Extended Synchronized Choice Nets

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ABSTRACT

SNC nets are extended to deal with nets with assymetric first order structures (FOS). They can be converted to a General Petri net (GPN) and existing theory can be applied to study its liveness and boundedness properties.

1.INTRODUCTION

Proving liveness or equivalently, solving the reachability problem for Petri nets is a difficult problem since it takes exponential time and space [6]. Unlike traditional classification by output conditions of places, Synchronized Choice nets (SNC) were defined as a new class of nets [2,3] characterized by bridges and handles satisfying requirements **R1 (R2):** If a circuit Ω has a TP- (PT-) handle, H', then H' is bridged to Ω through a PT- (TP-) bridge, B'. Figs. 1 to 4 are examples of SNC. Note the net in Fig. 4 is not an FC (free-choice) net. In Fig. 1, two handles H_1= [p_2 t_4 p_4 t_3] and H_2= [p_2 t_2 p_3] start from the same place p_2 but they join at a transition t_3; there is a bridge B_{12}= [t_4 p_3] from H_1 to H_2 and a bridge B_{21}= [t_2 p_4] from H_2 to H_1.

By examining the synthesis rules presented in [3], we find that synthesized nets and SNC nets are closely related. **R1** and **R2** involve nodes in a global fashion; the synthesis rules, nevertheless, involve nodes in a local fashion. Thus one can view the rules as localization of the two requirements that reduces the complexity of analysis. On one hand, the rules provide local conditions for a net to be SNC similar to that for an FC [11]; on the other hand, they are better than the two requirements by [3] which are global conditions. Note that any Ct (Consistent) & Cv (Conservative) FC is an SNC but not vice versa. An arbitrary SNC net may not be SL. However, any SNC net that is an FC, is structurally live (SL) which is not true for AC (asymmetric-choice).

A **first-order** structure (FOS) contains two directed paths with identical start (called n_s) and end nodes (n_e). In [2], we show that in an SNC, any FOS must be symmetrical; that is, both n_s and n_e must be of the same type: they are both transitions or are both places. An *asymmetrical FOS* with $n_s \in T(P)$ and $n_e \in P(T)$ may result in unboundedness (nonlive). To fix the problems, one way is to insert bridges into the structure. This results in SNC [2]. Another way is to have a second *asymmetrical* **FOS** with $n_s \in$ P(T) and $n_e \in$ T(P) to absorb or provide the extra tokens for the first *asymmetrical* **FOS**. An example is shown in Fig. 5

Fig. 5(a) shows an *asymmetrical* TP **FOS**; the TP-path [t1 p4 t4 p3], which injects an extra token into the circuit in each iteration, causing the Petri net unbounded. To consume the extra token, a PT **FOS** should follow the above FOS. In Fig. 5(b), an *asymmetrical* PT **FOS**; the PT-path [p2 t4 p4 t3], makes t3 nonlive, and thus results in a deadlock.

One way to fix the problem is to insert bridges into the above **FOS**. This results in a new class of nets called **SNC** whose properties have been studied in [2,3]**.** There is only one kind of bad siphon, causing an SNC to be not live. It is interesting because it is bounded and the condition for liveness is so simple that there exists an integrated algorithm for SNC and liveness detection. However, this class of nets is limited. For instance, there is no ordering of firings among a set of resource-sharing transitions that are exclusive to each other. Sometimes, these transitions must execute one by one. Also, if the synthesized net is initially safe, it stays to be safe for any reachable marking. It is marking monotonic, that is, it will not evolve into a deadlock by adding more tokens. The synchronic distance between any two transitions in a synthesized net with safe marking is either one or infinite.

To create classes of nets with more general properties, we have to find more alternatives to fix the problem. In one alternative, the two *asymmetrical* **FOS** should be combined as in Fig. 5(c).

Note that it must be that $n_{e1} \leftrightarrow n_{s2;}$ i.e., they are in a circle. Otherwise, the PT **FOS** cannot consume the extra token from the TP **FOS**. The net is live if the two output transitions of the n_s place of the PT **FOS** are synchronized to have synchronic distance of one.

