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Application of Optimal Boolean Filter in Document Image Processing

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Abstract

In this paper, the problem of designing optimal Boolean filter under the mean absolute error (MAE) criterion is solved completely. The applications of the optimal Boolean filter have been successively extended to document image enhancement and extraction of text characters from overlapping text/background image. The performance and operation theory of optimal Boolean filter against other competitive techniques are compared. The feasibility of training Boolean filter is also confirmed by experimental results in case that the original image is not available.

1. Introduction

The performance of nonlinear filter in image processing has been extensively studied for decades. However, the design of optimal nonlinear filter has not been well defined until the problem of optimal stack filter initialized by Coyle et al. Coyle et al. [1] introduced the class of stack filters and found the connection between the stack filter and positive Boolean function. In [2], Coyle and Lin defined the optimal stack filter under the mean absolute error criterion and the problem of designing optimal stack filter was transformed to that of finding optimal positive Boolean function. Since then many different approaches have been proposed to the finding of optimal stack filter [3][4][5][6]. Most of them belong to the method of training approach since the problem of finding optimal stack filter is considered as a Linear Programming problem.

An extension of stack filter called Boolean filter was introduced by [7]. The procedures to design optimal Boolean filter has been discussed by Tabus et al. [8]. In this paper, we will concentrate on the designing and extending the application of Boolean filters in image processing. The problem of designing optimal Boolean filter is defined under the MAE criterion. In our approach, the statistical measurement between the observed image and desired image is the dominated computational task. The MAE of Boolean filter is represented in terms of the error incurred by the input vectors. In this way, the optimal Boolean filter can be obtained immediately.

Usually, we are given a corrupted image to be enhanced and the original image is not available. Hence, it is impossible to find the optimal filter for this corrupted image. However, it is possible to find an optimal filter of a corrupted image from another given set of images assuming that the noise can be regenerated and is irrelevant to the content of images. The given set of images includes an original image and its corrupted version. This is the concept of training filter being considered as the practical usefulness of optimal filter theory. The feasibility of training filter is also investigated throughout the experiments.

The rest of this paper is organized as follows. In section 2, we will briefly review the definition of Boolean filter. In section 3, the problem of optimal Boolean filter is studied and the relationship of the MAE of a Boolean filter and the cost function of input vectors is derived. The applications of Boolean filter in the area of image processing including document image enhancement and text extraction from overlapping text/background image will be presented in section 4. Real images are tested to evaluate the performance of the Boolean filter. Finally, conclusions are given in section 5.

2. Boolean Filter

The Boolean filter is defined on Boolean function possessing the threshold decomposition property. Let $BF_f(.)$ denote a Boolean filter specified by Boolean function $f(\cdot)$, X be the input gray scale image, T be the thresholding function, and $T_k(X)$ be the thresholded binary image of X thresholded at gray level k. The threshold decomposition property of Boolean filter can be expressed as:

$$BF_f(X) = \sum_{k=1}^{M} f(T_k(X))$$

Owing to the threshold decomposition property, the design, analysis and realization of Boolean filter can then be reduced to binary domain.

In our work, the geometrical representation of Boolean function is adopted. According to the True and False entries of truth table, the 2^n input vectors of an n-variable Boolean function can be classified into two subclasses, one contains the input vectors corresponding to the True entries of truth table which is called on-set, another is called off-set which contains the input vectors corresponding to the False entries of truth table. The Boolean function can be completely specified by the on-set [9] and so does the Boolean filter.

3. Optimal Boolean Filter Under MAE Criterion

In this section, the problem of optimal Boolean filter is revisited and the mean absolute error (MAE) is adopted as the error criterion in determining the optimal Boolean filter. The MAE of a Boolean filter can then be represented in terms of the total error incurred by the input vectors of the on-set. In consequence, the optimal Boolean filter can be found immediately. Furthermore, the relationship between the Boolean filter, stack filter and WOS filter can be revealed based on the structure of the on-set. The optimal design complexities of these filters are also analyzed in this section.

For a Boolean filter BF_f where f is the Boolean function defining the Boolean algebra of the Boolean filter, the mean absolute error (MAE) of the Boolean filter is defined as the mean absolute error between the desired image Z and the output of Boolean filter with the observed image X serving as the input.

$$MAE(BF_f) = E[|Z - BF_f(X)|] \qquad (3.1)$$

where $E[\cdot]$ is the expectation operator.

