An Efficient Distributed Online Algorithm to Detect Strong Conjunctive Predicates

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Abstract

Detecting strong conjunctive predicates is a fundamental problem in debugging and testing distributed programs. A strong conjunctive predicate is a logical statement to represent the desirable event of the system. Therefore, if the predicate is not true, an error may occur because the desirable event do not happen. Recently proposed detection algorithms have the problem of unbounded state queue growth since the system may generate a huge amount of execution states in a very short time. In order to solve this problem, this paper introduces the notion of removable states that can be disregarded in the sense that detection results still remain correct. A fully distributed algorithm is developed in this paper to perform the detection in an online manner. Based on the notion of removable states, the time complexity of the detection algorithm is improved as the number of states to be evaluated is reduced.

Keywords: Conjunctive predicate, distributed debugging, distributed system, global predicate detection.

1 Introduction

With the rapid development of networks and distributed systems, programming in distributed environments is becoming more common. However, the difficulty of distributed programming is much higher than that of sequential programming. This is because distributed debugging requires the ability to analyze and control the execution of processors that run asynchronously. Also, within a distributed environment, stopping a program at a specific breakpoint is non-trivial.

It is well understood that distributed programs are usually designed to obey certain conditions [6]. For example, a distributed mutual exclusion program obeys the condition "at any time, the number of processes in the critical section is no more than 1". If it violates this condition, an error (two or more processes are in the critical section simultaneously) may occur. Typically, the conditions are formulated as boolean expressions, called *global predicates* [1, 5]. Detecting whether a given global predicate is satisfied is essential for debugging and testing distributed computations.

As the detection of general global predicate was proved to be NP-complete [2], most researchers restricted their research regarding a specific class of global predicates to detect in polynomial time. In this paper, the focus is on an important class of global predicates, known as *conjunctive predicate* [7, 8, 11, 12], which can be expressed as a conjunctive form of local predicates. The *local predicate* is a boolean expression defined by the local variables of the process. At any time, a process can evaluate its local predicate without communication.

In this paper, the problem of detecting whether a given conjunctive predicate Φ is *definitely true* [5, 12] is considered. Φ is definitely true if for *all* runs of the distributed program, Φ is true at some time. Intuitively, detecting this sort of global predicates is used to ensure that a certain *desirable* event occurs. For simplicity, we define predicate DEFINITELY(Φ) is true if and only if Φ is definitely-true. DEFINITELY(Φ) is called *strong* conjunctive predicate in [8].

In [12], Venkatesan and Dathan proposed a distributed algorithm to detect DEFINITELY(Φ). This algorithm performs an *offline* evaluation of the predicates, *i.e.* predicates are evaluated after the program execution is terminated. The analysis shows that their algorithm uses $O(p^3M_t)$ additional control messages with the size of each being only $O(1)$, where p is the number of processes and M_t is the total number of truth value changes of the local predicates. In [8], Garg and Waldecker proposed an algorithm that evaluates the predicate in an *online* manner, *i.e.* predicates are evaluated immediately following each instruction execution. This algorithm employs a central debugger that collects debug information from application processes and then performs the detection. The time complexity of the detection algorithm is $O(p^2m)$, where m is the maximum number of states in one application process. Compared with Venkatesan and Dathan's algorithm, this algorithm uses only $O(M_r)$ additional control messages with the size of each being $O(p)$, where M_r is the total number of messages that all application processes receive.

One disadvantage of the above algorithms is that the debugger evaluates execution states, which are collected from application processes, in a certain order. Restated, before evaluating certain states, all other states are queued. Since real systems can generate hundreds of states in a very short time, the queues may grow unbounded. To solve this problem, in this paper, the notion of *removable states* is introduced. By discarding the removable states, the space requirement for each process can be minimized to $O(p)$, where p is the number of processes. While minimizing the memory space, time complexity is also improved to $O(pm)$, because the number of states to be evaluated is reduced. Our algorithm does not require exchange of control messages during program execution, because all the debug information is piggybacked in normal application messages.

The remainder of this paper is organized as follows. In Section 2, we define model and notations. In Section 3, we introduce the notion of removable states and derive the condition of identifying removable states. Based on this result, Section 4 presents an efficient way to maintain non-removable states, and then discusses a new detection algorithm. Finally, a concluding remark is made in Section 5.

2 Model and Notations

A distributed system consists of p *processes* denoted by $P_1, P_2, \ldots, P_p$. These processes share no global memory and no global clock. Message passing is the only way for processes to communicate. The transmission delay of the communication channel between each pair of processes is random. However, we assume that no message in any channel is lost, altered, or spuriously introduced.

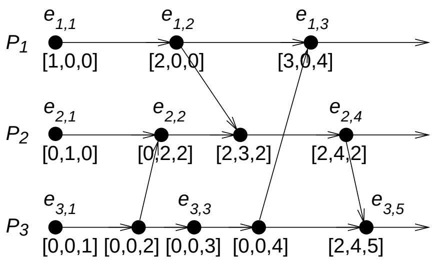


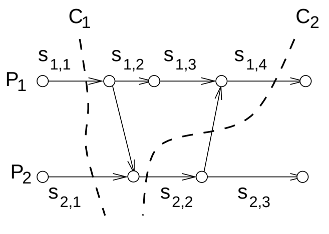
Figure 1: Events and their time vectors.

States and Events At a given time, the *state* of a process is defined by its variable's values. The *states* of processes can change only when *events* are executed. There are three kinds of events: an *internal event* which does a local computation, a *send event* which sends a message to another process, and a *receive event* which receives a message from another process via the channel.

The x^{th} event occurring in process P_i is referred to as $e_{i,x}$. The number x is called the *sequence number* of $e_{i,x}$. Figure 1 illustrates the events of the execution of a distributed program. Event $e_{i,x}$ *happens before* event $e_{j,y}$, denoted by $e_{i,x} \rightarrow e_{j,y}$, if and only if one of the following conditions holds [9]:

1. $i = j$ and $x < y$.
2. A message is sent from $e_{i,x}$ to $e_{j,y}$.
3. Another event $e_{k,z}$ exists such that $e_{i,x} \rightarrow e_{k,z}$ and $e_{k,z} \rightarrow e_{j,y}$.

