An Optimal Transform Image Coding

Cheng-Hsiung Hsieh and Jui-Lung Hsueh

Department of Electronic Engineering
Chien Kuo Institute of Technology
Changhua, TAIWAN, R.O.C.
E-mail: hsieh@ckit.edu.tw

Abstract

In this paper, an optimal transform image coding (OTIC) system is proposed. Note that the energy-invariant property in orthogonal transformation and that the mean squared error (MSE) of reconstructed image is proportional to the total energy of transform coefficients discarded in the coding process. The OTIC system is developed and is proved optimal in the sense of minimum average energy loss. Basically, the proposed coding system consists of three phases. First, the average energy image block is obtained from transform image blocks. Next, indices of average energy image block are sorted in descending order by energy. When the number of coefficients to be retained, *M*, in the coding process is determined, the first *M* elements in reordered indices form a set denoted as $\boldsymbol{S}_M$. Then a fixed mask $\boldsymbol{A}_M$ acquired from set $\boldsymbol{S}_M$ is used to select *M* significant coefficients in transform image blocks. Finally, the *M* selected coefficients are quantized and coded by the order as in $\boldsymbol{S}_M$. Simulations are provided to justify the optimality in the proposed OTIC system. Besides, the effectiveness of the proposed selection approach, which is based on $\boldsymbol{S}_M$, is compared with the zigzag scan used in JPEG. Simulation results indicate that the $\boldsymbol{S}_M$-based selection approach is superior, in terms of PSNR, to the zigzag scan in most of cases.

Keywords: Optimality, transform coding, image compression, coefficient selection, JPEG

1. Introduction

The objective of image compression is to reduce required memory capacity while having acceptable visual quality in the reconstructed image. One of popular image compression schemes is called transform coding. The idea of transform coding is to transform image blocks, from spatial domain to transform domain, i.e., the information of image block is converted in its corresponding transform coefficients. Then transform coefficients of significant energies are retained and the rest set to zero. By this doing, it achieves the goal of image compression. Note that the mean squared error (MSE) in a reconstructed image is proportional to the total energy of transform coefficients discarded in the coding process. Consequently, effectively selecting significant transform coefficients implies that better reconstructed image can be obtained.

In transform coding, it is clear that to reconstruct better image transform coefficients of large magnitudes should be selected and the others discarded. However, in general it requires large overhead indicating the coefficient selection on a block-to-block basis. Therefore, the selection of significant transform coefficients is still an active area in the field of transform coding. Up to present, several adaptive approaches to select significant transform coefficients have been proposed. Given a portion of total energy in the transform domain, Palau and Mirchandani [1] used several geometric shapes to search for the geometric zone, which contained the least number of coefficients having the specified portion of energy. Using equipotentials of energy in transform domain, Neto and Nascimento [2] proposed a modified zonal coding approach where the selection of transform coefficients was based a given signal-to-noise ratio. Crouse and Ramchandran [3] applied the optimization technique in finding coefficient thresholds and optimal Q-matrix used in JPEG. Ong and Ang [4] utilized the statistical property, cross-correlation, to choose transform coefficients. Tran and Safranek [5] included the perceptual masking threshold model into the framework of image coding. The mask was used in the selection of transform coefficients.

In this paper, an optimal transform image coding (OTIC) system is proposed. The motivation of OTIC system is based on the following two observations: First, the MSE of a reconstructed image is proportional to the total energy of transform coefficients discarded in the coding process. Second, the total energy of an image is invariant between spatial domain and transform domain if an orthogonal transformation is applied. By these observations, the OTIC system is developed. Basically, the proposed coding system consists of three stages. First, the average energy image block is obtained from transform image blocks. Second, indices of average energy image block are arranged in descending order by energy. Then the first M elements in reordered indices form a set denoted as $\boldsymbol{S}_M$ where M is the number of coefficients retained in the coding process. By set $\boldsymbol{S}_M$, a fixed mask $\boldsymbol{A}_M$ is found and used to select M significant coefficients in transform image blocks. Third, the M selected coefficients are quantized and coded by the order as in $\boldsymbol{S}_M$. The proposed OTIC system will be proved optimal in the sense of minimum average energy loss later in Section 3.

This paper is organized as follows: In Section 2, the OTIC system is described. Next, the optimality, in the sense of minimum average energy loss, of OTIC system is derived in Section 3. Simulations are then provided to verify the theoretical results in Section 4 where the effectiveness of the proposed selection approach is compared with the zigzag scan used in JPEG [8]. Finally, conclusions and further research are described in Section 5.