The optimization problem can be stated as the finding of a Boolean filter which minimizes Equation (3.1). According to the threshold decomposition property, the MAE of a Boolean filter can be reduced to the sum of the decision errors made by the Boolean filters on each level of thresholded binary images. That is,

$$MAE(BF_f) = E[|Z - BF_f(X)|]$$
$$= E[|\sum_{k=1}^{M} T_k(Z) - \sum_{k=1}^{M} f(T_k(X))|] = \sum_{k=1}^{M} E[|T_k(Z) - f(T_k(X))|]$$

The MAE can be further represented as the decision errors incurred by the input vectors. Now, let us define a cost function, $cost(\underline{x})$, as the decision error incurred by $f(\underline{x})$ for deciding a 1 when seeing input vector $\underline{x}$.

$$cost(\underline{x}) = C(0, \underline{x}) - C(1, \underline{x})$$

where $C(0, \underline{x})$ is estimated by the number of occurrences of window vector $\underline{x}$ which appears in the observed image when the desired output of this vector is 0, likewise, $C(1, \underline{x})$ is estimated by the number of occurrences of window vector $\underline{x}$ which appears in the observed image when the desired output of this vector is 1. If the window vector $\underline{x}$ is in the on-set of the Boolean filter, then $C(0, \underline{x})$ represents the total decision error of the filter made by $\underline{x}$. Similarly, if the window vector $\underline{x}$ is in the off-set of the Boolean filter, then $C(1, \underline{x})$ represents the total decision error of the filter made by $\underline{x}$. By definition, the MAE of a Boolean filter can be expressed as:

$$MAE(BF_f) = E\{\sum_{k=1}^{M}[\sum_{\underline{x}\in on(f)} C(0, \underline{x}) + \sum_{\underline{x}\in off(f)} C(1, \underline{x})]\}$$
$$= E\{\sum_{k=1}^{M}[\sum_{\underline{x}\in on(f)} C(0, \underline{x}) + \sum_{\underline{x}\in off(f)} C(1, \underline{x}) + \sum_{\underline{x}\in on(f)} C(1, \underline{x}) - \sum_{\underline{x}\in on(f)} C(1, \underline{x})]\}$$
$$= E\{\sum_{k=1}^{M}\{\sum_{\forall \underline{x}} C(1, \underline{x}) + \sum_{\underline{x}\in on(f)}[C(0, \underline{x}) - C(1, \underline{x})]\}\}$$
$$= E\{\sum_{k=1}^{M}[\sum_{\forall \underline{x}} C(1, \underline{x}) + \sum_{\underline{x}\in on(f)} cost(\underline{x})]\} \qquad (3.2)$$

where $\sum_{\forall \underline{x}} C(1, \underline{x})$ is a constant for a constant window size. Hence, the MAE of a Boolean filter is equal to the summation of a constant and the total cost of all input vectors in the on-set of the defining Boolean function. It is evident that the optimal Boolean filter which minimizes Equation (3.2) is specified by the Boolean function whose on-set is composed of all the negative cost input vectors. Hence, the optimal Boolean filter can be obtained immediately after the computation of cost function based on Equation (3.2).

4. Experiment Results

In this experiment, the optimal Boolean filter is applied to the applications of document enhancement and text extraction from overlapping text/background image. The window size of the optimal Boolean filter is 3×3. In the finding of optimal Boolean filter, the cost parameters of the 512 input vectors are computed from the original image and its corrupted version. All input vectors with negative cost can then specify the optimal Boolean filter. In this approach, the original image is a prerequisite in finding the optimal Boolean filter. However, it is impractical in real applications.

Hence, there is a question arisen: What to do if the original image is not available?

The concept of training stack filter in [10] is adopted in this experiment. The training stack filter is an image enhancement method that obtains an optimal stack filter from an image and its corrupted version and then applies this training stack filter to another corrupted image to suppress the noise.

