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MULTI-SCALE EDGE DETECTION BASED ON SPIRAL ARCHITECTURE

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ABSTRACT

Edge detection in computer vision and image processing is a process which detects one kind of significant feature in an image which is a resultant of large delta values in intensities. In this paper, we set up a numerical method for edge detection using a multi-resolution technique based on Gaussian Filters with Spiral Architecture. The detection algorithm reduces noise and unnecesary detail of the image from a coarse level to a fine level of resolution.

Keywords. Scale-space Theory, Image Processing, Edge Detection, Spiral Architecture.

1 INTRODUCTION

Edge detection plays a key role in computer vision, image processing and related areas. It is a process which detects the significant features that appear as large delta values in light intensities. As an early stage of computation in a large scale computer vision application, an edge map is determined from the original image. It contains major image information and only needs a relatively small amount of memory space for storage. If needed, a replica image can be reconstructed from its edge map.

In the past, various edge detection algorithms were proposed (e.g. [1], [2], [3] and [4]). In this paper, we present a numerical method for edge detection based on Gaussian Multi-scale Theory [5] and Spiral Architecture [6].

Spiral Architecture described by Sheridan [6] is a new data structure for computer vision. The image is represented by a collection of hexagons of the same size (in contrast with the traditional rectangular representation). The importance of the hexagonal representation is that it possesses special computational features that are pertinent to the vision process.

Muiti-scale theory introduced by Lindeberg [5] is a tool to remove image noise. The image brightness function is parameterized. A large change in image brightness over a short spatial distance indicates the presence of an edge. The image is blurred and noise is removed when the parameter is positive. We start from the edge map at a coarse resolution level (i.e., choose a positive parameter value). The final edge map will be approached from the coarse level to a fine level (i.e., as the parameter gets smaller and smaller).

The context of this paper is arranged as follows. The Spiral Architecture is introduced in Section 2 followed by the Multi-Scale Theory for edge detection in Section 3. We construct an edge detection algorithm in Section 4. A discussion about some of the properties of the approach is given in Section 5.

2 SPIRAL ARCHITECTURE

An image may be considered as the collection of pixels (picture elements). These elements correspond to the position of the photo receiving cells of the image capturing device. In the case of the human eye, these elements would represent the relative position of the rods and cones on the retina. The geometric arrangement of cones on the primate's retina can be described in terms of a hexagonal grid. This leads to the consideration of an image as the collection of hexagonal cells as displayed in Fig. 1.

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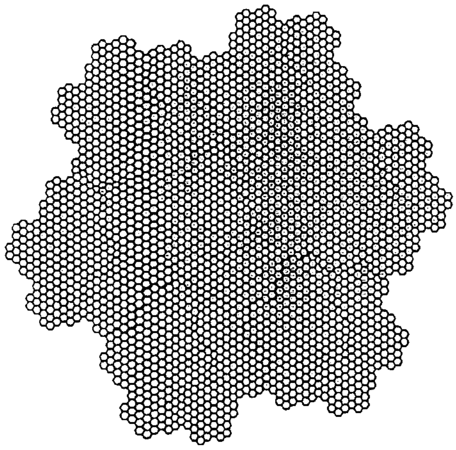

Figure 1: Collection of hexagonal cells.

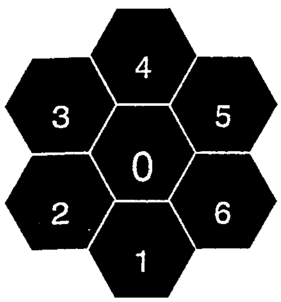

Figure 2: A collection of seven hexagons.

Each of the individual hexagons is labelled with a unique address as decribed in [6]. This is achieved by describing a process that begins with a collection of seven hexagons. Each of these seven hexagons is labelled consecutively with addresses 0, 1, 2, 3, 4, 5 and 6 as displayed in Fig. 2.
Dilate the structure so that six additional collections of seven hexagons can be placed about the addressed hexagons, and multiply each address by 10 (see Fig. 3).
For each new collection of seven hexagons, label each of the hexagons consecutively from the centre address as was done for the first seven hexagons (see Fig. 4).
The repetition of the above steps permits the collection of hexagons to grow in powers of seven with uniquely assigned addresses. It is this pattern of growth that generates the Spiral. Furthermore, the addresses are consecutive in base seven.
One really important feature of the Spiral arrangement is its powerfully computational advantages to computer vision [7].
As an example, our source image (Fig. 5) is coded on 256 grey levels and represented by 288 × 384