In this paper, the system is assumed to recognize the happen-before relationships by using *vector clocks* [10] (Figure 1). With this approach, each process P_i maintains an integer vector $vector_i[1..p]$. Initially, each process P_i sets $vector_i$ to $[0, 0, \ldots, 0]$ but $vector_i[i] = 1$. When a process P_i executes an internal event, it increases $vector_i[i]$ by 1. When process P_i sends out a message, it increases $vector_i[i]$ by



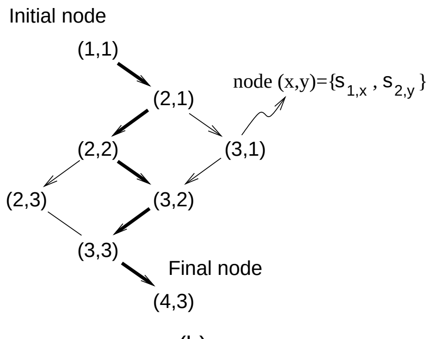


Figure 2: (a) Space-time diagram of a distributed program. (b) The lattice of (a).

1 and then associates $vector_i$ within the message. When process P_i receives a message associated with a vector, say v, it sets $vector_i[k] = max(vector_i[k], v[k]), \forall k$, and then increases $vector_i[i]$ by 1.

Let $vector(e_{i,x})$ to represent the value of $vector_i$ after executing $e_{i,x}$ and before executing $e_{i,x+1}$. The following properties can be seen from Figure 1: for event $e_{i,x}$, $vector(e_{i,x})[i] = x$ represents the sequence number of $e_{i,x}$, and $vector(e_{i,x})[j] = y$, $j \neq i$, represents the sequence number of event $e_{j,y}$ where $e_{j,y} \rightarrow e_{i,x}$ and $e_{j,y+1} \not\rightarrow e_{i,x}$. Therefore, the happen-before relationships can be determined in time $O(1)$ by using vector clocks, as shown in Theorem 2.1.

Theorem 2.1 ([10]) *For two events $e_{i,x}$ and $e_{j,y}$, $e_{i,x} \rightarrow e_{j,y}$ if and only if $vector(e_{i,x})[i] \leq vector(e_{j,y})[i]$.* ■

Global States and Global Predicates A global state is a collection of states, one from each process, in which no happen-before relationship occurs. (Note that the system can not enter a state with happen-before relationship because messages can not be received before they are sent.) For example, in Figure 2(a), C_1 is a global state but C_2 is not. The set of all global states within a distributed program forms a *lattice* [5]. In the lattice, a node (global state) S_1 is linked to another node S_2 if the system can proceed from S_1 to S_2 by executing only one event. Figure 2 shows the space-time diagram of a distributed program and the corresponding lattice. A possible run of a distributed program can be viewed as a path in the lattice from the initial node (initial global state) to the final node (final global state). For example, the path depicted by bold lines in Figure 2(b) represents a possible execution order of the events occurring in the program.

A *local predicate* is a boolean expression of the process states. At any time, the process can evaluate its local predicate without communication. A global predicate is a boolean expression, which involves the states of several processes. In this paper, we consider an important class of global predicates, known as *conjunctive predicate*, which can be expressed in a conjunctive form $LP_1 \wedge LP_1 \wedge \ldots \wedge LP_p$, where LP_i is the local predicate of process $P_i, i = 1, 2, ..., p$. For simplicity, we use either Φ or $LP_1 \wedge LP_1 \wedge \ldots \wedge LP_p$ to denote the conjunctive predicate.

In [12], Venkatesan and Dathan indicated that in a typical software development environment, developers may have occasion to use the conjunctive predicate Φ in one or more of the following ways:

- *DEFINITELY(Φ)* is true iff Φ is *definitely true*. Φ is *definitely true* if in every path from the initial node to the final node in the lattice, Φ holds in some node. Detecting this kind of global predicates is usually used to ensure a certain *desirable* event occurs. For example, we can consider a distributed two-phase commit protocol (see Figure 3). When the master decides to commit the transaction, it must assure that all the slaves are prepared to commit. Assume that there are two slaves, P_1 and P_2. Let $\Phi = LP_1 \wedge LP_2$, where $LP_1 = \{P_1 \text{ is committable}\}$ and $LP_2 = \{P_2 \text{ is committable}\}$. In Figure 3(b), a path (depicted by bold lines) exists in which Φ is not true in all nodes. Therefore, Φ is not definitely true. This implies that an error may occur because at least one slave process is not ready to commit during the program execution which corresponds to this bold path.

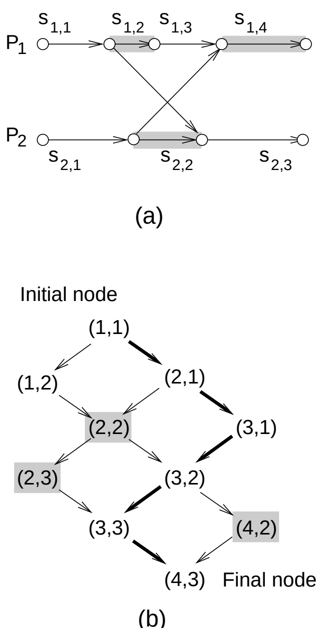


Figure 3: An example of two-phase commit protocol. Let $\Phi = LP_1 \wedge LP_2$, where $LP_1 = \{P_1 \text{ is committable}\}$ and $LP_2 = \{P_2 \text{ is committable}\}$. (a) In process P_1 (P_2), a state is shaded if the local predicate LP_1 (LP_2) holds within this state. (b) A global state is shaded if Φ holds (*i.e.* both P_1 and P_2 are prepared to commit) within this global state.