Note that in Fig. 5(c), one TP generation is immediately followed by a PT generation. In general, several TP (PT) FOS may be combined to form a composite structure so that more than one extra token generated (consumed). Also, there may be more than one TP FOS in combination with more than one PT FOS as shown in Fig. 6. The above conditions must be generalized and will be dealt with in Section 4. We have extended this to allow min-

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gling a number of TP and PT generations; thus, multiple TP generations are followed by multiple PT generations. Also PT FOS must consume multiple tokens generated from TP FOS in a synchronized manner. One way to do this is to link all relevant PT FOS with a regulation circuit (RC). Based on this, we will define a new class of net called ESNC (Expanded SNC). The class of synthesized nets hence is extended. Liveness condition for SNC is simple because there is only one kind of bad siphon (minimal deadlock with no traps). For ESNC, a new kind of bad siphon emerges. The marking condition for liveness will be derived based on results from [5].

Section 2 presents the preliminaries. Section 3 defines handles, bridges, composite FOS (CFOS) and ESNC. Section 4 derives the structure constraints for liveness and boundedness of ESNC by transforming it into general Petri nets (GPN) with multiple weights (called WSNC). They are derived first for a subclass (called WMG) of WSNC where the transformed GPN is a mark graph (MG). The same constraint also holds for WSNC. The marking constraint is derived first for RC with multiple tokens in Section 5 based on a new kind of bad siphon. Section 6 concludes the paper.

We assume that readers are familiar with the various terminologies of PN; references for which can be found in [7].

Definition 1[5]: For a Petri net (N,M), a non-empty subset D of places is called a deadlock if $\bullet D \subseteq D\bullet$, i.e., every transition having an output place in D has an input place in D. If M(D)= $\sum_{p\in D}$ M(p)=0, D is called a token-free deadlock at M. A deadlock D_m is said to be minimal if it does not contain a deadlock as a proper subset. Similar definitions apply to trap with the change that $\bullet D \subseteq D\bullet$ is replaced by $\bullet D \supseteq D\bullet$.

Note that *deadlock* is also referred to as *siphon* in some literatures and the sequel.

Lemma 1[5]**:** For a Petri net (N, M_0), if there does not exist any firable transition, then there exist a token-free deadlock at M_0.

Definition 2[3]: Given a Petri net N in which *N'* is a subnet of it and *N'* ($\subseteq$ *N*) is a T-component (P-component) of *N* iff *N'* is strongly connected marked graph (state machine) and $P' = \bullet T' \cup T'\bullet$, ($T' = \bullet P' \cup P'\bullet$).

In [3], we showed that any SNC could be decomposed into a set of T- and P- components.

Definition 3(S-invariant & T-invariant): A Y (X) vector is called a S- (T-) invariant iff Y(X)≠0 and AY=0 ($A^TX=0$) where A is the incidence matrix.

The existence of S-invariants is equivalent to the preservation of token load [5]. The y values of all places for the PN in Fig. 7 ensure that the y values are balanced at each transition.

Definition 4(P-semiflow & T-semiflow): Y (X) is called a P- (T-) semiflow if Y (X) is integral and nonnegative and AY=0 ($A^TX=0$).

Definition 5(Conservative): A Petri net N is called conservative iff there exists a positive integer vector Y > 0 such that $W(M)=M^TY=M_0^TY=W(M_0)$dΩM∈R(N,M_0ea

Lemma 2[3]: ΩPN, AY=0 iff Ωt ∈T, $Y(\bullet t) = Y(t\bullet)$, $Y(Q) = \sum_{p_i\in Q} y_i * w_i$, where l$Q=\bullet t$ or $t\bullet$ and w_i the weight between p_i and t. The above condition is defined as the **T-condition.**

This T-condition is useful for finding Y in a step-by-step fashion. We can assign arbitrary equal y values for input places of a transition and compute the equal y value of each of its output places. Continue this until all y values of the S-invariant have been computed. We then multiply or divide all y values by a factor such that all y are integers and their greatest common factor is one. Examples are shown in Fig. 7.

Definition 6: Let D be a deadlock of N. p_dΩD and t_dΩp∈ is called a *drain transition* if t_d∈↔D; otherwise, it is called a *trap transition*. p_d is called a *drain place*. T_D is the set of all such t_d.