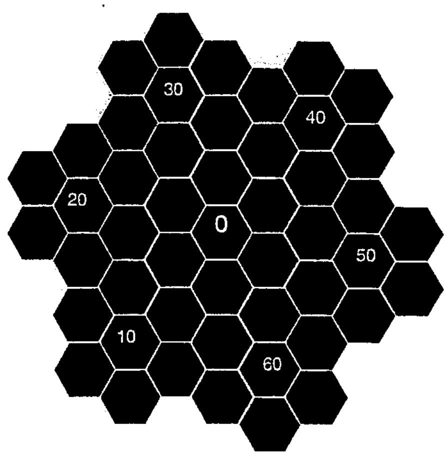

Figure 3: Dilated figure to fit more hexagons.

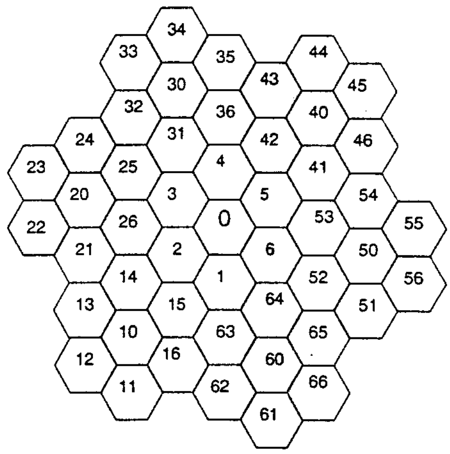

Figure 4: Collection of $7^2 = 49$ hexagons with labelled addresses.

rectangular pixels. Fig. 6 is the corresponding sample image represented by a collection of 7^5 hexagonal pixels.
Here, we use a set of four rectangular pixels to mimic a hexagonal pixel. Fig. 7 shows the arrangement of seven hexagonal pixels with addresses 0, 1, 2, 3, 4, 5 and 6. It is obvious that this approach preserves the important property of hexagon distribution that each such pixel has exactly six surrounding pixels.

3 MULTI-SCALE EDGE DEFINITION

Computer vision is a cross-disciplinary field with research methodologies from several scientific disciplines. Scale-space Theory was proposed by Lindeberg [5] to explain how certain aspects of image

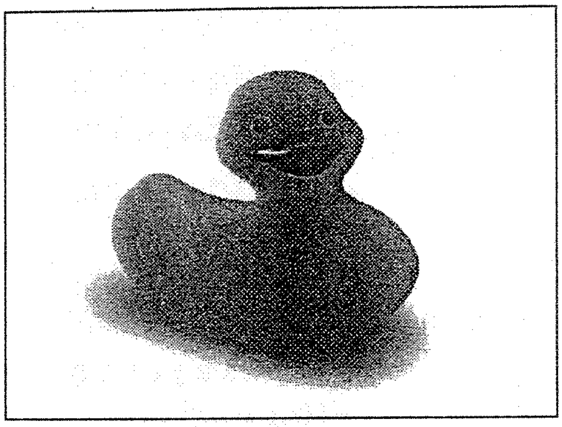

Figure 5: Source duck image in traditional space.

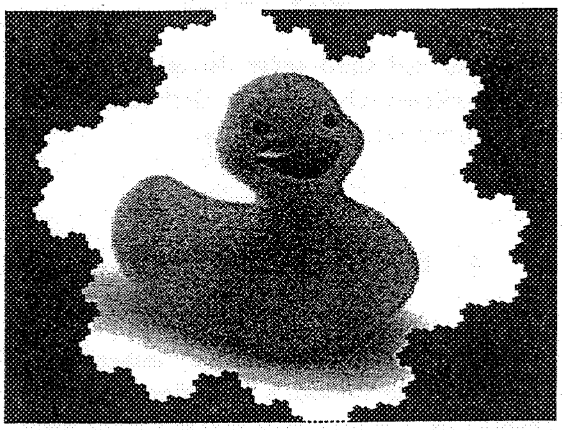

Figure 6: Sample image of the duck in spiral space.

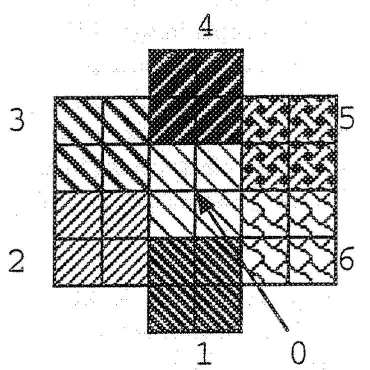

Figure 7: Distribution of 7 pixels constructed from rectangular pixels.

information can be represented and analysed at the earliest processing stages of a computer vision system. This theory can be used for edge detection as follows.