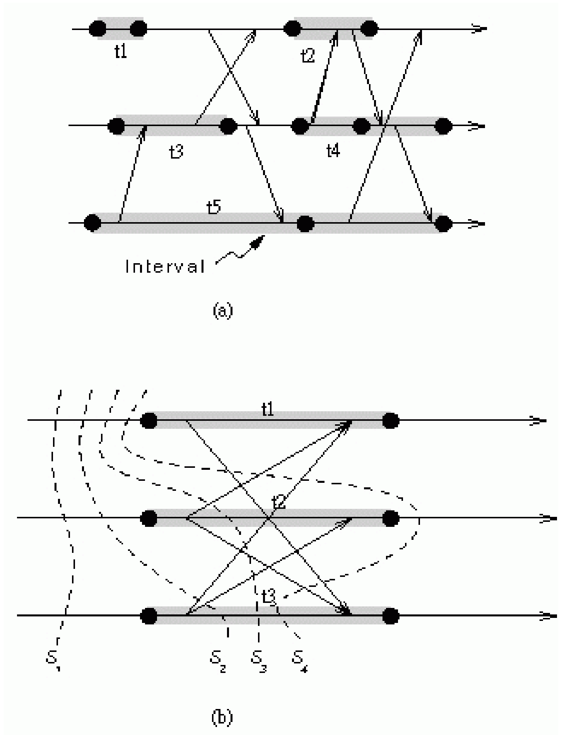


Figure 4: (a) Example of intervals. (b) A scenario that Φ is not definitely true in intervals.

- *POSSIBLY(Φ)* is true iff Φ is *possibly true*. Φ is *possibly true* if a path exists in the lattice such that Φ holds in some node. Detecting this kind of predicates is usually used to ensure that certain *undesirable* events do not occur. For example, consider a mutual exclusive program, which runs on a system with two processes P_1 and P_2. Let $\Phi = \{(P_1 \text{ is in the critical section}) \wedge (P_2 \text{ is in the critical section})\}$. If Φ is possible true, the undesirable event (both processes are in the critical section) may occur in some run of the program.

Intervals Researchers in [8, 12] proposed a necessary and sufficient condition of whether DEFINITELY(Φ) holds. This condition uses the notion of *intervals*. An interval t is a pair of events in the same process, in which $t.lo$ and $t.hi$ are referred to as its *beginning event* and *ending event* respectively. Furthermore, event $t.lo$ turns the truth value of the local predicate from false to true, events between $t.lo$ and $t.hi$ do not change the truth value, and event $t.hi$ turns the truth value from true to false. Two intervals t and t' are *overlapped* if $t.lo \rightarrow t'.hi$ and $t'.lo \rightarrow t.hi$. For example, in Figure 4(a), interval t_2 and t_4 are overlapped but t_2 and t_3 are not. A set of *overlapping global interval* (*OGI-set*) is a set of intervals, one from each process, in which each pair of intervals is overlapped. For example, interval set $\{t_2, t_4, t_5\}$ in Figure 4(a) is an OGI-set.

To simplify, the following notations for two interval sets I_1 and I_2 are defined:

- $I_1 = I_2$: For all $t \in I_1$ and $t' \in I_2$ in the same process, $t.lo = t'.lo$.
- $I_1 \preceq I_2$: For all $t \in I_1$ and $t' \in I_2$ in the same process, $t.lo \rightarrow t'.lo$ or $t.lo = t'.lo$. For example, in Figure 4(a), $\{t_1, t_3, t_5\} \preceq \{t_2, t_4, t_5\}$.
- $I_1 \rightarrow I_2$: For all $t \in I_1$ and $t' \in I_2$, $t.lo \rightarrow t'.hi$.

Figure 4(b) illustrates an interesting scenario that DEFINITELY(Φ) does not hold. In this figure, t_1 and t_2 are not overlapped since no message exists from $t_1.lo$ to $t_2.hi$. Thus, a program execution can be constructed such that Φ is true from global states S_1 to S_3 but is false in S_4. Theorem 2.2 generalizes this scenario.

Theorem 2.2 ([8, 12]) *For a distributed program, DEFINITELY(Φ) holds if and only if there exists an OGI-set.* ■

Distributed Online DEFINITELY(Φ) Detecting Problem In a distributed environment, processes collect the execution states of other processes by exchanging messages. In other words, when a process P_i executes an event $e_{i,x}$, all the events (and the associated states) that it can observe are those that happen before $e_{i,x}$. These events are denoted by $E_{i,x}$, *i.e.* $E_{i,x} = \{e_{j,y} | e_{j,y} \rightarrow e_{i,x}$ or $e_{j,y} = e_{i,x}\}$. $E_{i,x}$ is called the *E-set* of $e_{i,x}$. If $E_{i,x} \subseteq E_{j,y}$ then $E_{j,y}$ is called a *future* E-set of $E_{i,x}$. The following property can be verified easily:

P1 Event $e_{i,x} \rightarrow e_{j,y}$ if and only if $E_{i,x} \subseteq E_{j,y}$.

In this paper, the distributed online DEFINITELY(Φ) detecting problem is that whenever process P_i executes an event, say $e_{i,x}$, it tests whether DEFINITELY(Φ) holds for the debug information associated with E-set $E_{i,x}$.

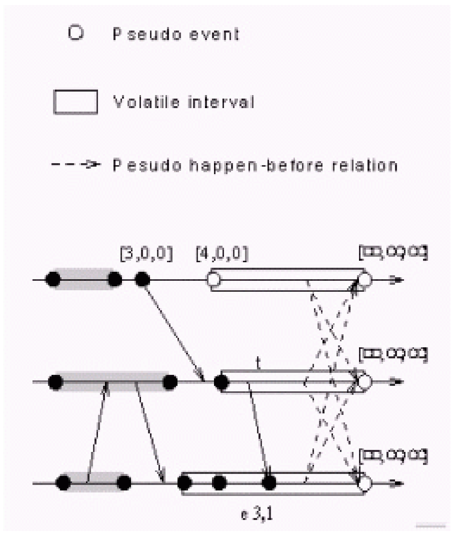


Figure 5: Volatile intervals in E-set $E_{3,7}$.

