

# TEXTURE SEGMENTATION BASED ON THE FRACTAL ANALYSIS

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## Abstraction

An efficient algorithm for segmenting a textured image into different regions is developed based on a set of fractal dimension estimated from calculated variogram. A simple and robust gradient operator is employed to detect texture boundary based on the multi-resolution framework including gradient pyramid construction followed by the reliable information obtained in the rough resolution as mask to constrain the calculation region in the rest resolution. Combined with mask and gradient in each resolution is propagating down to the finest resolution giving a more accurate boundaries estimation. The utility of the proposed method is demonstrated on a number of synthesis and natural textures.

## 1. INTRODUCTION

Segmenting an image into uniformly textured regions is a prerequisite for many computer vision and image understanding tasks [1]. Texture segmentation can be achieved either by detecting texture discontinuity between texture image regions or by extracting uniform texture regions. Therefore, segmentation methods are often categorized as boundary-based, region-based or as a hybrid of the two. The observation that a given neighbourhood of an image may both be seen as containing some definite structure, such as a edge feature, while also being seen as a part of a texture, shows that texture is a scale-dependent phenomenon. In this work, the selection of an appropriate analysis scale can be formulated in terms of the variogram in fractal dimensional approach [2, 3, 4]. The variogram is calculated from illumination variation versus with sets of both horizontal and vertical pair of distance. The regularity

of the texture element can be obtained from the periodicity shown in the variogram value corresponding to a particular set of pair of distance which is the scale of the texture element.

### 1.1. Self-similarity and scale

Textures exhibit two significant characteristics one is local property for measurement texture element and the other is self-similarity. For textures occurring in natural images generally show randomness or variability and exhibit fluctuations, ranging from the degradations occurring in the image formation process, surface irregularity and geometric warping caused by the 3-D to 2-D projection. Inevitably, these intrinsic properties affect the local texture measures used in analysis. Solutions often involve operations with local windows, which requires choosing the windowing function and window size. Choosing the right scale is a fundamental problem because texture is an area property [5]. There is seldom a systematic basis for choosing the window size. If the window is too big, for example, it may be difficult to identify region boundaries or adapt to variations within regions, while if it is too small it may give insufficient discrimination of the textures. In the aspect of self-similarity, textured image implicitly can be approximately described as a texture element distributed in a self-similarity manner.

## 2. TEXTURE SEGMENTATION

In general, there are two processes in the texture segmentation including texture feature extraction followed by the segmentation in the feature space.

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## 2.1. Feature extraction

Transformed domain approaches in feature extraction using Gabor filters with tuned direction and frequency to adapt the texture feature [8]. Some research adopt Multiresolution Fourier Transform and Wavelet transform to get transformed coefficient [7, 8] and segmentation is accomplished in feature domain. Gaussian random field uses the statistics of Gaussian distribution in various direction to describe the texture properties [9]. Gibbs random field is similar to that in Gaussian random field except that Gibbs random field uses exponential distribution [10]. These works have shown their well performance in texture feature extraction.

## 2.2. Texture Segmentation

Two major working stream in texture segmentation, region growing and boundary detection. Currently there are works hybrid both method into one cooperative framework [11]. The main issue centered around that the texture fluctuation affect the segmenting results, thus a smoothing operation is usually employed. In such a manner, the position resolution of the boundary has been sacrificed. This leads to a multiresolution framework to overcome this difficulty.

## 3. FRACTAL DIMENSION ESTIMATE AND TEXTURE SEGMENTATION

Tradition Euclidean geometric is hard to describe appearance of the nature object because that they are restricted in integer dimension. In 1975 fractal theory has emerged to resolve this difficulty allowing description of the object close to as it is.