In Fig. 7, p5 is a drain place in the D_m which includes all places with y>0 and $T_D=\{t_d^2, t_d^3\}$; t_d^1 is a trap transition,. The corresponding Y vector is an S-invariant. Synchronic distance is a concept closely related to the degree of mutual dependence between two events in a condition/event system. The definitions of Petri net language and synchronic distance are given as follows:

Definition 7(Language of a Petri Net): The language of a Petri net N, L(N,M), is the set of all firing sequences for the net with the initial marking M: L(N,M)={ ∑ |M[∑>M'].

Definition 8(Synchronic Distance): The synchronic distance between two transitions t1 and t2 in a Petri net N is defined as d_{12}= Max {∑(t1) - ∑(t2), ∑ΩL(N,M)}, where ∑(t) is the number of times t appears in ∑.

The deadlock-trap property is only a sufficient (but not necessary) condition for liveness. Hence in a live net, a minimal deadlock may not contain a trap; such a deadlock is called a *bad siphon*. Consider the case that there is only one drain place and its output transitions t1ΩD and t2↔D of the deadlock. If $d_{12} = \bullet$, then tokens in D will be drained completely by a certain number of firings of t2 without any firing of t1. This empty D will stay forever and the N is not live. The general case is similar. To prevent this, t1 and t2 must be synchronized with finite synchronic distance so that D cannot be empty.

3. HANDLES, BRIDGES, FIRST AND SECOND ORDER STRUCTURES

We follow [2] for the definitions of handles, bridges, XY-handles, and XY- bridges where X and Y can be T or P.

Definition 9: Let N=(P, T, F) and N_1, N_2 partial subnets of N. An elementary directed path H= [$n_s n_1 n_2$ $n_k n_e$], n_iΩP∪T, i=1, 2, , k, is called a handle of N_1 if H∩N_1={n_s,n_e}; n_s and n_e are called the start and the end

$n_s, n_e\}$; n_s and n_e are called the start and the end nodes of the handle H, respectively. Note that n_s and n_e may be identical. An elementary directed path $B=[n_1, \ldots, n_r]$, $r \geqq 2$, is a bridge from N_1 to N_2 iff $B \cap (P_1 \cup T_1) = \{n_1\}$ and $B \cap (P_2 \cup T_2) = \{n_r\}$. **p1= p2** if p1 and p2 are on an elementary circuit. **$n_1 \bullet$ n_2** if **n1= n2** and there is an elementary directed path from n_h to n_2 via n_1 where n_h is a reference node (initially marked) called a **home place**.

Definition 10: Handles H_1 and H_2 are said to be *mutually complementary* if they share the same n_s and n_e**;** i.e.**,** $H_1 \cap H_2 = \{n_s, n_e\}$. Let $p_i \Omega H_i$ (i=1 or 2), $p_i \sqsubseteq n_s$ and $p_i \sqsubseteq n_e$, then define $\sqsupseteq_i \neq H_i$ is a directed path on H_i from n_s to p_i, $n_s(p_1,p_2)=n_s$ if there exists at least one $\sqsupseteq_i$ on a certain H_i that contains no other n_s of another H_j; $n_s(p_1,p_2)$ can be defined in a dual fashion and $n_s(p_1,p_2)=n_e$ if there exists at least one ν_i on a certain H_i that contains no other n_e of another H_j; where $\nu_i \neq H_i$ is a directed path on H_i from p_i to n_e. (p_1, p_2) is called a TP-inconsistent pair of places if $\exists n_s(p_1, p_2)$ is a transition and $\exists n_e(p_1, p_2)$ is a place. (p_1, p_2) is called a PT-inconsistent pair of places if $\exists n_s(p_1, p_2)$ is a place and $\exists n_e(p_1, p_2)$ is a transition. Let $\Upsilon = H_1 \cup H_2$**,** $H_1 \neq N_1$ and $H_2 \neq N_2$, H_1 (H_2) is a *prime handle* to N_2 (N_1) (1) If there are no bridges *B* between N_1 and N_2 and Υ is defined to be a **first-order** structure (FOS). If $n_s \Omega X$ and $n_e \Omega Y$ where X,Y=T or P, then Γ(H, B) is said to be a XY-path (XY-handle, XY-bridge). If X=Y, then the **FOS** Υ is said to be *symmetrical;* otherwise it is *asymmetrical (AFOS).* (2) If B_1 (B_2) is a bridge from N_1 to N_2 (N_2 to N_1), then **φ=** $H_1 \cup H_2 \cup B_1 \cup B_2$ is defined to be a **second-order** structure (SOS). (3) A strongly connected net is **SNC** (Synchronized Choice net) if every TP-(PT-)handle to a certain circuit has a PT- (TP-)bridge from its complementary TP-handle to itself. If X=Y, then the **FOS** φ is said to be *symmetrical; otherwise it is asymmetrical (AFOS).*

