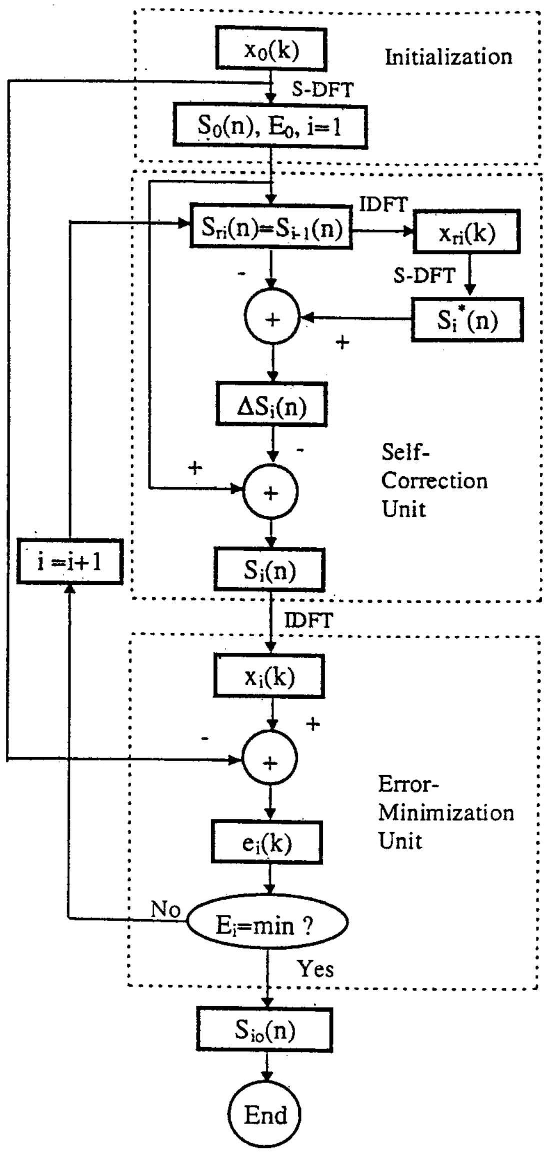


Fig. 4 Recursive self-correction algorithm for the discrete Fourier Transform

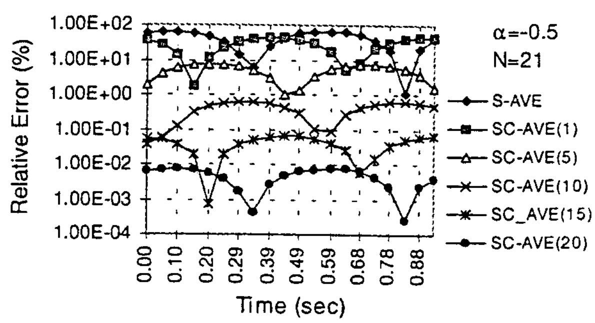
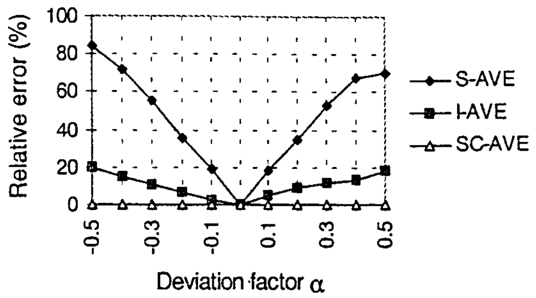
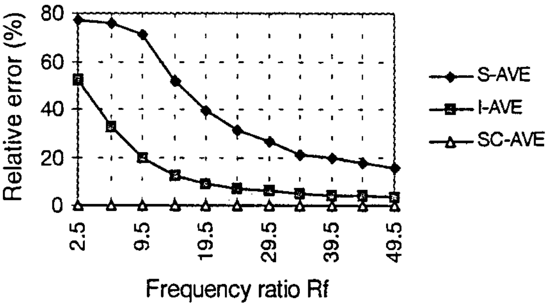


Fig 5 Approximation procedure of the recursive SC-averaging algorithm for sinusoid signal

increase with the reduce of the frequency ratio, while the error of the SC-AVE is nearly independent on the frequency ratio and is negligible in the range of $R_f \geq 2.5$.



(a) The relation between the relative error and the deviation factor α (calculation condition: N=10, M=7)

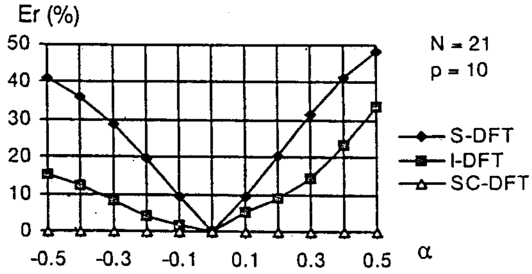


(b) The relation between the relative error and the frequency ratio R_f (calculation condition: M=7, α=0.5)