Fractal dimension is based on the self-similarity measurement to describe the object structure [3]. Give an object partitioned into  $N$  parts, the change rate of scale is denoted as  $P$ , then the fractal dimension  $D$  of the object can be defined as:

$$1 = N(P)P^D \quad (1)$$

It is equivalent to

$$D = \frac{\log(N(P))}{\log(1/P)} \quad (2)$$

Fractal dimension can be used as roughness measurement of an object [3]. However, it is not sufficient to decide the texture roughness in terms of fractal dimension only due to the texture surface is a stochastic phenomenon. Thus, fractal Brownian surface motion is copied to analysis texture structure [12]. For a given random variable  $B(m)$  is denoted as fractal Brownian

function, then the probability of scaled surface change rate within a distance  $\Delta m$  is a monotonic increasing function  $F(n)$ , that is

$$Pr\left[\frac{B(m + \Delta m) - B(m)}{\|\Delta m\|^H} < n\right] = F(n) \quad (3)$$

where  $H$  is a Hausdroff parameters [12]. If  $H$  equals to 1 then the measured surface is a flat plane, whileas if  $H$  is close to 0 then the surface appears as rocky. For the purpose of the illustration, figures 3 show the surface appearance in different values of  $H$ ,  $H$  in figures 3 top-left is 0.1, bottom-left is 0.3, top-right is 0.5 and bottom-right is 0.9 respectively.

Followed by the calculation of  $H$ , the fractal dimension  $D$  of the surface can be obtained as

$$D = E - H \quad (4)$$

where  $E$  is the Euclidean dimension.

### 3.1. The Algorithm

Variogram method There are a number of methods to calculate Fractal Dimension, FD [2]. For the natural image surface description, variogram exhibits a characteristics of Fractal Brown motion [3]. Thus, variogram is able to describe the texture structure. For an input image, variogram between intensity  $Z(m, n)$  and  $Z(m', n')$  are defined as

$$Var(Z(m, n), Z(m', n')) = k[d[(m, n); (m', n')]]^{2H} \quad (5)$$

where  $d$  is the distance of two pixels and  $k$  is a constant corresponding to the interception of the fitted linear curve of Richardson Plot [3],  $H$  is the parameter to define FD as

$$FD = 3 - H \quad (6)$$

where 3 is the topological dimension in three dimension. If define a curve  $V(d)$  to be the square of difference of gray level between two pixels, than  $V(d)$  is calculated with a set of pixel distance  $d$  along some direction  $\theta$

$$V(d) = \sum_d ((Z(m + d_\theta, n + d_\theta) - Z(m, n))^2) \quad (7)$$

Given a variable  $d$ , a curve  $V(d)$  can be obtained and plotted as a Richardson plot. When  $V(d)$  is small, it means that two neighboring pixel value is similar and vice versa. By observing the amplitude variation of  $V(d)$ , the structure of the image texture can be analyzed. A linear curve is used to fit the variation of the  $V(d)$  in terms of the slop and interception. For illustration purpose, figure 4 is a variogram plot of an

input reptile texture image varying regularly showing that the texture element scale can be estimated with some pixel number. Followed by the variogram curve linear fitting, FD can be estimated as following

$$FD = 3 - (b/2) \quad (8)$$

where  $b$  is the slope of the curve  $V(d)$  and is estimated by means of linear regression fitting.

### 3.2. Fractal Pseudo Image

From the linear fitting method, a linear curve is parameterized in terms of a slope and interception. Consequently, the estimated slope can be utilized to compute FD. In order to obtain the local texture properties in terms of the FD, a moving window is used. The size of the moving window is chosen based on the regularity of the variogram. For instance, in the reptile image, the variogram shown in figure 4 is varied every 11-13 pixel interval regularly such that the size of the moving window is chosen  $13 \times 13$  accordingly. By moving the window one pixel at each time to compute FD in a certain direction "a" and scale to 0 - 255 yielding a one directional fractal pseudo image.

$$F^a(x, y) = w(m - x, n - y) \star Z(m, n) \quad (9)$$

where  $\star$  is the FD calculation in equations (7) and (8).