4.1 Document image enhancement

Suppose that we get a noisy document image corrupted by impulsive noise during transmission. In this case, the noise cannot be eliminated by thresholding because the gray level of noise is the same as those of the text characters. Experimental results show that the optimal Boolean filter is efficient in eliminating the impulsive noise of document images. Let us compare the performance of optimal Boolean filter with that of median filter and a heuristic algorithm which enhances document images by deleting the isolated points. Fig. 4.1(a) is the original 256×256 document image. Fig. 4.1(b) is the testing noisy document image corrupted by adding 10% impulsive noise, Fig. 4.1(c) is the enhanced image generated by 3×3 median filter, Fig. 4.1(d) is the enhanced image generated by deleting isolated points. Fig. 4.1(e) is the enhanced image generated by 3×3 optimal Boolean filter. The results show that the performance of optimal Boolean filter is the best one. In addition, a more clear filtered image can be obtained as we applying filtering twice. Applying optimal Boolean filtering on image Fig. 4.1 (e) generates the image Fig. 4.1 (f) which shows a reduction on the impulsive noise. Besides, it is interesting to note that the median filter has very good performance in eliminating the impulsive noise of gray scaled images. However, it does not work well for the impulsive noise embedded in document images. The median filter eliminates not only the noise but also the text characters as shown in Fig. 4.1(c).

The training Boolean filter is also effective in this case. We apply the optimal Boolean filter trained by Fig. 4.1(a) and Fig. 4.1(b) to the noisy document image Fig. 4.2(a). Fig. 4.2(b) is the result generated by the training optimal Boolean filter which demonstrates the capability of the filter in noise removal while preserving text characters.

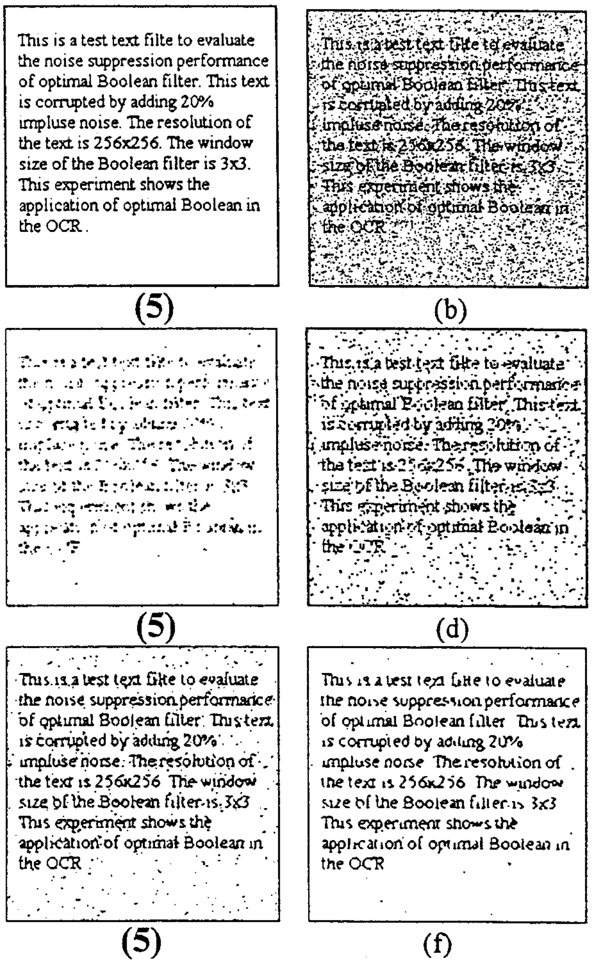


Fig. 4.1. Comparison of the performance of document image enhancement between different approaches. (a) The original document image; (b) the testing noisy document image corrupted by adding 10% impulsive noise; (c) the enhanced image generated by 3×3 median filter; (d) the resulting image generated by deleting isolated points; (e) the enhanced image generated by 3×3 optimal Boolean filter; (f) the enhanced image generated by applying 3×3 optimal Boolean filter twice.

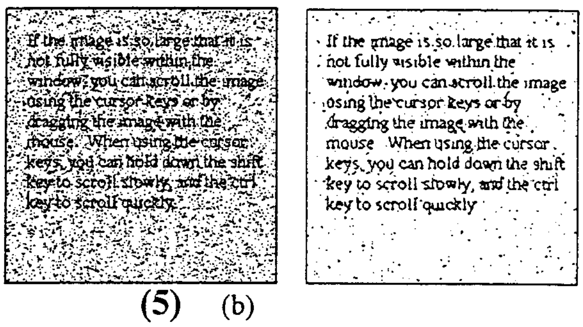


Fig. 4.2. The experimental result of training Boolean filter. (a) the testing noisy document image ; (b) the enhanced image generated by 3×3 training Boolean filter trained from Fig. 4.1(a).

