Performance Modeling of Mobile Prepaid and Priority Call Services

Ming-Feng Chang and Wei-Zu Yang
Dept. of Comp. Sci. & Info. Engr.
National Chiao Tung University
Hsinchu, Taiwan, R.O.C.
Fax: +886-3-5724176
Email: {mfchang, wzyang}@csie.nctu.edu.tw

Abstract

In this paper, we conduct simulation experiments to study a mobile phone system that provides priority and prepaid call services using service nodes. An analytic model for the non-prepaid and prepaid services has been developed to verify the computer simulations. We observe that the traffic ratio of prepaid calls has significant effect on the system performance. Our experiments also show that there exists a threshold point such that beyond this point, increasing the number of mobile switching center ports (service node ports or the radio channels) does not improve the system performance. Furthermore, two priority assignment schemes, namely, scheme PP (prepaid calls as the priority calls) and scheme NP (non-prepaid calls as the priority calls) are compared based on a revenue function. Our results indicate that PP is better than NP when the prepaid charge rate and the prepaid traffic ratio are higher than a threshold.

Keywords: prepaid service, priority call, service node.

I. Introduction

Recently, the prepaid service has grown rapidly in mobile phone service business. It is predicted that the revenue of prepaid service grows from US$1.3 billion in 1999 to US$14.2 billion by 2004 [10]. In USA, the prepaid calling market grew 56% to about two billion US dollars in 1998 and is expected to maintain a high growth rate to 2005 [11]. Alcatel has predicted that more than 40% GSM customers will subscribe to prepaid services by 2001 [11]. At the end of 2003, mobile prepaid service is expected to account for 62% of the cellular user base [9].

Prepaid service is a win-win solution for both the customers and the service providers. From the customer's point of view, prepaid service provides an immediate service without a long-term contract or regular bills. As the charge rate is understood and the credit is paid in advance, the prepaid customers have better control over their telephone fees than the postpaid customers. From the system provider's point of view, prepaid service enlarges the customer base and reduces operation overhead such as printing monthly bills and checking customer's credit before providing services. Since the customers pay before they use the service, prepaid service improves the cash flow for the operators. The revenue of prepaid service is received typically one and half month earlier than the postpaid.

Four billing technologies have been used in mobile prepaid service: hot billing approach, service node approach, intelligent network approach and handset-based approach. We have studied performance of the hot billing approach in [2] and the service node approach in [3]. The details and comparison of four approaches can also be found in [8]. Since the service node is widely deployed today, we use it as the platform for prepaid services. The architecture of the service node approach is depicted in Fig. 1. In

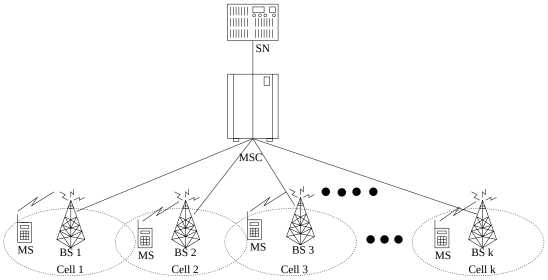


Figure 1: The architecture of the service node approach

this architecture, a service node (SN) is connected to a mobile switching center (MSC) using high-speed T1/E1 trunks and the MSC is connected to base stations (BSs). Each BS communicates with mobile stations through radio channels.

Two types of calls (i.e., non-prepaid calls and prepaid calls) exist in the mobile network. For each originating prepaid call, the MSC routes the call to the service node for call processing. After the service node performs service control functions (e.g., checking the credit), the prepaid call is routed back to the MSC and then to the called party. Thus, to set up a prepaid call, it requires one radio channel, four MSC ports and two SN ports. On the other hand, for a non-prepaid new call, the MSC directly routes the call to its destination. Hence, it requires one radio channel and two MSC ports to set up a non-prepaid call.

When a mobile initiates a call attempt and the resources are insufficient, the call is blocked. This is referred to as new call blocking. The probability that a call setup fails is referred to as the *new call blocking* probability. During the conversation, a mobile station can move from a cell to another cell. This procedure is referred to as call handoff. To support handoff, a radio channel of the new BS is assigned to the call and the radio channel of the old BS is released. If no free radio channel is available in the new BS, the call is forced to terminate. The probability that a handoff request is rejected is referred to as the *forced-termination* probability. The *call incompletion* probability is the probability that a call is either blocked or forced to terminate at handoff.