2. The OTIC System

When the total number of transform coefficients kept, M, is specified, the implementation steps of OTIC system are described in the following:

Step 1. Input original $L \times L$ image $\boldsymbol{O}$.

Step 2. Divide image $\boldsymbol{O}$ as $t \times t$ image blocks $\{\boldsymbol{b}_i, \text{for } 1 \le i \le N_b\}$ where L is a multiple of t and $N_b = (L/t)^2$ is the total number of image blocks. Then $\boldsymbol{b}_i$ is 128-level shifted, i.e., $\boldsymbol{b}_i = \boldsymbol{b}_i - 128$.

Step 3. Obtain $\boldsymbol{B}_i = DCT\{\boldsymbol{b}_i\}$ where $DCT\{\cdot\}$ denotes discrete cosine transform [8].

Step 4. Find the average energy image block $\overline{\boldsymbol{B}}$ as

$$\overline{\boldsymbol{B}} = \frac{1}{N_b}\sum_{i=1}^{N_b}\boldsymbol{B}_i^2 \tag{1}$$

where $\boldsymbol{B}_i^2 = \boldsymbol{B}_i .* \boldsymbol{B}_i$ and the operation $.*$ is an element-to-element multiplication.

Step 5. By descending sorting, find M elements of most significant energies in $\overline{\boldsymbol{B}}$ and let the indices of M selected elements denote as set $\boldsymbol{S}_M$.

Step 6. Obtain the corresponding mask of $\boldsymbol{S}_M$, $\boldsymbol{A}_M$, by setting $\overline{B}(k,l) = 1$ if $(k,l) \in \boldsymbol{S}_M$ and $\overline{B}(k,l) = 0$ otherwise, where $\overline{B}(k,l)$ is an element of $\overline{\boldsymbol{B}}$.

Step 7. By $\boldsymbol{A}_M$, $\boldsymbol{B}_i$ is modified as $\hat{\boldsymbol{B}}_i = \boldsymbol{A}_M .* \boldsymbol{B}_i$.

Step 8. Quantize $\hat{\boldsymbol{B}}_i$ as $\hat{\boldsymbol{B}}_i = \hat{\boldsymbol{B}}_i ./\boldsymbol{Q}$ where the operation $./$ is an element-to-element division and $\boldsymbol{Q}$ is a quantization matrix with appropriate dimension. Then code the selected coefficients $\hat{B}(k,l)$ of $\hat{\boldsymbol{B}}_i$ in the order as in $\boldsymbol{S}_M$.

Step 9. Decode $\hat{B}(k,l)$ and reform $\hat{\boldsymbol{B}}_i$ as $\hat{\boldsymbol{B}}_i = \hat{\boldsymbol{B}}_i .* \boldsymbol{Q}$.

Step 10. By inverse DCT (IDCT), find reconstructed image block $\hat{\boldsymbol{b}}_i = IDCT\{\hat{\boldsymbol{B}}_i\} + 128$.

Step 11. Obtain reconstructed image of $\boldsymbol{O}$, $\hat{\boldsymbol{O}}$, through $\hat{\boldsymbol{b}}_i$.

Step 12. Calculate peak signal-to-noise (PSNR) of $\hat{\boldsymbol{O}}$ as

$$PSNR = 10\log\frac{255^2}{MSE} \tag{2}$$

where

$$MSE = \frac{1}{L^2}\sum_{i=1}^{L}\sum_{j=1}^{L}[O(i,j) - \hat{O}(i,j)]^2 \tag{3}$$

and $O(i,j)$ and $\hat{O}(i,j)$ are elements of $\boldsymbol{O}$ and $\hat{\boldsymbol{O}}$, respectively.

Three points about OTIC system should be pointed out here. First, only several bytes are required to indicate the coefficient selection since mask $\boldsymbol{A}_M$ is fixed for all transform image blocks in the coding process. Therefore, the OTIC system is simple in the selection of transform coefficients. Second, note that $\boldsymbol{S}_M$ is an optimal set in the sense of minimum average energy loss, which will be proved in Section 3. Consequently, the reconstructed image obtained from the *M* selected elements in $\boldsymbol{S}_M$ is of minimum average energy loss. Third, since the elements of $\boldsymbol{S}_M$ are sorted in descending order by energy, thus the order in $\boldsymbol{S}_M$ is the significance order of the selected elements as well. In other words, the optimal feature selection problem, is solved accordingly. When *K*, for $1 \le K \le M-1$, transform coefficients need to be discarded further in the coding process, there is no need to find $\boldsymbol{S}_{M-K}$ but simply discard last *K* elements in $\boldsymbol{S}_M$. Note that set $\boldsymbol{S}_{M-K}$ is still optimal in the sense of minimum average energy loss. Consequently, the OTIC system is effective in the selection of transform coefficients.