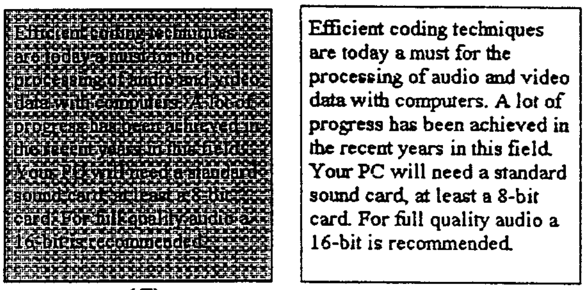
4.2 Text extraction from overlapping text/background image

Occasionally, texts will be printed over uniformly distributed graphical background to beautify the layout of articles or attract the attention of readers. Though it is not difficult for humans to read these characters, the extraction of the characters from overlapping background image is still a prerequisite to facilitate the process of Optical Character Recognition (OCR). A morphological approach [11] that based on mathematical morphology has been successively used to solve this kind of problem. We found that optimal Boolean filter is also effective in separating graphic background and text characters. A testing document image with the background composed of uniformly dots as shown in Fig. 4.3(a). Fig. 4.3(b) shows that the optimal Boolean filter can extract the text characters completely. Besides, the training Boolean filter is also feasible in performing this task. The optimal Boolean filter is also efficient in extracting the text when the background is made up by periodically patterns as is shown in Fig. 4.5. Fig. 4.5(b) is the output image generated by the training Boolean filter of Fig. 4.3. Fig. 4.6(b) is the output image generated by the training Boolean filter of Fig. 4.4.

Comparing to the approach of morphology, the advantages of the approach of optimal Boolean filter are:

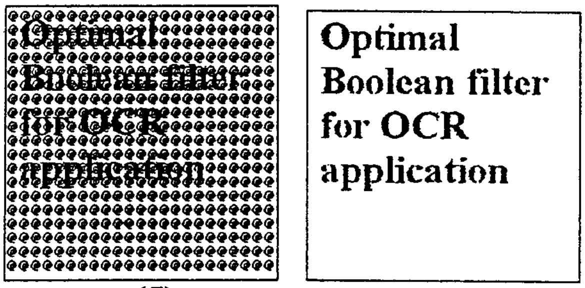
1. The design method of optimal Boolean filter is simpler and the extraction process is faster.
2. The extraction of text characters outperforms the approach of morphology. Take Fig. 4.5(a) as an example, the result generated by the morphological approach contains more noise on the characters as shown in Fig. 4.7(a). Another example as shown in Fig.4.7(c) is the result of text extraction by using morphological approach to extract the overlapped image of Fig. 4.7(b).

The common property of the two approaches is that they have to detect the pixel distribution of the background graphic symbol. However, the detecting algorithm is different. The optimal Boolean filter detects the background distribution by learning from an overlapping text/background image and the same text image without background symbols. The optimal Boolean filter is represented by an on-set which is like a look-up table deciding what kind of input vectors should be "1" and what kind of input vectors should be "0". Therefore, the text extraction performance of optimal Boolean filter should not be affected by the defects in the periodicity of background symbols. The results as shown in Fig. 4.8 verify this opinion. On the contrary, the morphological approach detects the background distribution by computing the distribution frequency of the background graphic symbols. The regularity and periodicity of distributed background symbols are prerequisites for the morphological approach. The text extraction performance of the morphological approach, as shown in Fig. 4.9, will be degraded by the defects in the periodicity of background symbols.



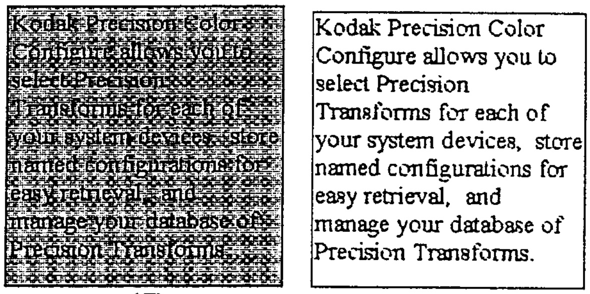
(5) (b)

Fig.4.3. The result of optimal Boolean filter in the extraction of text characters from overlapping text/background image. (a) a testing document image with overlapping text/background; (b) the output image generated by the optimal Boolean filter.