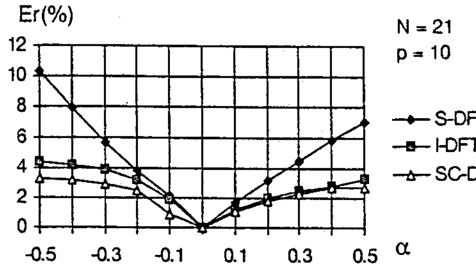
Fig. 6 Simulation results of three averaging algorithms using a sinusoid signal (S-AVE: standard averaging, I-AVE: interpolation averaging, SC-AVE: Self-correction averaging)

Fig. 7 shows the relative errors of the DFT algorithms on the deviation factor α for sinusoid series and triangle signal wave-forms respectively, with N=21 as the number of sampling points and P=10 as the highest series order. For all the three DFT algorithms the relative errors increase with the growth of |α|. The errors are greatly influenced by a large |α| for both sinusoid series and triangle signal wave-forms, which are analyzed with the S-DFT and the I-DFT. With the help of the SC-DFT the error is reduced to less than 3% for the triangle signal. For the sinusoid series the error approaches to be negligible, too.

Fig. 8 shows the errors as the function of the frequency ratio R_f under the condition: α=-0.5 and P=10. The Reference in Fig. 8(b) means the reconstruction error calculated by the S-DFT under the condition α = 0 due to the *spectral leakage*. We can find that the errors decrease with the increase of the frequency ratio R_f for all three DFTs. Similarly, the calculation error of the SC-DFT is negligible for calculating sinusoid series and is less than those of the other two DFTs for calculating triangle signal. The reconstruction error for higher frequency ratio

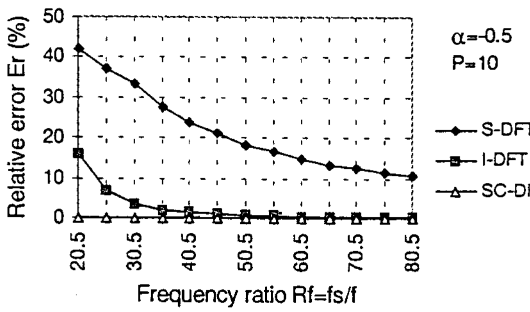


(a) Sinusoid series

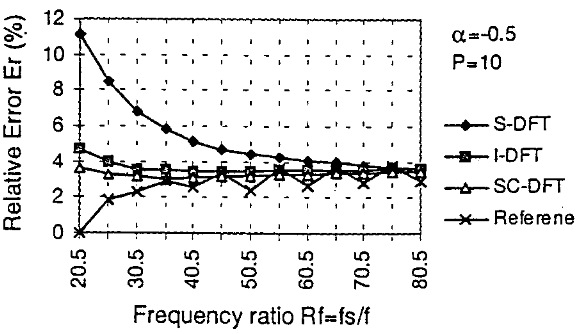


(b) Triangle

Fig. 7 The relation between the relative error and the deviation factor α by using DFTs (S-DFT: Standard DFT, I-DFT: Interpolation DFT, SC-DFT: Self-correction DFT)



(a) Sinusoid series



(b) Triangle

Fig. 8 The relation between the relative error and the frequency ratio R_f by using DFTs (S-DFT: Standard DFT, I-DFT: Interpolation DFT, SC-DFT: Self-correction DFT) results from the leakage (loss) of the higher harmonics because the highest order is fixed at $P=10$.

Another important advantage of the SC-DFT is that the error is nearly constant in the whole frequency range determined by the *sampling theorem.* Thus a minimal sampling frequency can be used for data acquisition in sciences and engineering. The SC-DFT algorithm is used for the measurement of electrical quantities [3, 4, 5] and contributes to the accuracy improvement.

4. CONCLUSIONS

The recursive self-correction algorithm is examined by the simulation of the averaging and DFT of periodic signals. From the analytical and simulation results we can draw the following conclusions:

- The recursive self-correction algorithm contributes to the accuracy improvement for processing discrete signals sampled non-synchronously. This algorithm is based on the corresponding standard algorithm. The calculation error of the standard algorithm is self-corrected by the recursive self-calibration and error minimization units. This algorithm is easily applied to discrete signal processing.
- Using the example algorithm SC-AVE an precise averaging calculation is realized for sinusoid signals without the limitation of the calculation conditions. The convergence condition for sinusoid signals is easily fulfilled for the practical uses.
- The amplitude and phase as well as reconstruction data of sinusoid signals can exactly be calculated by the example algorithm SC-DFT. For other periodic signals the calculation accuracy of the SC-DFT is much higher than that of the S-DFT and I-DFT, especially for discrete data processing in the case of lower frequency ratio.
- The error of the SC-DFT is dependent on the deviation factor α and nearly independent upon the frequency ratio R_f. Therefore, The SC-DFT can be applied to data processing under the minimal frequency ratio determined by the sampling theorem. This algorithm is very suitable for data processing of higher frequency signals.
- The application of the recursive self-correction algorithms enables measuring and data processing systems to simplify the system structure and to improve the calculation accuracy.

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