Many priority solutions based on the queuing schemes have been proposed to reduce the call blocking and incompletion rates [4, 12]. In the queuing schemes, when the resources are not enough, a priority call is placed in a queue until the resources are sufficient. On the other hand, a call request without the priority service is blocked if the resources are insufficient. We have studied the priority call service in [12]. In this paper, we use computer simulations to study the performance of a mobile network that provides priority and prepaid call services.

This paper is organized as follows. Section 2 presents an analytic model for the prepaid and non-prepaid services to verify the correctness of computer simulations. Section 3 presents the numeric results and the conclusions are given in Section 4.

II. An Analytic Model

In this section, we use the analytic technique developed in [6] to study the performance of prepaid and non-prepaid call services. This technique uses an iterative algorithm where the Kelly model [5] is utilized in each iteration to compute the blocking probabilities. Since the Kelly model does not consider the queuing effect of the priority calls, the results of numeric analysis were used only to verify the correctness of computer simulations that support both prepaid and priority services.

Assume that the MSC connects to k BSs. BS i is equipped with c_i radio channels for communication. Let the number of ports of the MSC and the SN be $2 \times C_{MSC}$ and $2 \times C_{SN}$, respectively. The call arrivals to BS i are assumed to be Poisson with rate $\lambda_{o1,i}$ for non-prepaid new calls and with rate $\lambda_{o2,i}$ for prepaid new calls. The call holding times of both non-prepaid and prepaid calls are assumed to be exponentially distributed with mean $1/\mu$. Assume that the cell residence time of a mobile station has a general distribution with mean $1/\eta$.

The network for mobile prepaid services can be viewed as a network of nodes and links. The nodes represent the BSs, the MSC and the SN shown in Fig. 1; a link represents a radio channel between an MS and a BS, the circuit between a BS and the MSC or the circuit between the MSC and the SN.

In the Kelly model, a route is a fixed path that accepts some calls in progress. Assume that $\Re$ denote the set of these fixed routes. Let $r_{1,i}$ $(1 \leq i \leq k)$ denote the route of non-prepaid calls served by BS i; $r_{1,i}$ includes the link between an MS and BS i and the link between BS i to the MSC. Let $r_{2,i}$ $(1 \leq i \leq k)$ denote the route of prepaid calls served by BS i; $r_{2,i}$ includes the link between an MS and BS i, the link connecting BS i to the MSC and the link connecting the MSC to the SN. Thus, all routes in a mobile prepaid service network can be described by $\Re = \{r_{1,1}, r_{1,2}, \ldots, r_{1,k}, r_{2,1}, r_{2,2}, \ldots, r_{2,k}\}$. Note that a non-prepaid new call on route $r_{1,i}$ requires one radio channel and two MSC ports. A prepaid new call on route $r_{2,i}$ requires one radio channel, four MSC ports and two SN ports. Both non-prepaid and prepaid calls compete for the radio channels and the MSC ports. The SN ports are used only by the prepaid calls.

Let $\mathbf{n} = [n_{r_{1,1}}, n_{r_{1,2}}, \ldots, n_{r_{1,k}}, n_{r_{2,1}}, n_{r_{2,2}}, \ldots, n_{r_{2,k}}]$ be a $|\Re| \times 1$ matrix, where $n_{r_{1,i}}$ $(n_{r_{2,i}})$ is the number of non-prepaid (prepaid) calls in progress on the route $r_{1,i}$ $(r_{2,i})$, $\forall r_{1,i}$ $(r_{2,i}) \in \Re$. Therefore, vector $\mathbf{n}$ denotes the state of a stochastic process on the network. Since the total amount of resources (channels and MSC/SN ports) used by the outstanding calls should be no more than the link capacities of the routes, a legal state in the state space S must satisfy the following inequalities:

$$n_{r_{1,i}} + n_{r_{2,i}} \leq c_i, \text{ for } 1 \leq i \leq k \qquad (1)$$

$$\sum_{i=1}^{k} n_{r_{1,i}} + 2\sum_{i=1}^{k} n_{r_{2,i}} \leq C_{MSC} \qquad (2)$$

$$\sum_{i=1}^{k} n_{r_{2,i}} \leq C_{SN} \qquad (3)$$

Inequality (1) indicates that total number of non-prepaid and prepaid calls in a BS should be no more than the number of radio channels in the BS. Inequality (2) indicates that total number of MSC ports used by non-prepaid and prepaid calls should be no more than the capacity of the MSC. Inequality (3) indicates that total number of SN ports used by the prepaid calls should be no more than the capacity of the SN. The state space S of the stochastic process is expressed as

$$S = \{\mathbf{n} | n_{r_{1,i}} \text{ and } n_{r_{2,i}} \text{ satisfy inequalities } (1), (2) \text{ and } (3)\}$$