Note that for PT-inconsistent pair, it must be that every $n_s(p_1, p_2)$ is a place. Otherwise, it may no longer be irreversible (Fig. 3). For TP-inconsistent pair, however, as long as there exists a transition $n_s(p_1, p_2)$, it is not live.

$[p_2\ t_2\ p_3]$ and $[p_2\ t_4\ p_3]$ in Fig. 1 are two *prime handles* complementary to each other; $n_s = p_2$ and $n_e = p_3$**.** Note that there are no bridges interconnecting them; hence, they together form an FOS. Since X=Y=P, it is *symmetrical.* p_1 and p_2 in Definition 16 are inconsistent because they are concurrent (exclusive) and the tokens in them will flow to a set of mutually exclusive (concurrent) places. Note that n_s(p1, p2) and n_e(p1, p2) do not exist if p1 and p2 is on an elementary circuit; instead **we define p1= p2**.

Note that any pair of places (excluding n_s and n_e) in an AFOS is also inconsistent. This leads [2] to an integrated algorithm to detect SNC and liveness for an arbitrary net.

Definition 11: A **composite first-order structure** (FOS) Z is a set of first-order structures $\Psi_1, \Psi_2, .. \Psi_k$, $k \geq 2$, that are (1) interconnected; that is, $\forall \Psi_i$, $\exists\ \Psi_j$ such that $\Psi_i \cap \Psi_j \sqsubseteq \phi$, if $i \sqsubseteq j$, (2) $\forall$pair of (n_s^i, n_e^i) and (n_s^j, n_e^j), if $\Psi_i \cap \Psi_j = \phi$, then $n_s(n_s^i, n_s^j) \Omega T$ (P), and (3) $\forall Z_i$, Z_j, $1 \geq |Z_i \cap Z_j|$. n_s (n_e) of Z: $\forall\ n_s^i$ (n_e^i) of Ψ_i, either $n_s = n_s^i$ or $n_s \bullet\ n_s^i$ ($n_e = n_e^i$ or $n_e \bullet\ n_e^i$). If all Ψ_i is of TP (PT) type, then it is a **TP (PT) composite first-order structure, CFOS** with symbol Z^T (Z^P)**;** |Z| is the maximum number of tokens that can be generated (consumed) at n_e after all transitions have been fired once. Note $Z = \Psi_1 \cup \Psi_2 .. \cup \Psi_k$. Z is said to be covered by $\Psi_1, \Psi_2, .. \Psi_k$, which is a minimum cover of Z if no proper subset of itself is a cover of Z. A pure CFOS is one where except for all n_s^i and n_e^i, the rest nodes have a single input and a single ouput node; i.e., $\forall n \Omega Z$, $n \nleftrightarrow RC$, $n \sqsubseteq n_s^i$ and $n \sqsubseteq n_e^i$ for any $n_s^i \Omega Z$, $n_e^i \Omega Z$, $|\Subset n| = |n \Subset| = 1$ where RC is a regulation circuit explained below. A tokenless Z is a Z with no tokens. (We assume, unless otherwise stated, every Z is tokenless.)

Examples of CFOS are shown in Figs. 6-7. A PT CFOS will cause the net not live. One way to make it live is to add a regulation circuit (RC), $[t_d^1\ p1\ t_d^2\ p2\ t_d^3\ p3]$ in Fig. 7, across all t_d (also Fig. 5(d)). Such a structure is no longer an FOS; however, for brevity, we shall still refer to it CFOS in the rest of the paper. It is pure if the PT CFOS is pure.