### 3.3. Initial Boundary Detection and Refinement

In the fractal pseudo image, a Sobel edge detector is employed to extract the texture boundaries.

$$\nabla F = G_m \otimes F_m \quad (10)$$

where  $G_m$  is the intensity change in  $m$  axis. For the single directional fractal pseudo image existing various fluctuations which affect the boundary estimate, a median filter is used to smooth the fractal pseudo image. Furthermore, the single directional fractal pseudo image has difficulty in describing texture structure uniquely. Because two visually different textural images may have identical FD while two similar texture images may have different FD. A multi-fractal approach is usually employed to resolve this problem. In this work, four directional fractal pseudo images are generated to achieve this target. These fractal pseudo images are created based on moving the window in north-south, east-west and two diagonal directions respectively followed by averaging them to yield a multi-fractal like image.

$$F(m, n) = \frac{1}{4} \sum_a F^a(m, n) \quad (11)$$

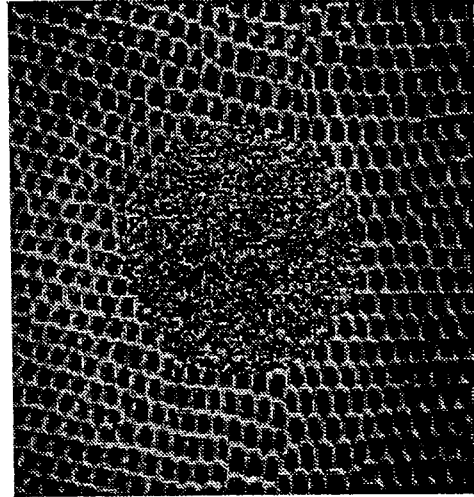


Figure 1: Generated structured texture pair, reptile and grass, as testing image.

where  $a$  means four directions and  $F$  is one-direction fractal image. Followed by the initial edge detection over the averaged fractal pseudo image, a series of the refinement process over different resolution levels based on the pyramid structure is employed. The process is efficient as that in normal multiresolution framework because reliable information estimated from the level above to constrain the boundary detection within current level.

## 4. EXPERIMENTS AND RESULTS

A number of experiments have been conducted to demonstrate the efficacy of the proposed algorithm. The algorithm is implemented on the Matlab and the testing images are all size of  $256 \times 256$  shown in figure 1.

In figure 2 shows the fractal pseudo image exiting variation of the input image.

- Single Directional Fraction Dimension The single directional fractal dimension is shown in figure 5.
- Averaging Directional Fraction Dimension By averaging the four direction fractal pseudo images, the boundary detection can be enhanced significantly.
- Multiresolution Estimated Boundary Refinement In the figure 6 shows boundary detection within single resolution. Some of the emerged false edges affect the boundary estimate. The refinement result is shown in figure 7.

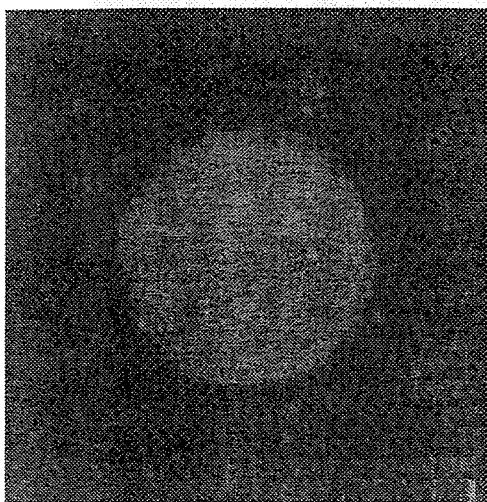


Figure 2: Fractal pseudo image of testing image.

## 5. CONCLUDING REMARKS AND FUTURE WORK

A texture segmentation based on the fractal analysis is presented. The difficulty of the texture fluctuation in normal texture segmentation is resolved by means of the multiresolution framework and the promise has been achieved to obtain an accepted boudoury estimate. Some of the detection technique within cross resolution level is being left out. Because the idea of this presented work is emphasized on the performance of the fractal dimension. However those existing segmentation techniques can be implemented readily to enhance the speed of the proposed work and being currently under investigated.