Let $\rho_{1,i}$ and $\rho_{2,i}$ be the offered load on the route $r_{1,i}$ and $r_{2,i}$, respectively. Let $P_{o,1}(i)$ be the new call blocking probability of non-prepaid calls in BS i and $P_{o,2}(i)$ be that of prepaid calls in BS i. Let $P_{f,1}(i)$ and $P_{f,2}(i)$ denote the forced-termination probability of non-prepaid and prepaid calls in BS i, respectively. From the offered load equation derived in [7] for the PCS handoff model, $\rho_{1,i}$ and $\rho_{2,i}$ can be expressed as

$$\begin{aligned}\rho_{1,i} &= \left(\frac{\lambda_{o1,i}}{\mu+\eta}\right)\left[1 + \frac{\eta(1-P_{o,1}(i))}{u+\eta P_{f,1}(i)}\right] \\ \rho_{2,i} &= \left(\frac{\lambda_{o2,i}}{\mu+\eta}\right)\left[1 + \frac{\eta(1-P_{o,2}(i))}{u+\eta P_{f,2}(i)}\right], \\ & \text{for } 1 \leq i \leq k \qquad (4)\end{aligned}$$

According to Kelly model, the stationary probability of the state $\mathbf{n} = (n_{r_{1,1}}, \ldots, n_{r_{1,k}}, n_{r_{2,1}}, \ldots, n_{r_{2,k}})$ can be computed as

$$p(\mathbf{n}) = G^{-1}\left[\prod_{i=1}^{k} \frac{\rho_{1,i}^{n_{r_{1,i}}}}{(n_{r_{1,i}})!}\right]\left[\prod_{i=1}^{k} \frac{\rho_{2,i}^{n_{r_{2,i}}}}{(n_{r_{2,i}})!}\right] \qquad (5)$$

where G is a normalized factor to ensure that $\sum_{\mathbf{n}\in S} p(\mathbf{n}) = 1$. Let S_1^i denote the state space where a non-prepaid new call is blocked in BS i. A non-priority new call is blocked if either no radio channel is available or the capacity of the MSC is insufficient. A state $\mathbf{n} \in S_1^i$ if $\mathbf{n}$ satisfies one of the following equations.

$$\begin{aligned} n_{r_{i,1}} + n_{r_{i,2}} &= c_i \\ \sum_{i=1}^{k} n_{r_{1,i}} + 2\sum_{i=1}^{k} n_{r_{2,i}} &= C_{MSC} \qquad (6)\end{aligned}$$

From (6), we have

$$P_{o,1}(i) = \sum_{\forall \mathbf{n}\in S_1^i} p(\mathbf{n}) \qquad (7)$$

Let S_2^i denote the state space where a prepaid new call is blocked in BS i. A prepaid new call is blocked if either no radio channel is available or the capacity of the MSC/SN is insufficient. A state $\mathbf{n} \in S_2^i$ if $\mathbf{n}$ satisfies one of the following equations or inequality.

$$\begin{aligned} n_{r_{i,1}} + n_{r_{i,2}} &= c_i \\ C_{MSC} - 2 < \sum_{i=1}^{k} n_{r_{1,i}} + 2\sum_{i=1}^{k} n_{r_{2,i}} &\leq C_{MSC} \\ \sum_{i=1}^{k} n_{r_{2,i}} &= C_{SN} \end{aligned} \tag{8}$$

From (8), we have

$$P_{o,2}(i) = \sum_{\forall \mathbf{n} \in S_2^i} p(\mathbf{n}) \tag{9}$$

Let S_3^i denote the state space where a handoff call is blocked in BS i. A handoff call request (i.e., either non-prepaid or prepaid) is rejected if no radio channel is available in the new BS. A state $\mathbf{n} \in S_3^i$ if $\mathbf{n}$ satisfies the following equation and inequalities.

$$\begin{aligned} n_{r_{i,1}} + n_{r_{i,2}} &= c_i \\ \sum_{i=1}^{k} n_{r_{1,i}} + 2\sum_{i=1}^{k} n_{r_{2,i}} &\leq C_{MSC} \\ \sum_{i=1}^{k} n_{r_{2,i}} &\leq C_{SN} \end{aligned} \tag{10}$$