3. Optimality in OTIC System

The optimality, in the sense of minimum average energy loss, in OTIC system is derived here. Suppose the original image $\boldsymbol{O}$ is of size $L \times L$ and is partitioned into $t \times t$ image blocks $\boldsymbol{b}_i$ where L is a multiple of t. Note that an orthogonal transformation like DCT is of the energy-invariant property. That is,

$$\sum_{i=1}^{N_b}\sum_{m=1}^{t}\sum_{n=1}^{t} b_i^2(m,n) = \sum_{i=1}^{N_b}\sum_{k=1}^{t}\sum_{l=1}^{t} B_i^2(k,l) \qquad (4)$$

where $N_b = (L/t)^2$ is the total number of image blocks, $b_i(m,n)$ is an element of $\boldsymbol{b}_i$ and $B_i(k,l)$ is an element of transformed $\boldsymbol{b}_i$, $\boldsymbol{B}_i$. Pre-dividing both sides of (4) by $1/N_b$, we have the average energy of $\boldsymbol{O}$, *E*, as

$$E = \sum_{m=1}^{t}\sum_{n=1}^{t}[\frac{1}{N_b}\sum_{i=1}^{N_b} b_i^2(m,n)] = \sum_{k=1}^{t}\sum_{l=1}^{t}[\frac{1}{N_b}\sum_{i=1}^{N_b} B_i^2(k,l)] = \sum_{k=1}^{t}\sum_{l=1}^{t}\overline{B}(k,l) \qquad (5)$$

where

$$\overline{B}(k,l) = \frac{1}{N_b}\sum_{i=1}^{N_b} B_i^2(k,l) \qquad (6)$$

In (6), $\overline{B}(k,l)$ is an element of $\overline{\boldsymbol{B}}$ which is the average energy of the (*k*, *l*) element in $\boldsymbol{B}_i$ for $1 \le i \le N_b$.

With elements $\overline{B}(k,l)$, the way to find the optimal set of elements in $\boldsymbol{B}_i$, in the sense of minimum average energy loss, is given in the following. Note that the average energy of original image $\boldsymbol{O}$ is related to the sum of $\overline{B}(k,l)$, for $1 \le k,l \le t$. When the total number of transform coefficients kept, *M*, is specified, it is obvious that the *M* selected elements in transform domain should be the *M* most significant elements in $\overline{\boldsymbol{B}}$. Let the indices of *M* selected elements be set $\boldsymbol{S}_M$. With set $\boldsymbol{S}_M$, the average energy of reconstructed image $\hat{\boldsymbol{O}}_M$, E_M, is then given as

$$E_M = \sum_{k=1}^{t}\sum_{l=1}^{t}[\frac{1}{N_b}\sum_{i=1}^{N_b}\hat{B}_i^2(k,l)] = \sum_{(k,l)\in S_M}\overline{B}(k,l) \quad (7)$$

where $\hat{B}_i^2(k,l)$ is the energy of $\hat{B}_i(k,l)$ and $B_i(k,l)=\hat{B}_i(k,l)$, if $(k,l)\in \boldsymbol{S}_M$, is applied in the second equality. Consequently, the average energy loss of $\hat{\boldsymbol{O}}_M$, $\tilde{E}_M$, is given as

$$\tilde{E}_M = E - E_M = \sum_{k=1}^{t}\sum_{l=1}^{t}\{\frac{1}{N_b}\sum_{i=1}^{N_b}[B_i^2(k,l)-\hat{B}_i^2(k,l)]\} = \sum_{(k,l)\notin S_M}\overline{B}(k,l) \quad (8)$$

Since E_M is of the *M* most significant energies in $\overline{\boldsymbol{B}}$, thus $\tilde{E}_M$ is of minimum average energy loss. In other words, the reconstructed image $\hat{\boldsymbol{O}}_M$ is optimal in the sense of minimum average energy loss.