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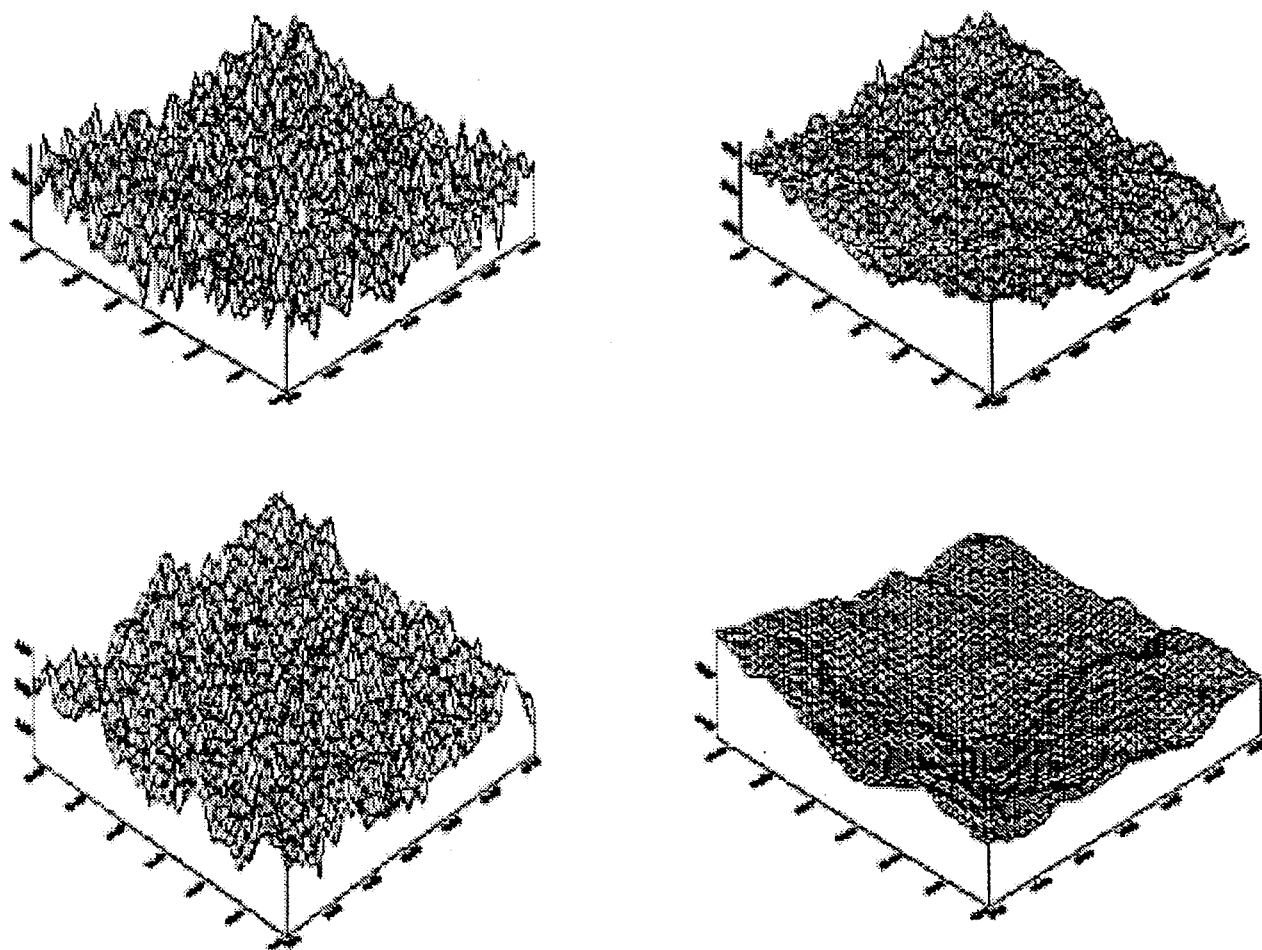


Figure 3: Fractal Brownian surface in different Hausdroff value.