For an ESNC to be well-behaved (WB); i.e., bounded and live, it must be correct in both static structure and dynamic marking behavior. In the sequel, we develop theory for both separately.

4. STRUCTURE CONSTRAINT

There are two cases: (1) only one token (2) multiple tokens in the RC. The first case is easier and the derived structure constraint also holds for (2).

Case (1): Single token in RC

The net (Fig. 6) can be transformed into a General Petri net (GPN) according to the following rule.

Rule of Transformation to GPN: Replacing every CFOS by a single arc with two ends being n_s and n_e and the arc weight being |Z|.

We consider only the case of weighted SNC (**WSNC**) where the OPN (ordinary PN) version of the GPN; i.e., by making all arc weights unity, is an SNC. Let the class of nets transformed into WSNC be defined as **ESNC** (**Expanded SNC**). The WB of WSNC does not, however, imply the WB of ESNC. An example is shown in Fig. 8 where two identical PT CFOS shares the same n_s and the WSNC is live. The tokens in n_s could be trapped in each PT CFOS without firing n_e causing the net not live. This token trapping does not happen, however, if the WSNC is a MG, called Weighted MG (WMG). We investigate first the properties of WMG in the sequel.

Theorem 1: An ESNC is WB iff the transformed net is a WMG and WB.

This theorem is significant because we can find how TP and PT-CFOS are combined by studying the transformed net that is a GPN. It does not hold if WMG is replaced by WSNC because as shown in Fig. 8, the WSNC is live but the corresponding ESNC is not.

The following theorem help determine the structuralliveness of WMG.

Theorem 2[8]: For a WMG N, the following statements are equivalent:

1. N is SL (structurally live) and SB (structurally bounded).
2. N is Ct (consistent) and Cv (conservative).
3. N is Ct and strongly connected.
4. N is strongly connected and rank(A) = |T| -1.

Theorem 2 is very useful because if we can find vector X for any strongly connected WMG satisfying the conditions in Theorem 2, the WMG is SL and SB.

We first consider the special case of a MG, followed by SNC. When a WMG is consistent, there exists a firing sequence to return the state to the original marking. During each iteration, each node n_i executes R_i (a finite number) times which constitutes a minimum nonzero T-semiflow denoted as a |T|-vector (|T| being the number of transitions of a PN) $R = [R_0\ R_1\ \ ...\ R_{|T|}]^T$.

Case (2): Multiple tokens in RC

The net (Fig. 7) cannot be transformed into a General Petri net (GPN). Since transitions in a RC can still fire one by one in the same fashion as Case (1), the structure constraint remains, however, the same as case (1). This is to avoid continuous token generation and token absorption. Such a constraint will be referred to as the *arc-constraint*. Another source for being not live comes from inappropriate token distribution among output transitions of a place. In other words, there exist bad siphons. Thus, the marking constraints for WSNC and multiple tokens in RC can be treated together based on Lemma 6.

For WSNC, the existence of TP-inconsistent pair of places implies that of a bad siphon similar to SNC with unit weights. However, the presence of multiple weights may not empty a bad siphon as shown in Fig. 9. This does not imply live transitions in the bad siphon. The following lemma shows WSNC is not SL if there exists TP-inconsistent pair of places.

Lemma 3:d A WSNC is not SL if there exists TP-inconsistent pair of places.

In the sequel, WSNC°deSNC°addenotes WSNC (SNC) without TP-inconsistent pair of places.

lod MARKING CONSTRAINT

Lautenbach's marking condition (Definition 12) can be applied to provide a more generalized liveness condition for sequential mutual exclusion (SME). Examples will be provided. Case (1) includes Case (2). Hence we will treat case (2) first and degenerate to case (1) subsequently.

The marking constraint for WSNC with multiple tokens in RC is now investigated. A WSNC° still has D_m. Instead of loop, we consider a P-component Ω in the equivalent WSNC that is both a minimal siphon (or deadlock) and a trap. In the WSNC, Ω is expanded to Ω' that contains D_m.