3 Identifying Removable Intervals

According to Theorem 2.2, DEFINITELY(Φ) holds if and only if at least one OGI-set exists. To detect DEFINITELY(Φ) efficiently, the main idea of this paper is to derive only the *minimum* OGI-set and treat the others as *removable*. An OGI-set I in E-set $E_{i,x}$ is minimum if $I \preceq I'$ for all OGI-sets I' in $E_{i,x}$. The minimum OGI-set in $E_{i,x}$ is given by $F(E_{i,x})$.

To simplify our presentation, *pseudo event* and *volatile interval* are defined in Definition 3.1.

Definition 3.1 *For an E-set $E_{i,x}$, define the volatile interval $\widehat{t}$ for each process P_j as follows:*

1. *If an interval t exists in P_j, it satisfies $t.lo \in E_{i,x}$ but $t.hi \notin E_{i,x}$* (e.g. *interval t in Figure 5), then $\widehat{t}.lo = t.lo$, and $\widehat{t}.hi$ is a pseudo event with $vector(\widehat{t}.hi) = [\infty, \infty, \ldots, \infty]$.*
2. *Otherwise, let $e_{j,y} \in E_{i,x}$ be the last event from P_j* (e.g. *event with vector clock* $[3, 0, 0]$ *in Figure 5), both $\widehat{t}.lo$ and*

$\widehat{t}.hi$ are pseudo events where $vector(\widehat{t}.lo) = vector(e_{j,y})$ but $vector(\widehat{t}.lo)[j] = vector(e_{j,y})[j] + 1$, and $vector(\widehat{t}.hi) = [\infty, \infty, \ldots, \infty]$.

The interval without pseudo events is called nonvolatile. *This E-set contains all events in $E_{i,x}$ and its pseudo events are denoted by $\widehat{E}_{i,x}$. Notably, $E_{i,x} \subseteq \widehat{E}_{i,x}$. The vector of volatile intervals in $\widehat{E}_{i,x}$ is denoted by $f(\widehat{E}_{i,x})$.* ■

An E-set $E_{i,x}$ may not contain an OGI-set. However, $\widehat{E}_{i,x}$ always contains an OGI-set because intervals in $f(\widehat{E}_{i,x})$ are pairwisely overlapped. This is due to $vector(v.hi) = [\infty, \infty, \ldots, \infty]$ for any interval $v \in f(\widehat{E}_{i,x})$. Intuitively, $f(\widehat{E}_{i,x})$ is a candidate of OGI-sets within future E-sets. The following property is useful in the remainder of this paper:

P2 Let E-set $E_{i,x} = E_{j,y} \cup E_{k,z}$. The events in $E_{i,x}$ is from either $E_{j,y}$ or $E_{k,z}$. Hence, the interval t is nonvolatile in $E_{i,x}$ if and only if t is nonvolatile in either $E_{j,y}$ or $E_{k,z}$.

Given an E-set $E_{i,x}$, intervals are said to be removable if they do not belong to the minimum OGI-sets of all the future E-sets of $E_{i,x}$, because deriving the minimum is our only concern. Specifically, an interval $t \in E_{i,x}$ is $E_{i,x}$-*removable* if $t \notin F(E_{j,y})$ for all $E_{j,y}$, where $E_{i,x} \subseteq E_{j,y}$. Otherwise t is $E_{i,x}$-*nonremovable*. Note that a removable interval must be nonvolatile. By this definition, the following property holds:

P3 If interval t is $E_{i,x}$-removable then t is $E_{j,y}$-removable for all $E_{j,y}$ where $E_{i,x} \subseteq E_{j,y}$.

Next, the necessary and sufficient condition to identify removable intervals is derived in Theorem 3.1.

Theorem 3.1 *In an E-set $E_{i,x}$, a nonvolatile interval t is $E_{i,x}$-removable if and only if $t \notin F(\widehat{E}_{i,x})$.*

Proof. ($\Rightarrow$) If t is $E_{i,x}$-removable, since $\widehat{E}_{i,x}$ is a future E-set of $E_{i,x}$, $t \notin F(\widehat{E}_{i,x})$ can be derived (Property P3).

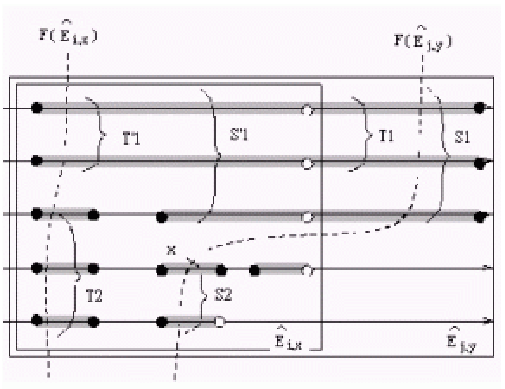


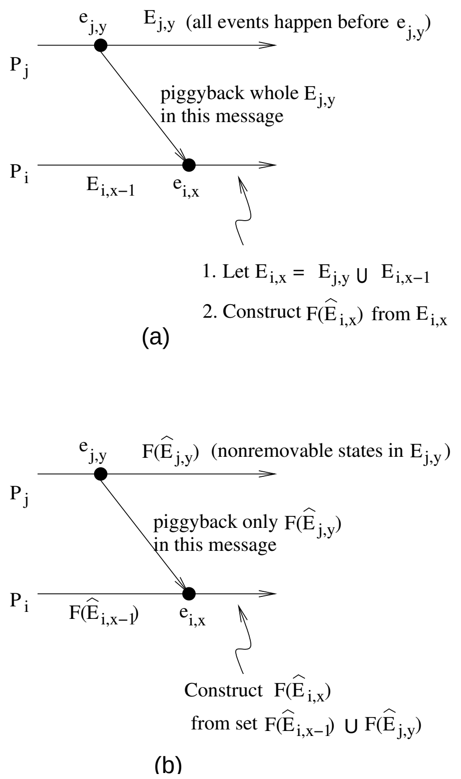
Figure 6: Illustration of the minimum OGI-sets of E-set $\widehat{E}_{i,x}$ and $\widehat{E}_{j,y}$, where $E_{i,x} \subset E_{j,y}$. Note that the messages are not drawn for clarity in this figure.