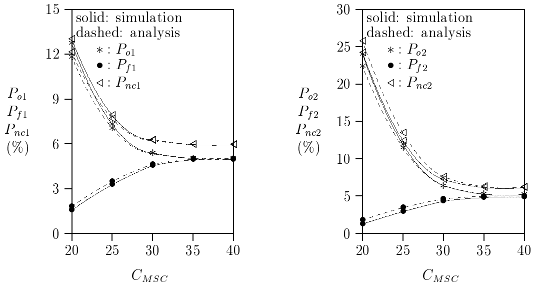
From (10), we have

$$P_{f,1}(i) = P_{f,2}(i) = \sum_{\forall \mathbf{n} \in S_3^i} p(\mathbf{n}) \tag{11}$$

Let $P_{nc,1}(i)$ and $P_{nc,2}(i)$ be the call incompletion probability of non-prepaid and prepaid calls, respectively. From Eq. (14) in [7], $P_{nc,1}(i)$ and $P_{nc,2}(i)$ can be expressed as

$$\begin{aligned} P_{nc,1}(i) &= P_{o,1}(i) \\ &+ \frac{\eta(1 - P_{o,1}(i))\,[1 - f_1^*(\mu)]\,P_{f,1}(i)}{\mu\,[1 - f_1^*(\mu) + P_{f,1}(i) f_1^*(\mu)]} \\ P_{nc,2}(i) &= P_{o,2}(i) \\ &+ \frac{\eta(1 - P_{o,2}(i))\,[1 - f_1^*(\mu)]\,P_{f,2}(i)}{\mu\,[1 - f_1^*(\mu) + P_{f,2}(i) f_1^*(\mu)]} \end{aligned} \tag{12}$$

where $f_1^*(\mu)$ is the Laplace transform of the density function of the residence time.



(a) P_{o1}, P_{f1} and P_{nc1}

(b) P_{o2}, P_{f2} and P_{nc2}

Figure 2: Effects of C_{MSC} ($k = 4$, $c = 6$, $\mu = 1\ \mathrm{min}^{-1}$, $\lambda_{o1} = \mu$, $\lambda_{o2} = 2\mu$, $\eta = \mu/5$ and $C_{SN} = 15$)

We consider a homogeneous case where $c_i = c$, $\lambda_{o1,i} = \lambda_{o1}$, $\lambda_{o2,i} = \lambda_{o2}$, $\rho_{1,i} = \rho_1$ and $\rho_{2,i} = \rho_2$, for $1 \leq i \leq k$. In this case, the blocking probabilities, forced-termination probabilities and call incompletion probabilities for all BSs are the same. We use the notations $P_{o1} = P_{o,1}(i)$, $P_{o2} = P_{o,2}(i)$, $P_{f1} = P_{f,1}(i)$, $P_{f2} = P_{f,2}(i)$, $P_{nc1} = P_{nc,1}(i)$ and $P_{nc2} = P_{nc,2}(i)$, for $1 \leq i \leq k$. The values of P_{o1}, P_{o2}, P_{f1}, P_{f2}, P_{nc1} and P_{nc2}, can be computed by using the iterative algorithm in [7] with Eqs. (7), (9), (11) and (12) until all values of blocking probabilities converge.

III. Numeric Results

Based on the analytic model described in Sec. 2, we have validated our simulation model. Each simulation experiment was repeated 4,000,000 times to ensure stable results. We consider a homogeneous network with four BSs (i.e., $k = 4$) and every BS is equipped with six radio channels (i.e., $c = 6$).

Figs. 2 (a) and (b) plot P_{o1}, P_{f1}, P_{nc1}, P_{o2}, P_{f2} and P_{nc2} as functions of C_{MSC}. In these figures, $\mu = 1\ \mathrm{min}^{-1}$, $\lambda_{o1} = \mu$, $\lambda_{o2} = 2\mu$, $\eta = \mu/5$, $20 \leq C_{MSC} \leq 40$ and $C_{SN} = 15$. The solid curves represent the simulation results, and the dashed curves are the analytic results. These figures indicate that the analytic results are consistent with the simulation results. Figs. 2 (a) and (b) also show that $P_{o1} \approx P_{f1}$ and $P_{o2} \approx P_{f2}$ when $C_{MSC} \geq 35$, indicating that the radio channels are bottleneck resources.