3.1 Scale-space representation

Let $f : \Re^2 \to \Re$ be a brightness function of an image which maps the coordinates of a pixel, (x, y) to a value in light intensities. The scale-space representation $L : \Re^2 \times [0, +\infty) \to \Re$ is defined such that the representation at 'zero scale' is equal to the original signal, i.e.,

$$L(\cdot; 0) = f(\cdot), \quad (1)$$

and the representation at 'coarser scales' is the convolution

$$L(\cdot; t) = g(\cdot; t) * f(\cdot),$$

where $g : \Re^2 \times (0, +\infty) \to \Re$ is the Gaussian kernel

$$g(x, y; t) = \frac{1}{2\pi t} e^{-\frac{(x^2+y^2)}{2t}}.$$

In fact, L is the solution of the *isotropic diffusion equation*

$$\partial_t L = \frac{1}{2}\Delta^2 L = \frac{1}{2}(\partial_{xx} + \partial_{yy})L. \quad (2)$$

with the input image f taken as initial condition, i.e., Eq. 1.

Scale-space representation is used to suppress and remove unnecessary and distorting details so that later stage processing tasks can be simplified. This can be explained in that the signal which is L becomes gradually smoother as t increases. Eq. 2 gives a direct physical interpretation of the smoothing transformation. The scale-space representation can be understood as the result of letting an initial heat distribution f evolve over time t in a homogeneous medium. Hence it can be expected that fine-scale details will disappear, and images become more diffuse when the scale parameter t increases.

3.2 Edge definition in continuous case

A natural way to define edges from a continuous grey-level image $L : \Re^2 \times [0. + \infty) \to \Re$ is as the set of points for which the gradient magnitude assumes a maximum in the gradient direction [5]. To give a differential definition of this concept, introduce a curvilinear coordinate system (u, v), such that at every point the v-direction is parallel to the gradient direction of L, and at every point the u-direction is perpendicular to the v-direction. Moreover, at any point $P = (x, y) \in \Re^2$, let $\partial_{\bar{v}}$ denote the directional derivative operator in the gradient direction of L at P and $\partial_{\bar{u}}$ the directional

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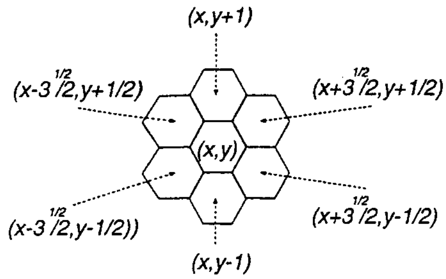

Figure 8: Ordinary coordinates of a cluster of 7 hexagons.

derivative operator in the perpendicular direction. Then at P the gradient magnitude is equal to $\partial_{\bar{v}} L$, denoted by $L_{\bar{v}}$, at that point. Assuming that the second and third order directional derivatives of L in the v-direction are not simultaneously zero, the condition for P_0 to be a gradient maximum in the gradient direction may be stated as

$$L_{\bar{v}\bar{v}} = 0 \text{ and } L_{\bar{v}\bar{v}\bar{v}} < 0. \quad (3)$$

Note that

$$\begin{aligned} \partial_{\bar{u}} &= (\sin\beta)\partial_x - (\cos\beta)\partial_y, \\ \partial_{\bar{v}} &= (\cos\beta)\partial_x + (\sin\beta)\partial_y, \end{aligned}$$

where $(\cos\beta, \sin\beta)$ is the normalized gradient direction of L at P_0. Hence, Eq. 3 is equivalent to

$$\begin{aligned} \tilde{L}_{\bar{v}\bar{v}} &= L_{\bar{v}}^2 L_{\bar{v}\bar{v}} \\ &= L_x^2 L_{xx} + 2L_x L_y L_{xy} + L_y^2 L_{yy} = 0, \\ \tilde{L}_{\bar{v}\bar{v}\bar{v}} &= L_{\bar{v}}^3 L_{\bar{v}\bar{v}\bar{v}} \\ &= L_x^3 L_{xxx} + 3L_x^2 L_y L_{xxy} \\ &\quad +3L_x L_y^2 L_{xyy} + L_y^3 L_{yyy} < 0. \end{aligned}$$

By reinterpreting L as the scale-space representation of a signal f, it follows that the edges in f at any scale t can be defined as the points on the zero-crossing curves of $\tilde{L}_{\bar{v}\bar{v}}$ for which $\tilde{L}_{\bar{v}\bar{v}\bar{v}}$ is strictly negative. Note that with the above formulation there is no need for any explicit estimate of the gradient direction.