($\Leftarrow$) Let $E_{j,y}$ be a future E-set of $E_{i,x}$. We will prove this direction by showing that if $t \in F(\widehat{E}_{j,y})$ then $t \in F(\widehat{E}_{i,x})$. As illustrated in Figure 6, partition $F(\widehat{E}_{j,y})$ into $S_1 \cup S_2$, where $S_2 \subseteq \widehat{E}_{i,x}$ and $S_1 = F(\widehat{E}_{j,y}) \setminus S_2$ (note that $S_2 \neq \{\}$ since $x \in S_2$). Since $S_1 \rightarrow t$, we can derive that there exists a set of volatile intervals in $\widehat{E}_{i,x}$, say S_1', such that the set of beginning events of S_1 and S_1' are identical.

The set ${S_1}' \cup S_2$ is an OGI-set in $\widehat{E}_{i,x}$ since ${S_1}' \rightarrow S_2$ and $S_2 \rightarrow {S_1}'$ (because the intervals in S_1' are volatile). Next, we show that ${S_1}' \cup S_2$ is the minimum OGI-set of $\widehat{E}_{i,x}$ *i.e.* $S_1' \cup S_2 = F(\widehat{E}_{i,x})$. By contradiction, assume that ${S_1}' \cup S_2 \preceq F(\widehat{E}_{i,x})$ and ${S_1}' \cup S_2 \neq F(\widehat{E}_{i,x})$. As shown in Figure 6, partition $F(\widehat{E}_{i,x})$ into ${T_1}' \cup T_2$, where ${T_1}'$ and T_2 are sets with volatile and nonvolatile intervals, respectively. Through Figure 6, we can derive that ${T_1}' \subseteq {S_1}'$ because $\{{T_1}' \cup T_2\} \preceq \{{S_1}' \cup S_2\}$ and a volatile interval must be the last interval in the process. Let T_1 represent the set containing nonvolatile intervals in S_1, as shown in Figure 6. (Note that the set of beginning events of T_1 and T_1' are identical.) Then, $T_1 \cup T_2$ is an OGI-set in $E_{j,y}$ because

1. $T_1 \rightarrow T_2$ in $E_{j,y}$ since ${T_1}' \rightarrow T_2$ in $F(\widehat{E}_{i,x})$.
2. $T_2 \rightarrow T_1$ in $E_{j,y}$ since $T_2 \rightarrow S_2$, $S_2 \rightarrow S_1$, and $T_1 \subseteq S_1$.

Therefore, $T_1 \cup T_2$ is an OGI-set in $E_{j,y}$ and $\{T_1 \cup T_2\} \preceq \{S_1 \cup S_2\}$. This derivation con-



$e_{j,y}$
$E_{j,y}$ (all events happen before $e_{j,y}$)
P_j
piggyback whole $E_{j,y}$ in this message
P_i
$E_{i,x-1}$
$e_{i,x}$
1. Let $E_{i,x} = E_{j,y} \cup E_{i,x-1}$
2. Construct $F(\widehat{E}_{i,x})$ from $E_{i,x}$
(a)
$e_{j,y}$
$F(\widehat{E}_{j,y})$ (nonremovable states in $E_{j,y}$)
P_j
piggyback only $F(\widehat{E}_{j,y})$ in this message
P_i
$F(\widehat{E}_{i,x-1})$
$e_{i,x}$
Construct $F(\widehat{E}_{i,x})$ from set $F(\widehat{E}_{i,x-1}) \cup F(\widehat{E}_{j,y})$
(b)

Figure 7: (a) Illustration of the detection without the notion of removable states. (b) Illustration of the detection that discards the removable states.

tradicts that $S_1 \cup S_2$ is the minimum OGI-set in $E_{j,y}$. Therefore, the theorem holds. ■

The following corollary extended from this theorem is useful for the later parts of this paper.

Corollary 3.1 *If E-set* $E_{j,y} \subseteq E_{i,x}$ *then* $F(\widehat{E}_{j,y}) \preceq F(\widehat{E}_{i,x})$.

Proof. Based on Theorem 3.1, this corollary is accurate because removable intervals in the E-set $E_{j,y}$ must be also removable in its future E-sets. ■

4 Distributed Online Algorithm to Detect DEFINITELY(Φ)

4.1 Using the Notion of Removable States

This subsection shows the difference between the detection algorithms with and without using the notion of removable states. Figure 7 illustrates the detection scenarios. In part (a) of this figure, when process P_i receives a message by executing event $e_{i,x}$, it first constructs the new E-set $E_{i,x}$ from its old E-set $E_{i,x-1}$ and the new received E-set $E_{j,y}$, *i.e.* $E_{i,x} = E_{i,x-1} \cup E_{j,y}$. Then, it computes set $F(\widehat{E}_{i,x})$ from set $E_{i,x}$. Unfortunately, this simple approach is intractable, since the E-sets grow each time whenever an instruction is executed. Part (b) of Figure 7 illustrates how the notion of removables can be applied to improve the detection. It employs only the minimum OGI-set rather than employing whole E-set, based on the concept depicted in Corollary 4.1. Clearly, this approach incurs a very low overhead to the distributed system, because an OGI-set contains only $O(p)$ members.