4. Simulation Results and Discussions

In this section, the proposed OTIC system is verified and compared with the baseline JPEG through simulation results. Since the OTIC system is optimal, the PSNR of reconstructed image should be no less than that in JPEG if the number of selected transform coefficients is same. In other words, the coefficient selection of OTIC system should be as least as effective as that in JPEG. The theoretical result will be justified by simulation results which are then discussed.

Test images used in the simulation are Lena, Baboon, Jet, and House. All images are of size 512×512 and partitioned into 8×8 image blocks. For the convenience of presentation, the 8×8 two-dimensional index is mapped to one-dimensional index. The conversion is given in Figure 1. Let $M=16$. The optimal sets $\boldsymbol{S}_{16}^L$, $\boldsymbol{S}_{16}^B$, $\boldsymbol{S}_{16}^J$, and $\boldsymbol{S}_{16}^H$, for images Lena, Baboon, Jet, and House, are recorded in Table 1, respectively, where the superscripts *L, B, J,* and *H* in $\boldsymbol{S}_{16}$ are for images Lena, Baboon, Jet, and House, respectively.

The effectiveness of coefficient selection in OTIC system is demonstrated by comparing with the zigzag scan (ZZS) used in JPEG. The optimal coefficient selection approach in OTIC system is called $\boldsymbol{S}_M$-based scan (SBS) in the following discussion. The coding system used to compare the effectiveness of coefficient selection in ZZS and SBS is shown in Figure 2 where DCPM stands for differential pulse code modulation [8]. Quantization matrix $\boldsymbol{Q}$, DC coefficient coding, and AC coefficient coding in Figure 2 use the default settings in baseline JPEG [8]. In Figure 2 only the block of scan scheme can be either ZZS or SBS since what we concern is the effectiveness in the coefficient selection. In fact, Figure 2 is the block diagram of baseline JPEG when the ZZS is used. The comparison results on PSNR, with various *M*, are given in Table 2. Consider the case of $M = 4$. Only 4 coefficients are selected either by ZZS or SBS. For ZZS, the selected coefficients in transform image blocks for all images are elements 1, 2, 9, and 17 as shown in Figure 1. However, for SBS elements 1, 2, 9, and 3 are selected for image Lena while elements 1, 9, 2, and 17 are selected for other images as given in Table 1. Note that the elements selected by ZZS and SBS for images Baboon, Jet, and House, are same except in different order. Therefore, the PSNR are expected to be identical which is verified in Table 2. However, the order to code the selected coefficients is different. Thus, the number bytes used in coded image may be varied in general. This explains why different bit rate (bit/pixel) is obtained in Table 3 for the case of $M = 4$ and other similar cases. Since SBS is optimal in the sense of minimum average energy loss, it is expected to have better PSNR than ZZS. From Table 2, it is clear that PSNR obtained from SBS is no less than that from ZZS for all cases which is consistent with the theoretical results as expected. The maximum improvement on PSNR for images Lena, Baboon, Jet, and House, are 1.0373 dB ($M = 12$), 0.7012 dB ($M = 32$), 0.2264 dB ($M = 20$), and 0.8937 dB ($M = 8$), respectively. Simulation results indicate that SBS is more effective than ZZS.

The comparison results on bit rate, with various *M*, are given in Table 3. Note that the overhead to indicating coefficient selection is $M \times \lceil \log_2(8 \times 8) \rceil$ bits. Table 3 indicates that for image Lena the bit rate for SBS is less than that for ZZS for all *M* except $M = 28$. Besides, the bit rate for SBS is a little bit higher than that for ZZS in images Baboon, Jet, and House except the case $M = 8$ for image Baboon. It should be noted that the higher bit rates shown in Table 3 does not necessarily imply that the OTIC system trades bit rate with better PSNR shown in Table 2 which in fact comes from the effectiveness of SBS since PSNR in Table 2 are compared under the condition of identical *M*. One possible reason for higher bit rates is that the ZZS is generally advantageous to the variable length coding (VLC) [8] since large amount of coefficients after quantization reduce to zero. This benefits the use of VLC on which AC coefficient coding in Figure 2 is based. However, VLC is not suitable for SBS in general. Since coefficients are coded in descending order by average energy, it generally reduces the possibility to have a sequence of zeros in the coding process and therefore the application of VLC in SBS is not as good as in ZZS. In others words, it may cause a higher bit rate in most of cases. Consequently, simulation results in Table 3 suggest that an appropriate coefficient coding approach should be sought such that the bit rate can be reduced in OTIC system.