A. Effect of the Traffic Ratio of Prepaid Calls

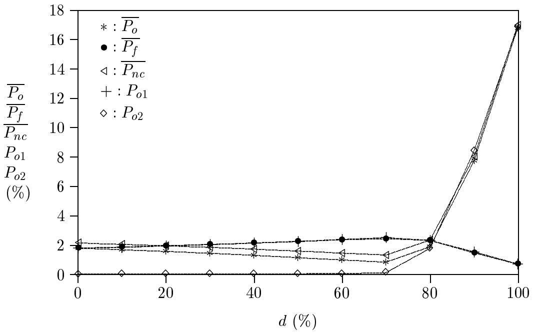


Figure 3: Effect of d ($\lambda_o = 5\mu$, $k = 36$, $c = 10$, $\mu = 1 \text{ min}^{-1}$, $\eta = \mu/5$, $C_{MSC} = 360$, $C_{SN} = 150$ and $T_\theta = 30$ secs)

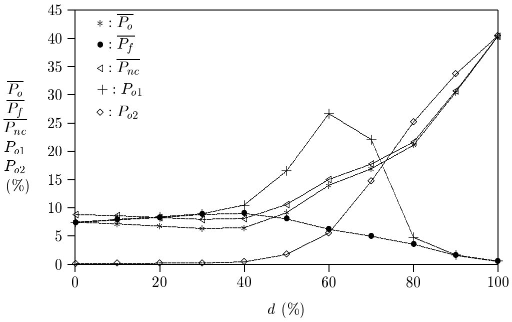


Figure 4: Effects of d ($\lambda_o = 7\mu$, $k = 36$, $c = 10$, $\mu = 1 \text{ min}^{-1}$, $\eta = \mu/5$, $C_{MSC} = 360$, $C_{SN} = 150$ and $T_\theta = 30$ secs)

In this subsection, we consider the case where the prepaid new calls are queued if there is no idle channel or sufficient MSC/SN ports. The queued calls can wait in the queue for T_θ seconds and can be served if the resources become available before the waiting timer expires and before the mobiles leave the cell.

Let λ_o be the total new call arrival rate of prepaid and non-prepaid calls (i.e., $\lambda_o = \lambda_{o1} + \lambda_{o2}$). Let d be the ratio of the new call arrival rate of prepaid calls over that of total new calls (i.e., $d = \lambda_{o2}/\lambda_o$). Let $\overline{P_o} = (1-d)P_{o1} + dP_{o2}$, $\overline{P_f} = (1-d)P_{f1} + dP_{f2}$ and $\overline{P_{nc}} = (1-d)P_{nc1} + dP_{nc2}$. Probabilities $\overline{P_o}$, $\overline{P_f}$ and $\overline{P_{nc}}$ provide the net effect of the output measures for both non-prepaid calls and prepaid calls.

Fig. 3 plots the effect of d on $\overline{P_o}$, $\overline{P_f}$, $\overline{P_{nc}}$, P_{o1} and P_{o2}, where $\lambda_o = 5\mu$, $k = 36$, $c = 10$, $\mu = 1 \text{ min}^{-1}$, $\eta = \mu/5$, $C_{MSC} = 360$, $C_{SN} = 150$ and $T_\theta = 30$ secs. Note that since a handoff call (prepaid or non-prepaid) needs only one radio channel in the new BS, we have $P_{f1} = P_{f2} = \overline{P_f}$. The figure shows that there exists a threshold point d^* such that $\overline{P_o}$ and $\overline{P_{nc}}$ are minimized and $\overline{P_f}$ is maximized. In this figure, $d^* \approx 70\%$. The phenomenon is explained as follows.

As d increases, the number of queued calls increases. The figure shows that for $d < d^*$, P_{o2} remains very low due to the queuing effect. The queued calls have priority to use the radio channels and MSC ports over the non-priority calls and handoff calls. Thus, P_{o1} and $\overline{P_f}$ increase as d increases when $d < d^*$. In addition, we observe that $P_{o1} = \overline{P_f}$. This phenomenon indicates that the MSC ports are not the bottleneck resources in this case. When the MSC ports are sufficient, the non-prepaid new calls compete with the handoff calls only for the radio channels, and thus $P_{o1} = \overline{P_f}$. Since the total traffic to the system is fixed, as d increases, the queuing effect of the prepaid new calls becomes significant. Thus, $\overline{P_o}$ decreases as d increases when $d < d^*$.

When $d > d^*$, the number of prepaid new calls increases and the service node becomes the bottleneck. As d increases, more prepaid new calls are dropped due to the lack of SN ports, and thus P_{o2} increases rapidly. This leads to more radio channels and MSC ports available to non-prepaid calls and handoff calls. Hence, P_{o1} and $\overline{P_f}$ decreases as d increases when $d > d^*$. Since the probabilities $\overline{P_o}$ and $\overline{P_{nc}}$ are dominated by P_{o2}, both probabilities increase rapidly as d increases.