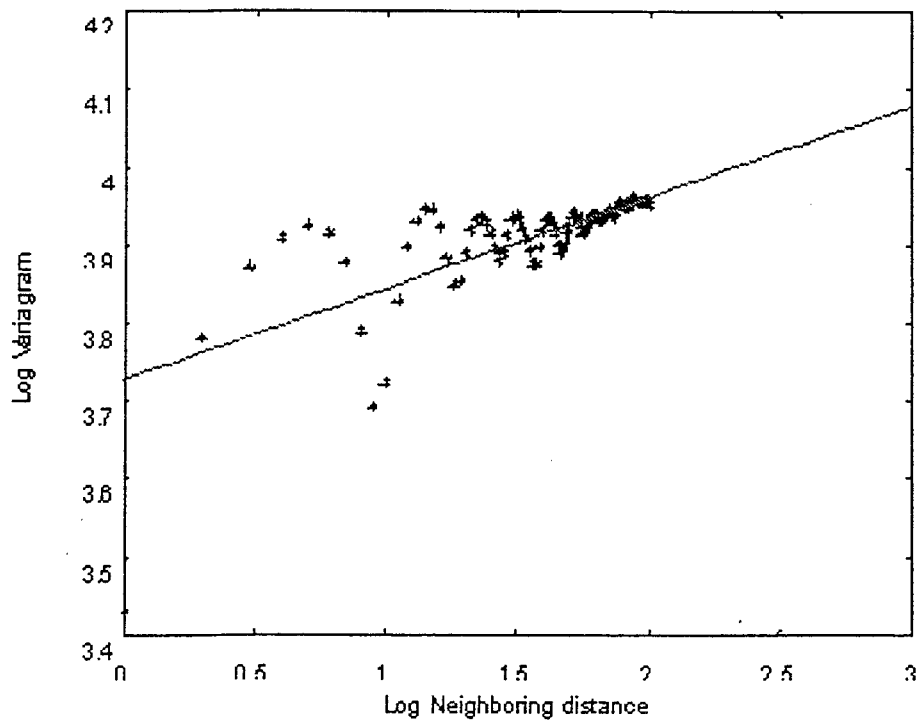


Figure 4: Variogram plot: square of neighboring pixel value difference versus neighboring distance.

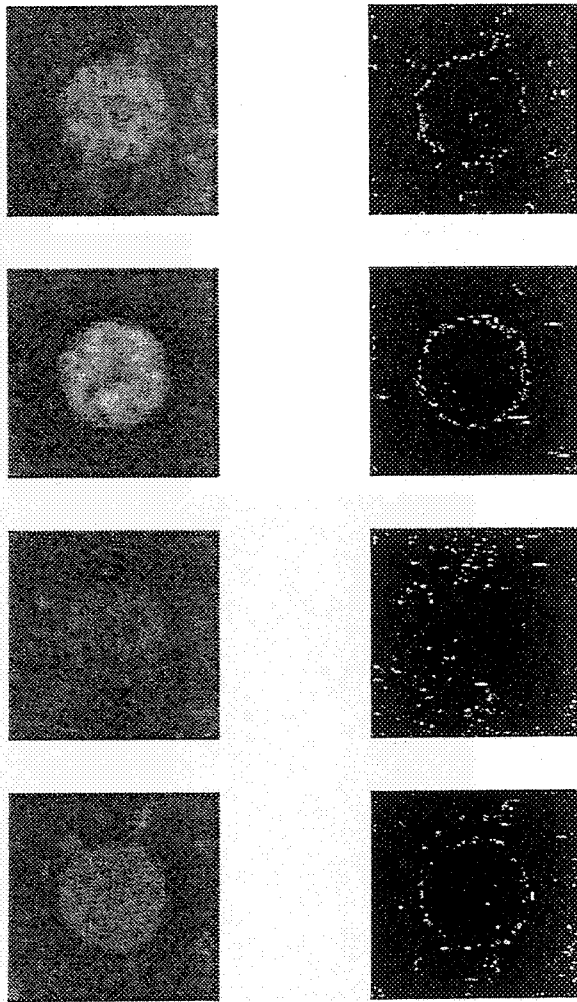


Figure 5: Boundary detection of single direction fractal pseudo image of testing image.

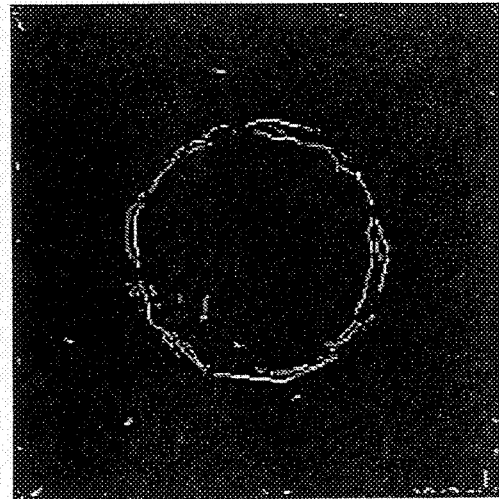


Figure 6: Initial Boundary detection of testing image.