5. Conclusions and Further Research

In this paper, an optimal transform image coding (OTIC) system is proposed. Based on the following observations, the OTIC system is developed. First, under orthogonal transformation the total energy of an image is invariant between spatial domain and transform domain. Second, the MSE of a given reconstructed image is proportional to the total energy of transform coefficients discarded in the coding process. The OTIC system is proved optimal in the sense of minimum average energy loss. The theoretical results have been verified by simulation results where images Lena, Baboon, Jet, and House, are used as examples.

In the simulation, the effectiveness of coefficient selection in OTIC system is compared with the zigzag scan (ZZS) used in JPEG. Simulation results indicate that $\boldsymbol{S}_M$ -based scan (SBS) is more effective than ZZS in terms of PSNR. The improvements on PSNR are as high as 1.0373 dB, 0.7012 dB, and 0.8937 dB for images Lena, Baboon, and House, respectively. However, the bit rate for images Baboon, Jet, and House, obtained from SBS is a little bit higher than that from ZZS for most of cases when quantization matrix $\boldsymbol{Q}$, DC coefficient coding, and AC coefficient coding use the default settings in baseline JPEG. The reason for higher bit rates may be that the AC coefficient coding approach in JPEG is not appropriate for OTIC system. Consequently, an appropriate coefficient coding approach for SBS will be sought in further research such that lower bit rate can be obtained in OTIC system.

References

[1] A. Palau and G. Mirchandani, “Image Coding with Discrete Cosine Transforms Using Efficient Energy-Based Adaptive Zonal Filtering,” *IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 337-340, 1994.

[2] Sebastião do Nascimento Neto and Fransisco Assis de Oliverira Nascimento, “Hybrid Adaptive Image Coding：Transform Coding and Vectorial Classification Approach for Bit Allocation,” *The 38th Midwest Symposium on Circuits and Systems*, pp. 846-849, Aug. 1995.

[3] M. Crouse and K. Ramchandran, “Joint Thresholding and Quantizer Selection for Transform Image Coding: Entropy-Constrained Analysis and Applications to Baseline JPEG,” *IEEE Trans. on Image Processing*, Vol. 6, No. 2, pp. 285-297, Feb. 1997.

[4] M. Bi, S.H. Ong and Y.H. Ang, “Coefficients Grouping Method for Shape-Adaptive DCT,” *Electronics Letters*, Vol. 32, No. 3, pp. 201-202, Feb. 1996.

[5] T.D. Tran and R. Safranek, “A Locally Adaptive Perceptual Masking Threshold Model for Image Coding,” *IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 1882-1885, 1996.

[6] J.Y. Nam and K.R. Rao, “Image Coding Using a Classified DCT/VQ Based on Two-Channel Conjugate Vector Quantization,” *IEEE Trans. on Circuits and Systems for Video Technology*, Vol. 1, No. 4, pp. 325-336, 1991.

[7] Y. Huang, H.M. Dreizen and N.P. Galatsanos, "Prioritized DCT for Compression and Progressive Transmission of Images," *IEEE Trans. on Image Processing*, Vol. 1, No. 4, Oct. 1992.

[8] K.R. Rao and J.J. Hwang, *Standards for Image, Video, and Audio Coding*, Pretice Hall, 1996.

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) & (1,7) & (1,8) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) & (2,7) & (2,8) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) & (3,7) & (3,8) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) & (4,7) & (4,8) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) & (5,7) & (5,8) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) & (6,7) & (6,8) \\ (7,1) & (7,2) & (7,3) & (7,4) & (7,5) & (7,6) & (7,7) & (7,8) \\ (8,1) & (8,2) & (8,3) & (8,4) & (8,5) & (8,6) & (8,7) & (8,8) \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\ 17 & 18 & 19 & 20 & 21 & 22 & 23 & 24 \\ 25 & 26 & 27 & 28 & 29 & 30 & 31 & 32 \\ 33 & 34 & 35 & 36 & 37 & 38 & 39 & 40 \\ 41 & 42 & 43 & 44 & 45 & 46 & 47 & 48 \\ 49 & 50 & 51 & 52 & 53 & 54 & 55 & 56 \\ 57 & 58 & 59 & 60 & 61 & 62 & 63 & 64 \end{bmatrix}$$