3.3 Discrete approximation

Given discrete data, note that the six neighbouring hexagons of the hexagon at (x,y) have ordinary coordinates $(x, y-1)$, $(x-\sqrt{3}/2, y-1/2)$, $(x-\sqrt{3}/2, y+1/2)$, $(x, y+1)$, $(x+\sqrt{3}/2, y+1/2)$ and $(x+\sqrt{3}/2, y-1/2)$ (Fig. 8).
The derivatives operators can then be obtained in finite difference form:

$$\begin{aligned} L_x(x,y;t) &= \frac{1}{\sqrt{3}}[L(x+\sqrt{3}/2, y+1/2;t) \\ &\quad +L(x+\sqrt{3}/2, y-1/2;t)] \\ &- \frac{1}{\sqrt{3}}[L(x-\sqrt{3}/2, y+1/2;t) \\ &\quad +L(x-\sqrt{3}/2, y-1/2;t)] \end{aligned}$$

and

$$\begin{aligned} L_y(x,y;t) &= \frac{1}{2}[L(x+\sqrt{3}/2, y+1/2;t) \\ &\quad +L(x-\sqrt{3}/2, y+1/2;t) \\ &\quad +L(x, y+1;t)] \\ &- \frac{1}{2}[L(x+\sqrt{3}/2, y-1/2;t) \\ &\quad +L(x-\sqrt{3}/2, y-1/2;t) \\ &\quad +L(x, y-1;t)]. \end{aligned}$$

The second and third order derivatives can be obtained respectively from the first and second order derivatives in the same way.

4 EDGE DETECTION ALGORITHM

An edge map is defined as a binary image represented by 0's and 1's with a resolution parameter t, denoted by $E(x,y;t)$. $E(x,y;t) = 1$ if the pixel (x,y) is an edge point, otherwise $E(x,y;t) = 0$. An edge detection algorithm can now be constructed in this section.

1. **Edge map of sample image.** Edge map of the initial sample image (Fig. 6) plays an important role in our research. One will find that the edge map at each resolution level is a subset of this initial edge map. This map consists of edge points defined in the previous section with the parameter $t = 0$. Fig. 9 is the map.

2. **Edge map at coarse level.** Using the Gaussain filter introduced in the previous section, the initial sample image is blurred with a pre-determined parameter value. The blurred image has lower resolution level than the initial image. This process makes the object structures blurred, the edge shape changed, and the noise smoothed. The edge map of this blurred image is then formed in the way stated in the previous step. Fig. 10 is the blurred image of Fig. 6 when $t = 5$ and Fig. 11 is the corresponding edge image.

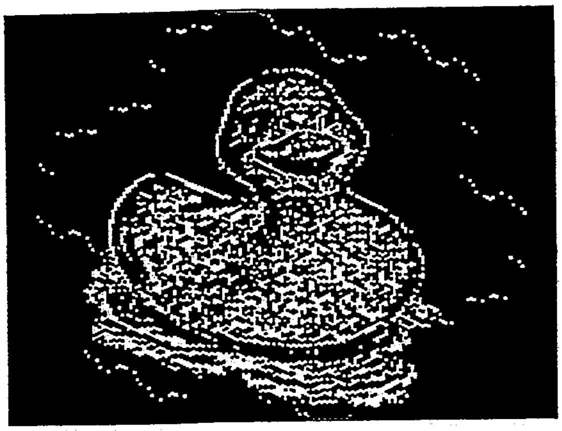

Figure 9: Edge image of Fig. 6.

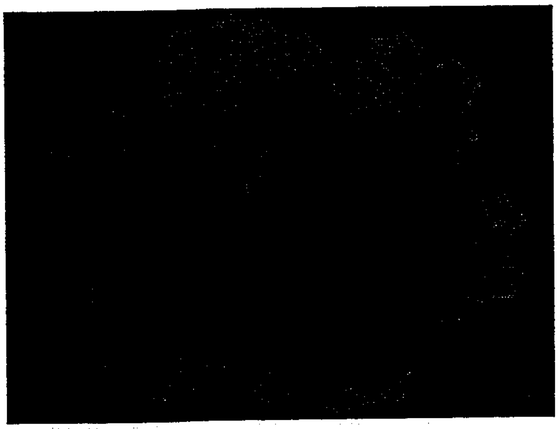

Figure 10: Blurred image of Fig. 6 when $t = 5$.