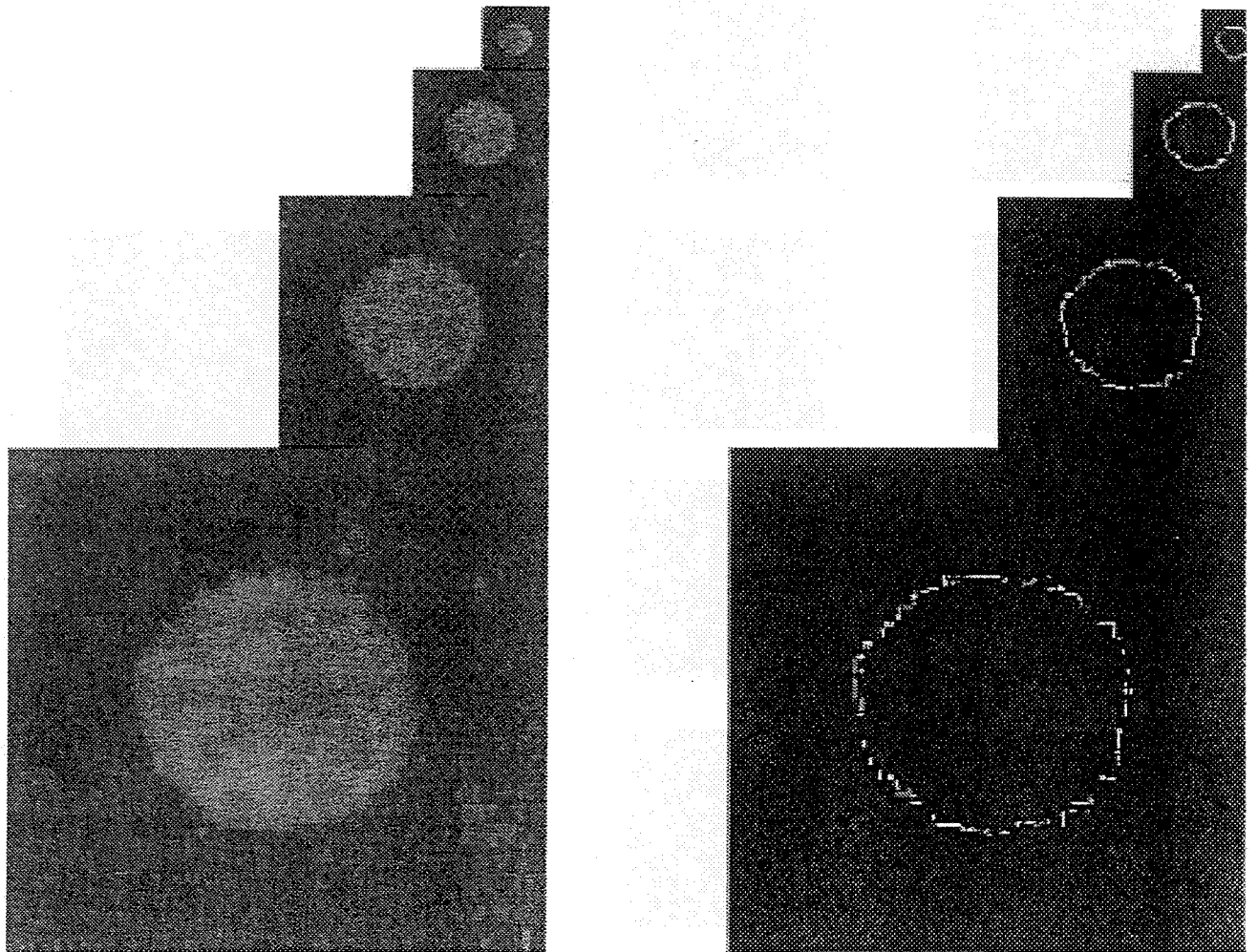


Figure 7: Multiresolution Boundary detection of testing image.