Figure 1. 2D-to-1D index conversion

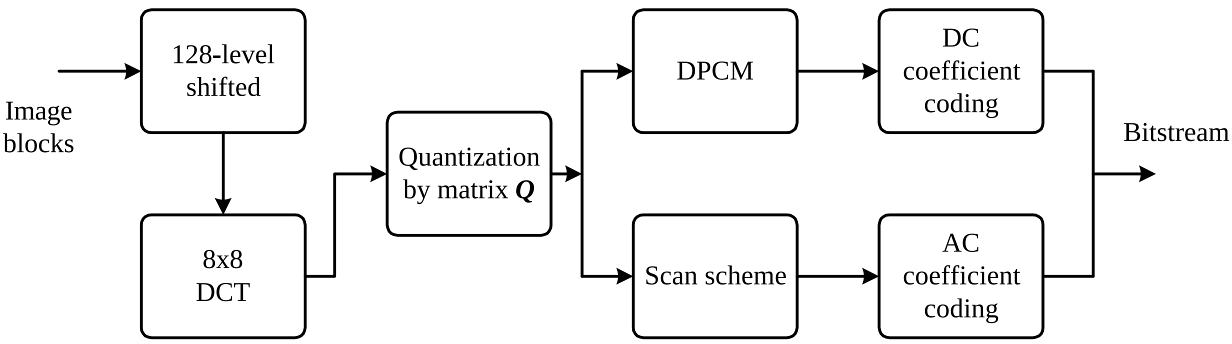

Figure 2. The coding system for effectiveness comparison of ZZS and SBS

Table 1. Elements of $\boldsymbol{S}_{16}^{L}$, $\boldsymbol{S}_{16}^{B}$, $\boldsymbol{S}_{16}^{J}$, and $\boldsymbol{S}_{16}^{H}$ in descending order by energy

Ranking index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\boldsymbol{S}_{16}^{L}$	1	2	9	3	10	11	4	17	18	19	12	5	20	25	26	27
$\boldsymbol{S}_{16}^{B}$	1	9	2	17	10	25	18	33	3	26	11	19	41	34	27	49
$\boldsymbol{S}_{16}^{J}$	1	9	2	17	3	10	25	11	33	18	4	19	41	5	12	26
$\boldsymbol{S}_{16}^{H}$	1	9	2	17	10	25	18	3	26	33	11	4	19	34	41	27

Table 2. Comparison results on PSNR for ZZS and SBS

Value M	**Lena**		**Baboon**		**Jet**		**House**	
	ZZS	**SBS**	**ZZS**	**SBS**	**ZZS**	**SBS**	**ZZS**	**SBS**
4	28.0255	28.7821	20.9326	20.9326	26.0713	26.0713	24.2117	24.2117
8	31.5424	31.5707	21.8509	22.1831	29.0938	29.2375	26.0780	26.9717
12	32.6627	33.7038	23.2180	23.3032	31.3858	31.5177	28.8781	28.8781
16	34.4988	34.9689	23.8157	24.2938	33.1688	33.3279	29.9644	30.4227
20	35.6150	35.7904	24.6818	25.2366	34.3248	34.5512	31.1134	31.4837
24	35.6973	36.0854	25.8493	26.0046	35.2322	35.3554	32.2362	32.2362
28	36.2173	36.3278	26.3234	26.6117	35.7608	35.7608	32.6015	32.6455
32	36.3556	36.3603	26.5090	27.2102	35.8307	35.8640	32.7031	32.7763

Table 3. Comparison results on bit rate for ZZS and SBS

Value M	**Lena**		**Baboon**		**Jet**		**House**	
	ZZS	**SBS**	**ZZS**	**SBS**	**ZZS**	**SBS**	**ZZS**	**SBS**
4	0.2611	0.2602	0.3053	0.3055	0.2606	0.2647	0.3147	0.3225
8	0.4032	0.3881	0.5376	0.5372	0.4048	0.4184	0.5184	0.5490
12	0.4719	0.4690	0.7464	0.7615	0.5112	0.5225	0.7039	0.7158
16	0.5316	0.5218	0.8985	0.9098	0.5799	0.5962	0.8174	0.8359
20	0.6085	0.6064	1.0303	1.0627	0.6721	0.6913	0.9079	0.9287
24	0.6614	0.6530	1.1453	1.1625	0.7276	0.7570	0.9845	0.9914
28	0.6953	0.6957	1.2263	1.2381	0.7693	0.7912	1.0331	1.0563
32	0.7205	0.7202	1.2625	1.3309	0.7860	0.8134	1.0574	1.0813