Corollary 4.1 *Assume that process P_j sends a message, say m, to process P_j, where the send and receive event of m are $e_{j,y}$ and $e_{i,x}$, respectively. The intervals in $F(\widehat{E}_{i,x})$ must be from either $F(\widehat{E}_{i,x-1})$ or $F(\widehat{E}_{j,y})$.*

Proof. E-set $E_{i,x}$ is a future E-set of $E_{i,x-1}$ and $E_{j,y}$ since $E_{i,x} = E_{i,x-1} \cup E_{j,y}$. According to Theorem 3.1, the intervals in $F(\widehat{E}_{i,x})$ must be from either $F(\widehat{E}_{i,x-1})$ or $F(\widehat{E}_{j,y})$. ■

A naive approach to compute $F(\widehat{E}_{i,x})$ in Figure 7(b) is to apply the algorithms that were proposed in [8, 12] and let $F(\widehat{E}_{i,x-1})$ and $F(\widehat{E}_{j,y})$ be the inputs of the algorithms. Their algorithms work by testing overlap between intervals and remove useless intervals systematically. However, in worst case, in each run it performs $O(p^2)$ testing to ensure that all the p intervals (one from each process) are pairwisely overlapped. The total time is $O(p^2m)$ if the maximum number of events in one process is m. In this section, a more efficient detection algorithm that runs in time $O(pm)$ is proposed. In Subsection 4.2 we present an efficient approach to maintain the minimum OGI-sets. Based on this result, Subsection 4.3 presents our new detection algorithm and analysis its complexity and correctness.

4.2 Maintain Minimum OGI-sets Efficiently

Let X, Y, and Z be the E-sets satisfy the condition $Z = X \cup Y$. This section describes how to derive the minimum OGI-set of Z by given the minimum OGI-sets of subsets X and Y. Before describing our approach, some notations used in this section are defined as follows. To identify one interval t in different E-sets, let $t^{(A)}$ refer t in E-set A (*i.e.* $t.lo = t^{(A)}.lo$). An interval $t^{(A)}$ is said B-removable if $t^{(B)}$ exists in B and is B-

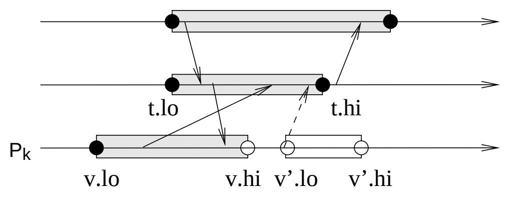


Figure 8: A volatile interval v becomes removable implies that all the nonvolatile intervals become removable.

removable.

The minimum OGI-set of Z is derived by finding those intervals not removable in X or Y, but becomes removable in Z $(= X \cup Y)$. The following Lemma demonstrates that the nonvolatile intervals remain nonremovable until some volatile interval becomes removable.

Lemma 4.1 *Let $Z = X \cup Y$ be the E-sets as described above. All the nonvolatile intervals in $F(\widehat{X})$ become Z-removable if and only if some volatile interval in $F(\widehat{X})$ becomes Z-removable.*

Proof. ($\Leftarrow$) Consider a nonvolatile interval $t^{(X)}$ and a volatile interval $v^{(X)}$ in $F(\widehat{X})$. Assume that $v^{(X)}$ is in process P_k. Figure 8(a) illustrates that v is the last interval in P_k that can overlap with t. (Otherwise, an interval v' exists in P_k such that $v.hi \rightarrow v'.lo \rightarrow t.hi$, contradicts with v is volatile in X.) This implies that if v becomes Z-removable then t also becomes Z-removable.

($\Rightarrow$) By contradiction, assume that all volatiles in $F(\widehat{X})$ are Z-nonremovable whereas the nonvolatile t is Z-removable. $F(\widehat{X}) = F(\widehat{Z})$ is proved as follows. For each ordered pair of $t_1, t_2 \in F(\widehat{X})$ (note that $t_1^{(X)} \rightarrow t_2^{(X)}$), the property $t_1^{(Z)} \rightarrow t_2^{(Z)}$ holds because:

- If $t_2^{(X)}$ is nonvolatile (Figure 9(a)) then $t_1^{(X)} \rightarrow t_2^{(X)}$ implies $t_1^{(Z)} \rightarrow t_2^{(Z)}$.
- If $t_2^{(X)}$ is volatile and $t_2^{(Z)}$ remains volatile (Figure 9(b)), clearly, $t_1^{(Z)} \rightarrow t_2^{(Z)}$ (see Definition 3.1).
- If $t_2^{(X)}$ is volatile but $t_2^{(Z)}$ becomes nonvolatile (Figure 9(c)), the property $t_1^{(Z)} \rightarrow t_2^{(Z)}$ is satisfied because: $t_1^{(X)} \in F(\widehat{X})$, $t_2^{(Z)} \in F(\widehat{Z})$ (this is because $t_2^{(X)}$ is volatile in $F(\widehat{X})$ and thus is Z-nonremovable by assumption), and $F(\widehat{X}) \preceq F(\widehat{Z})$ (see Corollary 3.1).

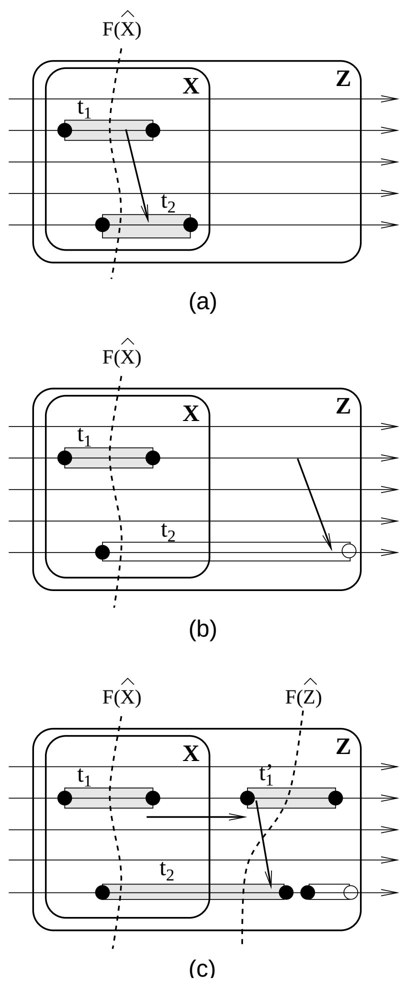


Figure 9: Illustration of proof of Lemma 4.1.