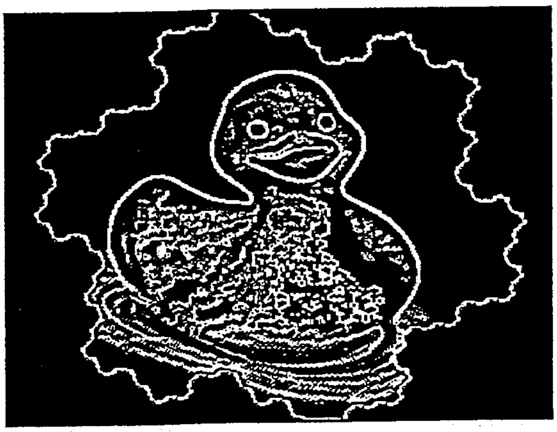

Figure 11: Edge image of Fig. 10.

3. **New edge map of the initial image.** An edge map obtained by a blurring scale has less noise than the edge map of the initial image. However, the initial edge map has more precise edge locations because it is not affected by the Gaussian operator. This step compares the two edge maps to obtain a new edge map. The new map consists of those initial edge points of which at least one neighbouring hexagonal pixel is an edge point of the blurred image. Fig. 12 shows the new edge map obtained from the initial map (Fig. 9) and the blurred map (Fig. 11).

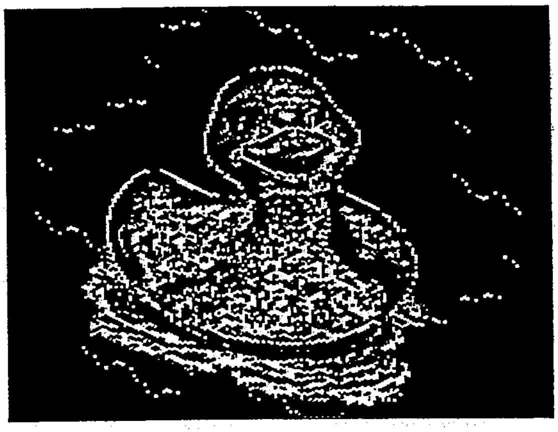

Figure 12: Edge map from Fig. 9 and Fig. 11.

4. **Repeating and halting.** Compare the new edge map with the old edge map (which is the edge map of the initial image at this stage). If they are not much different, then halt. Otherwise, construct a new sample image from the initial sample image by copying only those pixels of which each is a neighbouring pixel of an edge point of the new edge map. Then, create a coarse level edge map of this new sample image as shown in Step 2. Note that the parameter t is chosen to be slightly smaller than previous one. Repeat Step 3, a newer edge map is obtained. Fig. 13 is the new sample image after first recurrence. We set $t = 4$, $t = 3$ and $t = 2$ for the 2nd, 3rd and 4th recurrences respectively. TABLE 1 shows the numbers of rectangular pixels on the edge maps of our sample image after first four recurrences. Fig. 14, Fig. 15 and Fig. 16 are the edge maps of the sample image after 2nd, 3rd and 4th recurrences respectively.

5. **Final edge map.** The final edge map is constructed by including the points on the latest edge map and those edge points of blurred image after 1st recurrence, of which each is a neighbourhood of one of the latest edge points. Fig. 17 is the final edge map of the sample image.

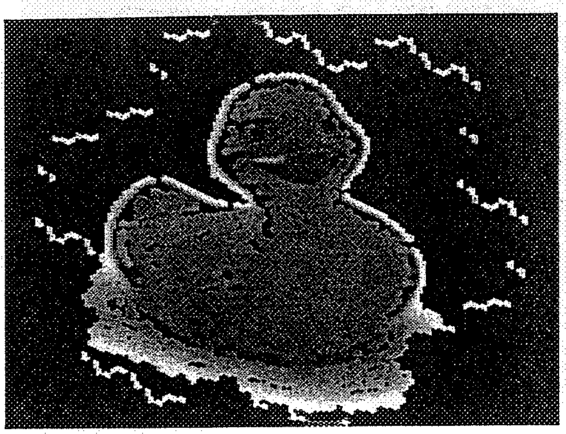

Figure 13: Sample image after 1st recurrence

Table 1: No. of rectangular pixels on the edge after each recurrence and difference between old and new edge map.