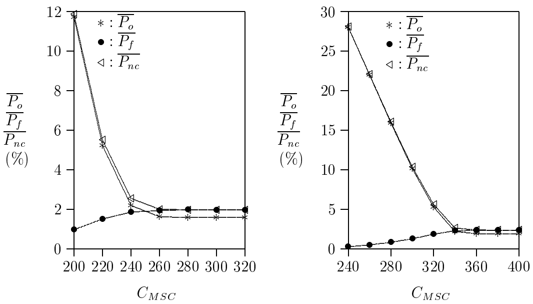
Fig. 4 plots $\overline{P_o}$, $\overline{P_f}$, $\overline{P_{nc}}$, P_{o1} and P_{o2} as functions of d, where the total new call arrival rate is 7μ. Compared with Fig. 3, the total traffic to the system in Fig. 4 increases from 5μ to 7μ and the value of d^* shifts from 70% to 30%. When $d \leq d^*$, the number of MSC/SN ports is sufficient. The non-prepaid new calls compete the radio channels with the handoff calls. We have $P_{o1} = \overline{P_f}$. Since a prepaid new call requires more MSC ports than a non-prepaid new call, more MSC ports are consumed as d increases. When $d > d^*$, the competition for the MSC ports between non-prepaid and prepaid new calls becomes significant and more new calls are dropped. Since the prepaid new calls have priority over the non-prepaid new calls, the probability P_{o1} increases rapidly and P_{o2} starts to increase. The increasing of P_{o1} and P_{o2} leads to more radio channels available for the handoff calls, and thus $\overline{P_f}$ decreases.

As d continues to increase, more SN ports are consumed by the prepaid new calls. When $d > 60\%$, the SN ports become the bottleneck and P_{o2} increases rapidly. As more prepaid new calls are dropped, more resources are available to the non-prepaid calls and handoff calls. Thus, P_{o1} starts to decrease and $\overline{P_f}$ continues decreasing when $d > 60\%$.

In 3G services, a prepaid customer may have a prepaid account and a normal post-usage billing account simultaneously [1]. The service profiles associated with each account do not have to be identical. When a customer originates a call, he/she can decide whether to activate the prepaid call service. Figs. 3 and 4 indicate that the traffic ratio of prepaid calls has significant effect on the system performance. Since the prepaid customers are sensitive to the call charges, in order to achieve the best system performance, the prepaid service provider should be careful in proposing their rate plans and service packages.

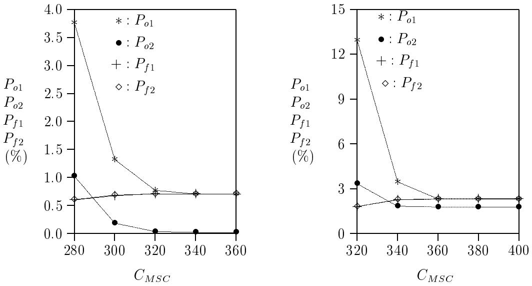
B. Effect of the Number of MSC ports

In this subsection, we consider the case where only the prepaid new calls are queued if there is no idle channel or sufficient MSC/SN ports. Fig. 5 plots $\overline{P_o}$, $\overline{P_f}$ and $\overline{P_{nc}}$ as functions of C_{MSC} for $d = 20\%$ and 80%, where $k = 36$, $c = 10$, $\mu = 1 \text{ min}^{-1}$, $\lambda_o = 5\mu$, $\eta = \mu/5$, $C_{SN} = 150$ and $T_\theta = 30$ secs. Intuition suggests that as C_{MSC} increases, the probability that a new call (non-prepaid or prepaid) is blocked decreases and $\overline{P_o}$ decreases. In addition, since the number of accepted new calls increases, the number of available channels decreases and $\overline{P_f}$ increases. Fig. 5 also shows that there exists a threshold point C^*_{MSC} such that beyond this point, increasing C_{MSC} does not improve the system performance. The value of C^*_{MSC} shifts from 260 to 340 for $d = 20\%$ (i.e., less prepaid calls) to 80% (i.e., prepaid calls dominate).



(a) $d = 20\%$ (b) $d = 80\%$

Figure 5: Effects of C_{MSC} for various d ($k = 36$, $c = 10$, $\mu = 1 \text{ min}^{-1}$, $\lambda_o = 5\mu$, $\eta = \mu/5$, $C_{SN} = 150$ and $T_\theta = 30$ secs)



(a) $\lambda_o = 4\mu$ (b) $\lambda_o = 5\mu$

Figure 6: Effects of C_{MSC} on the new call blocking and forced-termination probabilities ($k = 36$, $c = 10$, $\mu = 1 \text{ min}^{-1}$, $\eta = \mu/5$, $d = 80\%$, $C_{SN} = 150$ and $T_\theta = 30$ secs)