Therefore, $F(\widehat{X}) = F(\widehat{Z})$, contradicts with the fact that $t \in F(\widehat{Z})$ is Z-removable. ∎

Next, Lemma 4.2 and 4.3 demonstrate the condition for volatile intervals.

Lemma 4.2 *Let $Z = X \cup Y$ be the E-sets as described above. If neither $F(\widehat{X}) \preceq F(\widehat{Y})$ nor $F(\widehat{Y}) \preceq F(\widehat{X})$, then, $F(\widehat{Z}) = f(\widehat{Z})$.*

Proof. If $F(\widehat{X}) \npreceq F(\widehat{Y})$ and $F(\widehat{Y}) \npreceq F(\widehat{X})$, there exist intervals $t_1, t_2 \in F(\widehat{X})$ and $u_1, u_2 \in F(\widehat{Y})$ such that $\{t_1\} \prec \{u_2\}$ and $\{u_1\} \prec \{t_2\}$. Therefore, t_1 and u_1 are nonvolatile in Z and is Z-removable. From Lemma 4.1, all nonvolatile intervals in $F(\widehat{X})$ and $F(\widehat{Y})$ are Z-removable. Thus, $F(\widehat{Z}) = f(\widehat{Z})$. ∎

Lemma 4.3 *Let $Z = X \cup Y$ be the E-sets as described above. Furthermore, assume that $F(\widehat{Y}) \preceq F(\widehat{X})$. A volatile interval $t^{(X)} \in F(\widehat{X})$ becomes Z-removable if and only if $t^{(Y)}$ is nonvolatile and the following properties are satisfied:*

1. *$t^{(Y)}$ is Y-removable, or*
2. *$t^{(Y)}$ is Y-nonremovable and some volatile interval $u \in F(\widehat{Y})$ is X-removable.*

Proof. In this proof, only the second case, *i.e.* t is Y-nonremovable, is considered. (In the first case, clearly, t is Y-removable implies that t is Z-removable.)

($\Leftarrow$) If u is X-removable, from Lemma 4.1, all the nonvolatile intervals in $F(\widehat{Y})$, including $t^{(Y)}$, are Z-removable.

($\Rightarrow$) First, $t^{(Y)}$ must be nonvolatile from Property P2. By contradiction, assume that t is Z-removable but $t^{(Y)}$ is Y-nonremovable and all the volatile intervals $u \in F(\widehat{Y})$ are X-nonremovable. This assumption implies that $F(\widehat{X}) = F(\widehat{Y})$ (the detail is ignored since the proof is very similar to Lemma 4.1). Since $Z = X \cup Y$, $F(\widehat{X}) = F(\widehat{Y})$ is an OGI-set in Z. This implies that $F(\widehat{Z}) = F(\widehat{X}) = F(\widehat{Y})$, contradict with the fact that $t \in F(\widehat{X}) = F(\widehat{Z})$ is Z-removable. ■

4.3 The New Detection Algorithm

In our new detection algorithm, each process keeps only two p-tuple vectors F and f to represent the minimum OGI-set and the volatile interval set of this process, respectively. This algorithm consists of the following procedures that are executed at each process P_i:

- **Procedure** InternalEvent (to be called when P_i executes an internal event, say $e_{i,x}$):
  1. After execution of event $e_{i,x}$, the truth value of LP_i may change, as follows:
     (a) The truth value of LP_i is unchanged: Both F and f are remain unchanged.
     (b) The truth value of LP_i is in a false-to-true transition: Modify $f[i]$ to the new volatile interval whose beginning event is $e_{i,x}$.
     (c) The truth value of LP_i is in a true-to-false transition: In this case, the current volatile interval of process P_i is ended at event $e_{i,x}$. Modify $f[i]$ to the new volatile interval in which both beginning and end events are pseudo.

- **Procedure** SendEvent (to be called when P_i executes a send event, say $e_{i,x}$):
  1. Since the truth value of LP_i is unchanged, both F and f are also unchanged.
  2. Piggyback F and f in the message and then send it.

- **Procedure** ReceiveEvent (to be called when P_i executes a receive event, say $e_{i,x}$):
  1. Assume that the sent event of this message is $e_{j,y}$. Receive the message and extract data F' (represent $F(\widehat{E}_{j,y})$) and f' (represent $f(\widehat{E}_{j,y})$).
  2. Let $f[k] = \max(f[k], f'[k])$, $k = 1, 2, \ldots, p$.
  3. (Based on Lemma 4.2) If $F(\widehat{E}_{i,x-1}) \not\preceq F(\widehat{E}_{j,y})$ or $F(\widehat{E}_{j,y}) \not\preceq F(\widehat{E}_{i,x-1})$ then $F = f$. Go to Step 7.
  4. If $F(\widehat{E}_{j,y}) \preceq F(\widehat{E}_{i,x-1})$, determine whether an interval t is $E_{i,x}$-removable as follows:
     (a) (Based on Property P3) If $t \prec F(\widehat{E}_{i,x-1})$ then t is removable.
     (b) (Based on Lemma 4.3) If $t \in F(\widehat{E}_{i,x-1})$ and t is volatile:
        i. If the following properties are satisfied, mark t as removable:
           A. t is $E_{j,y}$-removable, or
           B. t is $E_{j,y}$-nonremovable and some volatile interval $u \in F(\widehat{E}_{j,y})$ is $E_{i,x-1}$-removable. For example, consider an interval t_2 illustrated in Figure 10. The interval t_2 is $E_{j,y}$-nonremovable and volatile interval $t_1 \in F(\widehat{E}_{j,y})$ is $E_{i,x-1}$-removable (Figure 10(a) and (b)). Hence, t_2 is $E_{i,x}$-removable (Figure 10(c)).
     (c) (Based on Lemma 4.1) If $t \in F(\widehat{E}_{i,x-1})$ and t is nonvolatile:

i. If any volatile interval is marked as removable in Step 4(b), mark t as removable.