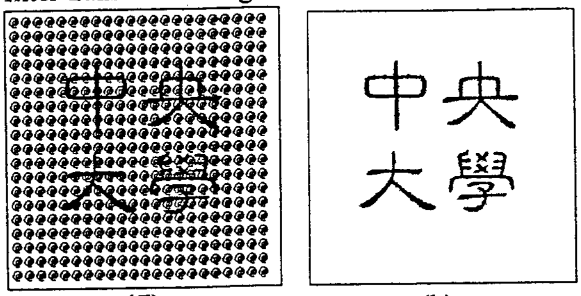
(5) (b)

Fig.4.4. The result of optimal Boolean filter in the extraction of text characters from overlapping text/background image. (a) a testing document image with overlapping text/background; (b) the output image generated by the optimal Boolean filter.



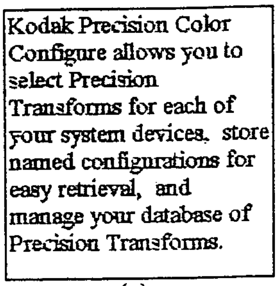
(5) (b)

Fig.4.5. The result of training Boolean filter in the extraction of text characters from overlapping text/background image. (a) a testing document image with overlapping text/background; (b) the output image generated by the training Boolean filter trained from Fig. 4.3.

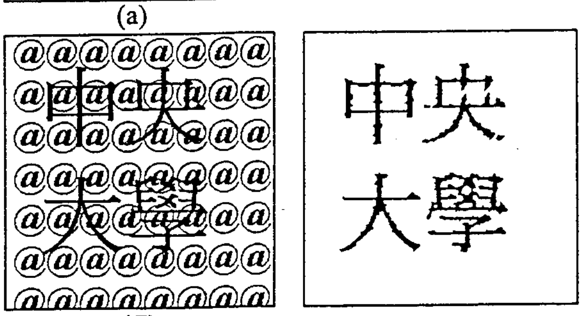


(5) (b)

Fig.4.6. The result of training Boolean filter in the extraction of text characters from overlapping text/background image. (a) a testing document image with overlapping text/background; (b) the output image generated by the training Boolean filter trained from Fig. 4.4

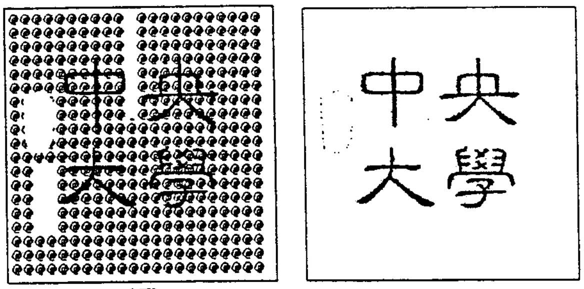


(a)



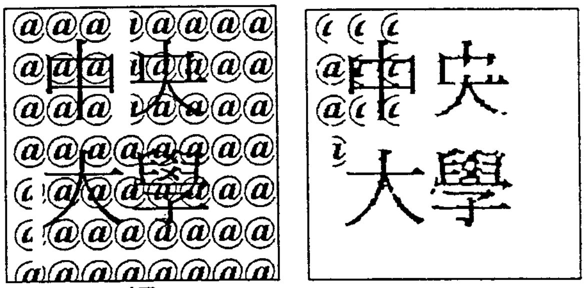
(5) (c)

Fig. 4.7. The result of text extraction by morphological approach (a) The result of text extraction for Fig. 4.5(a).; (b) an overlapping text/background image (c) The result of text extraction by morphological approach.



(5) (b)

Fig.4.8. The text extraction performance of optimal Boolean filter appraoch that is not affected by the defects in the periodicity of background symbols. (a) a testing document image with defects in background symbols; (b) the output image generated by the optimal Boolean filter approach.



(5) (b)

Fig. 4.9. The text extraction performance of morphological appraoch which will be degraded by the defects in the periodicity of background symbols. (a) a testing document image with defects in background symbols; (b) The result of text extraction by morphological approach.

5. Conclusions

In this paper, the design and applications of optimal Boolean filter is discussed. The good performance of Boolean filter demonstrated in this paper confirms the potential of Boolean filter in image processing. One observation from experimental results reveals the fact that the optimal Boolean filter is effective and robust in suppressing the impulsive signal no matter whether the impulsive signal is the noise being suppressed or is the background symbol being removed. Especially, the experimental results of text character extraction from overlapping text/background image reminds us that Boolean filter may be suitable for the image segmentation and texture analysis, which becomes our further work in the study of optimal Boolean filter.

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