Fig. 6 plots the effect of C_{MSC} on the new call blocking and forced-termination probabilities where $\lambda_o = 4\mu$, 5μ and $d = 80\%$. As C_{MSC} increases, the MSC ports are not the bottleneck resources. Fig. 6 (a) shows that, for $\lambda_o = 4\mu$, P_{o1} and P_{o2} stop to decrease when $C_{MSC} \geq 340$. The figure also shows that $P_{o1} = P_{f1} = P_{f2}$ when $C_{MSC} \geq 340$. The results indicate that the radio channels are the bottleneck resources when $C_{MSC} \geq 340$. Fig. 6 (b) shows that, for $\lambda_o = 5\mu$, P_{o2} stops decreasing when $C_{MSC} > 340$. On the other hand, the probability P_{o1} continues decreasing when $C_{MSC} > 340$ and stops decreasing when $C_{MSC} > 360$. The figure indicates that the SN ports are the bottleneck resources when $C_{MSC} > 340$. As $C_{MSC} > 360$, $P_{o1} = P_{f1} = P_{f2}$. This implies that the radio channels become the bottleneck resources when $C_{MSC} > 360$.

Our results indicate that an improper expansion of the MSC capacity may not im-

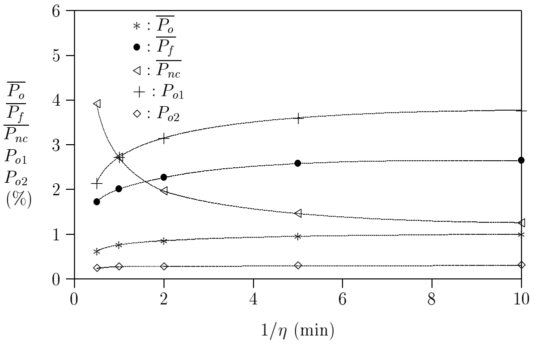


Figure 7: Effects of η ($k = 36$, $c = 10$, $\mu = 1\ \mathrm{min}^{-1}$, $\lambda_o = 5\mu$, $d = 80\%$, $C_{MSC} = 360$, $C_{SN} = 3000$ and $T_\theta = 30$ secs)

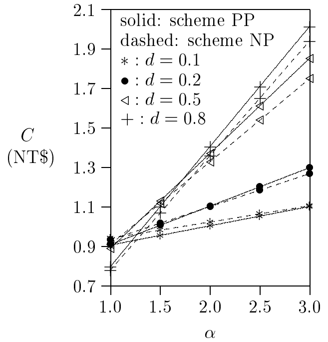


Figure 8: Effect of α ($k = 36$, $c = 10$, $\mu = 1\ \mathrm{min}^{-1}$, $\lambda_o = 7\mu$, $\eta = \mu/5$, $C_{MSC} = 360$, $C_{SN} = 150$, $T_\theta = 30$ secs and $\sigma = 0.5$)

prove the system performance. Similar phenomenon can also be observed in the effect of increasing the number of SN ports and the radio channels.

C. Effect of the Mobility

In this subsection, we consider the case where the prepaid new calls are the priority calls. Fig. 7 plots $\overline{P_o}$, $\overline{P_f}$, $\overline{P_{nc}}$, P_{o1} and P_{o2} as functions of the mean of cell residence time ($1/\eta$) for $1/\eta = 1/2$, 1, 2, 5 and 10 minutes, where $k = 36$, $c = 10$, $\mu = 1\ \mathrm{min}^{-1}$, $\lambda_o = 5\mu$, $d = 80\%$, $C_{MSC} = 360$, $C_{SN} = 3000$ and $T_\theta = 30$ seconds. Note that the ports of service node are not the bottleneck resources when $C_{SN} = 3000$. The figure indicates that $\overline{P_{nc}}$ decreases and then converges. $\overline{P_o}$ and $\overline{P_f}$ increase slowly and converge as $1/\eta$ increases.

As the mean of cell residence time ($1/\eta$) increases, the call incompletion probability ($\overline{P_{nc}}$) decreases. This is because a call experiences less number of handoffs as $1/\eta$ increases and the call is less likely to be forced to terminate. Moreover, as $1/\eta$ increases, $\overline{P_o}$ and $\overline{P_f}$ increase. This can be explained as follows. Since $\overline{P_{nc}}$ decreases when $1/\eta$ increases, the decreasing of $\overline{P_{nc}}$ leads to a higher offered load of the system. As a result, a new call or handoff call is more likely to be blocked as the offered load increases.