	No. of pixels on the edge	Difference
Original image	14342	
After 1st recurrence	12486	1856
After 2nd recurrence	11808	678
After 3rd recurrence	11611	197
After 4th recurrence	11455	166

5 DISCUSSION

1. In the edge detection algorithm presented in the previous section, we use the blurring method to identify essential structures in an image. Blurring itself is not disirable, but it is used as a mean to filter away noise and unnecessary detail.

2. In Step 4 of the algorithm in the previous section, Gaussian operator only acts on the edge points obtained at the previous recurrence and their neighbouring hexagonal pixels. This greatly increases the detection speed.

3. The positions of edge points on the final edge map will be at most one hexagonal pixel away from the original edge map. This guarantees the accurancy of the edge map being detected.

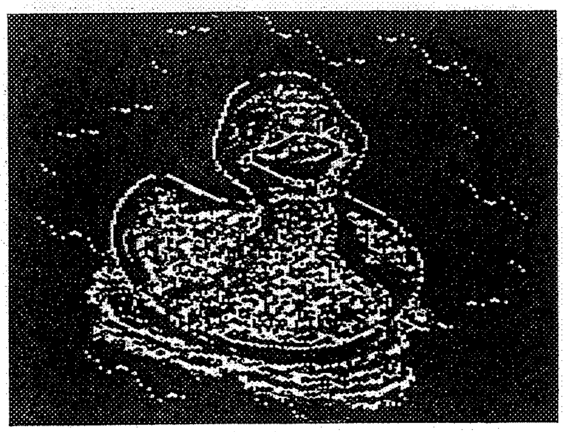

Figure 14: Edge map after 2nd recurrence

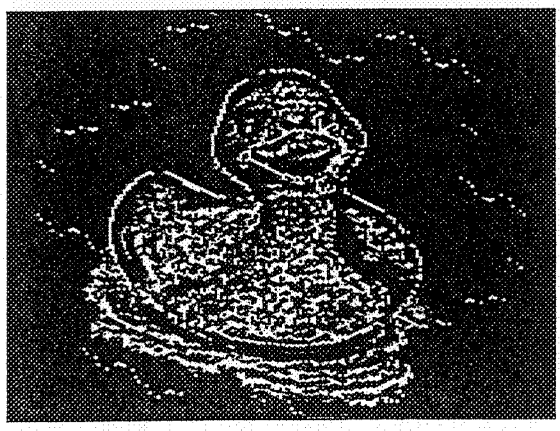

Figure 15: Edge map after 3rd recurrence

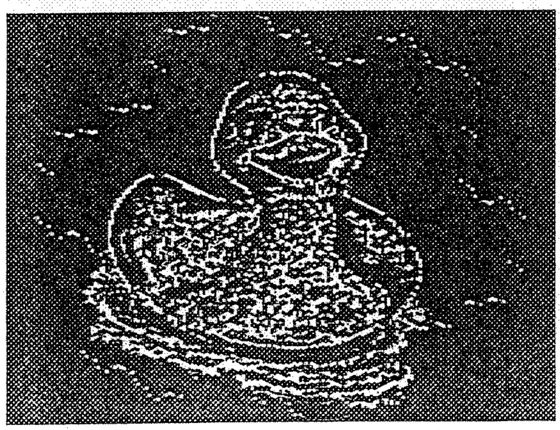

Figure 16: Edge map after 4th recurrence

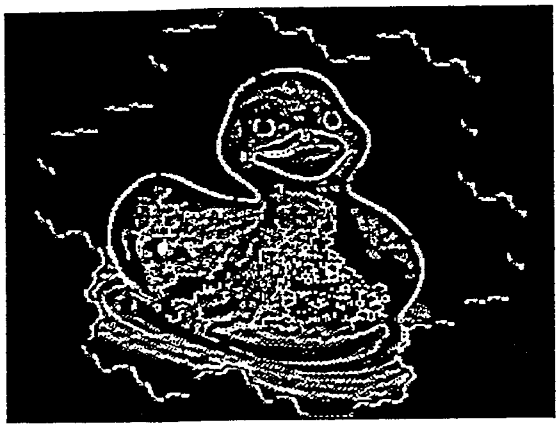

Figure 17: Final edge map of Fig. 6

4. The number of pixels decreases as the number of recurrences increases. This is because after each recurrence, some noise is removed.

5. The edge map detected can be improved when a better t value is chosen.

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