5. If $F(\widehat{E}_{i,x-1}) \preceq F(\widehat{E}_{j,y})$, repeat Step 4 except that the roles of $F(\widehat{E}_{i,x-1})$ and $F(\widehat{E}_{j,y})$ are swapped.
6. Let $F(\widehat{E}_{i,x})$ be the set of intervals that not have been marked as removable in previous steps.
7. If $F(\widehat{E}_{i,x})$ contains no volatile then DEFINITELY(Φ) is true. Otherwise, DEFINITELY(Φ) is false.

In this algorithm, a process cannot evaluate the global predicate if there are no messages sent from other processes to carry the debug information. To solve this problem, if DEFINITELY(Φ) is still false at the end of the program execution, p extra messages are sent among all p processes in a circular way to pass the debug information. However, as compared with the cost of the entire distributed computation, these p messages incur a very low overhead.

The correctness of this algorithm can be verified easily based on the theorems presented in Subsection 4.2. Before analyzing the complexity of the algorithm, the implementation has to be explained. First, vectors F and f are implemented by using vectors of integers: when process P_i executes the event $e_{i,x}$, the value $F[j]$ (resp. $f[j]$) equals the sequence number of the beginning event of interval $F(\widehat{E}_{i,x})[j]$ (resp. $f(\widehat{E}_{i,x})[j]$). The operations of the algorithm is implemented as follows:

- $F(\widehat{E}_{j,y}) \preceq F(\widehat{E}_{i,x-1})$ iff $F'[k] \leq F[k]$, for all k (assume that F' refers to $F(\widehat{E}_{j,y})$ and F refers to $F(\widehat{E}_{i,x-1})$). This operation takes $O(p)$ time.
- Interval $F[k]$ is volatile iff $F[k] = f[k]$. This operation takes $O(1)$ time.
- Interval t with $t.lo = e_{k,z}$ is $E_{i,x}$-removable if t is nonvolatile and $F[k] \neq z$, (assume that F refers to $F(\widehat{E}_{i,x})$). This operation takes $O(1)$ time.

Based on the above implementation, each invoke of the procedures (InternalEvent, SendEvent, and ReceiveEvent) requires $O(p)$ time. Assume that

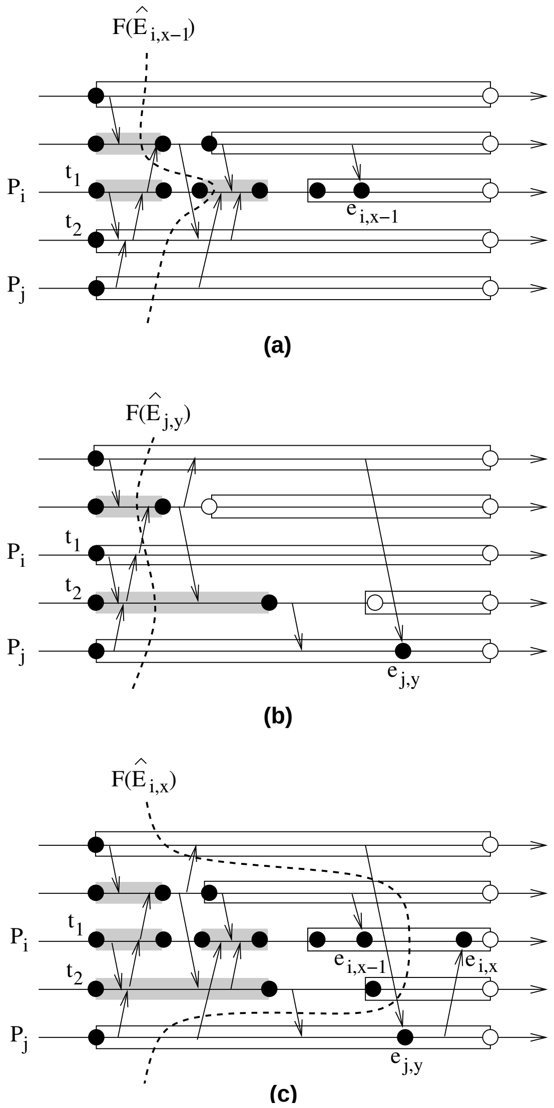


Figure 10: Illustration of (a) $F(\widehat{E}_{i,x-1})$, (b) $F(\widehat{E}_{j,y})$, and (c) $F(\widehat{E}_{i,x})$.

there are m_i events for process P_i, the total time complexity for the process is $O(pm_i)$.

5 Discussion

This paper investigates the problem of detection of definitely true conjunctive predicates (DEFINITELY(Φ)). To solve the problem of unbounded queue growth of previous detection algorithms, in this paper, the notion of removable states is introduced. By discarding the removable states, the space requirement for each process can be minimized to $O(p)$, where p is the number of processes. While bounding the memory space, this analysis shows that the time complexity of the proposed algorithm is only $O(pm)$, faster than previous algorithms are, by a factor of p.

Another related detection problem regarding to the distributed debugging is to detect what possibly true conjunctive predicates (POSSIBLY(Φ)) [7, 11, 12]. To enhance the performance of POSSIBLY(Φ) detection algorithms, Chiou and Korfhage [3] presented two algorithms to remove some useless states for the detection. However, their algorithms run in a centralized environment and identify only partial useless states. For the distributed detection of POSSIBLY(Φ), finding an efficient approach to identify the removable states is still a research topic.

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