D. The Revenue Function C

In this subsection, we propose an output measurement C that computes the total charge of the non-prepaid and prepaid calls. Two priority assignment schemes are compared: scheme PP where the prepaid new calls have priority, because they are charged at a higher rate than the non-prepaid new calls; scheme NP where the non-prepaid new calls have priority, since they consume less resource than the prepaid new calls.

Let $C = (1-d)[(1-P_{nc1})T_{c1} + \sigma P_{i1}T_{i1}] + \alpha d[(1-P_{nc2})T_{c2} + \sigma P_{i2}T_{i2}]$, where T_{c1} (T_{c2}) is the expected holding time of a complete non-prepaid (prepaid) call, σ ($0 \le \sigma \le 1$) is the discount factor for an incomplete call, P_{i1} (P_{i2}) is the probability that a non-prepaid (prepaid) call is connected but is eventually forced to terminate, T_{i1} (T_{i2}) is the expected holding time of an incomplete non-prepaid (prepaid) call and α is the normalized charge of a prepaid call. According to a report of FarEastone (a mobile company in Taiwan) in 1999, $d \approx 40\%$ and $\alpha \approx 2$. Fig. 8 plots C as a function of α under two priority assignment schemes. It is clear that if the charge of a prepaid call is much higher than that of a non-prepaid call (e.g., $\alpha \ge 2$), scheme PP provides higher revenue than scheme NP. On the other hand, the scheme should be selected to favor the calls that have higher traffic ratio. From Fig. 8, we observe that scheme NP is better than scheme PP for $d = 0.1$, and for $d = 0.2$ when $\alpha < 2$. We conclude that both d and α have significant effect on the choice of priority assignment schemes.

IV. Conclusions

In this paper, we studied a mobile phone system that provides priority and prepaid call services based on the service node approach. We investigated how the traffic ratio of prepaid calls, the number of MSC ports and the user mobility affect the system performance. Our results show that the base stations, the mobile switching center and the service node may become the bottleneck under various traffic ratios of prepaid calls over total traffic. In this paper, we show how these bottlenecks can be identified and the system performance can be further improved. In addition, we observed that there exists a threshold point such that beyond this point, increasing the number of mobile switching center ports does not improve system performance. Similar phenomenon can be observed in the effect of increasing the number of service node ports and the radio channels.

Next, we investigate the effect of user mobility. As the mean of cell residence time increases, the average call incompletion probability decreases and converges. On the other hand, the average new call blocking probability and forced-termination probability slowly increase and then stabilize as the cell residence time increases. Finally, a revenue function is proposed to compare two priority assignment schemes. Our experiments show that this function can be used as a guideline to select the priority assignment scheme.

References

[1] 3GPP2, 3rd Generation Partnership Project 2 (3GPP2). Prepaid Charging Revision 1. 3GPP2 N.S0018 version 1.0.0. *3GPP2*, Jul. 2000.

[2] Chang, M.-F., Lin, Y.-B. and Yang, W.-Z. Performance of hot billing mobile prepaid service. *Computer Networks Journal*, 36(2):269–290, Jul. 2001.

[3] Chang, M.-F., Yang, W.-Z. and Lin, Y.-B. Performance of service-node based mobile prepaid service. *Accepted and to appear in IEEE Transactions on Vehicular Technology.*

[4] Katzela, I. and Naghshineh, M. Channel assignment schemes for cellular mobile telecommunication systems: a comprehensive survey. *IEEE Personal Communications*, 3(3):10–31, June 1996.

[5] Kelly, F. P. Loss networks. *The Annals of Applied Probability*, 1(3):319–378, 1991.

[6] Lai, W.-R. and Lin, Y.-B. Effects of cell residence times on wireless PBX systems. *ACM Wireless Networks*, 5(6):479–488, 1998.

[7] Lin, Y.-B. Performance modeling for mobile telephone networks. *IEEE Network Magazine*, 11(6):63–68, November/December 1997.

[8] Lin, Y.-B., Rao, C. H. R. and Chang, M.-F. Mobile prepaid phone services. *IEEE Personal Communications Magazine*, 7(3):6–14, Jun. 2000.

[9] Martin, J. Wireless prepaid billing: a global overview. *Telecommunications*, Sep. 1999.

[10] Schwartz, S. AAA adds to prepaid woes. *Billing World and OSS Today*, May 2001.

[11] Tracey, L. V. Prepaid calling: market update. *Telecommunications*, Apr. 1999.

[12] Yang, W.-Z., Chang, M.-F. and Lin, Y.-B. Priority call service for PCS networks. *SCS Trans. Special Issue on Wireless Networks*, 16(3):102–112, 1999.