Definition 12[5]: Let (N,M_0) be a net-system, let i be an S-invariant and let D∈P be a deadlock of N. The deadlock is called controlled by the S-invariant i under M_0 iff the *marking condition* is satisfied: $W(M_0)=i^T*M_0>0\wedge\leftrightarrow s\sum P\backslash D: i(s)=0$.

Note that $W(M_0)=i^T*M_0$ can be separated into two parts: $\bullet_D$ and $\bullet_C$ associated with D and i\D respectively $(W(M_0)=\bullet_D-\bullet_C)$.

Lemma 4[5]: Let (N,M_0) be a net-system and let D∈P be a deadlock of N. If D is controlled by the S-invariant i under M_0, then $\leftrightarrow M\sum[M_0>$: D is marked under M.

Lemma 5[5]: Let (N,M_0) be a net-system. N is weakly live under M_0 if every minimal deadlock D_m is controlled by an S-invariant i under M_0.

Lemma 6[5]: Let N be a net which is bounded and covered by an elementary T-invariant j. If N is weakly live, then N is live.

Note that the invariant is not strongly connected (see Fig. 7). This is due to the fact that some y values in it are negative. If we reverse all the arcs in the invariant incident to places with negative y values, then it will become strongly connected.

Let p_d be a drain place and T_D the set of output transitions to p_d that are not in the bad siphon. To derive the marking constraint, we study the physical meanings of the y values in a D_m and $i\backslash D_m$.

Lemma 7: Let t be a transition in a marked ESNC N, t is not live, iff there exists a minimal deadlock D_m containing a place p∑⊏t and a reachable marking M such that D_m is empty in M.

This lemma implies that *if every minimal deadlock is never empty for all reachable markings in a WSNC, then every transition is live.* The only minimal deadlocks that can turn from nonempty to empty are bad siphons, i.e., deadlocks that are not traps. Without TP-inconsistent pair of places in the equivalent WSNC, there is only one kind of bad siphon shown in Fig. 7. In the sequel, we develop theory for the *marking condition* (see Theorem 3) for a bad siphon that cannot be emptied. We will show that for the above bad siphon, if i*M>0, then the D_m cannot be emptied; otherwise, if 0⊒i*M, the D_m can be emptied. Hence, if $\leftrightarrow D_m$, i*M>0 implies the liveness of the WSNC and we have

Lemma 8: Let (N,M_0) be a net-system. N is live under M_0 if every minimal deadlock D_m is controlled by an S-invariant i under M_0.

Remark: The liveness depends on the existence of S-invariant which is ensured by the satisfaction of the structure constraint. From [2], a WSNC can be decomposed into a set of T- and P-components. For each P-component Ω, we can obtain a P-semiflow of N as follows. We apply the procedure for R computation for Ω' (reverse of Ω by interchanging places with transitions). R_i for t_i∑Ω' corresponds to y_i (>0) for p_i∑Ω The S-invariant corresponding to the P-semiflow is obtained in Appendix A.

Lemma 8 implies that we can consider each D_m alone. In the equivalent WSNC^o, we can consider each P-component alone similar to the WMG case where each loop is considered alone. As long as each loop (P-component) in a WMG (WSNC^o) is live, the net is live. To check i*M>0, we resort to the physical meanings of the component $\bullet_C$ in i*M. Since every D_m must be checked, it implies that when we look at a specific D_m, we can ignore all other D_m and the tokens in this D_m can flow freely independent of tokens in other D_m. This way, we can figure out the physical meanings of $\bullet_C$.

The physical meaning of y for places in i\\$D_m$. We find y values for places in i\\$D_m$ based on the T-condition. This way, we can also see the physical meaning of y. Let $p_i \Sigma$RC, $p_i \Sigma \subseteq t_d^i$, then $y_i=y_{i-1}+y_d$ where y_d is the y value for p_d. In Fig. 7, where there are 3 t_d and p_d=p5, y_d=4, then

$$y_i=y_{i-1}+y_d, \qquad (1)$$

y_1=-8, and y_2=-4. For a token at p_1 to fire t_d^1, it must fire t_d^2 and t_d^3 first, and each firing of which consumes one token at p_d. The total number of tokens consumed is $2=|y_1/y_d|$. The physical meaning of y is now clear:

> **For every token at p_i, $\exists_i=|y_i/y_d|$ (absolute value of y_i/y_d) is the maximum number of tokens in (sucked from) p_d to be consumed by firing t_d not in D_m without firing the t_d in D_m.**

The maximum number of tokens sucked from $p_d^{\,j}$ due to a single RC_k, where m_i is the token at p_i in RC_k. Note that there may be more than one RC associated with $p_d^{\,j}$. Hence, the maximum number of tokens sucked from $p_d^{\,j}$ is

$\bullet_j \; \nu \neq_k \; \bullet_j^{\,k}$. This is the maximum number of tokens to be consumed from $p_d^{\,j}$ without firing any $t_d \Sigma p_d^{\,j} \subseteq$ in D_m. The following lemma is due to Eq. (1)

Lemma 9: Let (1) RC_i be the regulation circuit and $P_r^{\,i}=\{p_{ij} \mid p_{ij} \Sigma\ RC_i, j=1,2,..,k\}$ be the set of all places in RC_i; that is $RC_i=[p_{i1}\, t_{i1}\, p_{i2}\, t_{i2}\, p_{i3}\, t_{i3}\, ..\, p_{ik} t_{ik} p_{i1}]$ where $k=a_o(p_d)$; p_d is the drain place for RC. (2) H_e ($P(H_e) \in D_m$) be the PT-handle associated with p_{ie} whose $n_s=p_d$ and $t_{ie}\Sigma H_e$. (3) $p_{oe}\Sigma H_e$ and $p_{oe}\Sigma t_{ie}\subseteq$ The following y values for p_{oe} and places in RC_I satisfy the T-condition:

(1)$y_i=y_{i-1}+y_d$, (if i-1=0, $y_{i-1}=y_k$). (2) $y_{e-1}=0$, $y_l=-y_d*(k- l +e-1)\%k$, l=1,2,…,k. (3)$\leftrightarrow p\Sigma H_e$, $p\Upsilon p_d$, $y(p)= k*y_d$,

See Fig. 7 for the y values based on this lemma.

Corollary 1: Let D_m^1, D_m^2, …, D_m^k be the set of D_m associated with the RC in Lemma 9 and y_l^b (b=1,2,…,k) the y value for $p_l\Sigma$RC and $D_m=D_m^b$, then $\cup$ constant e, e$\supseteq$1, $y_l^b=-y_d*(l+e)\%k$ and e-b is a constant for every b and l.

Theorem 3[8]: dck2d[$_D$3d]$\bullet_1$:d$\bullet_2$:oo:d$\bullet_v$ ^{T}dbe a marking for a P-component Ω in a WSNC^{o}dwith every RC containing tokens, then

ᴥodWeM_Da3$\bullet_D$$\cup\!\!\bullet_C$30o

2odeΩ:d[$_0$) is deadlocked, iff $M_0=M_D$, where Ω is the equivalent of Ω ind the ESNC of the WSNC^{o}o

3od[=[$_D$ ifdeΩ, M) is deadlocked.

4odeΩ:d[$_0$) is live if $W(M_0) > W(M_D)$.

Note that there are $|p_d^{\,i}|$ possible values for $\bullet_i$. Each $\bullet_i$ corresponds to a D_m including a $t_d\Sigma\ p_d^{\,i}\subseteq$ The maximal $\bullet_i$ induces a maximal marking at $p_d^{\,i}$ and its computation for RC with multiple tokens is provided in Appendix B.

This theorem implies that Lautenbach's marking condition is both a sufficient and necessary condition for liveness.

In the special case of the WSNC being a WMG, $M_D(p_i)=\bullet_i=a_o(p_i)$-1, and we have the following theorem:

Theorem 4[20]:d Let M_D3]$a_o$ep$_1$a$\cup\!\bullet$:d $a_o$ep$_2$a$\cup\!\bullet$:oo: $a_o$ep$_K$a$\cup\!\bullet$ ^{T}dbe a marking for loop L, then

1.(L, M_Da is deadlocked,

2o dΩd$_D$ if (L, M) is deadlocked,

ea (L, M_0) is live if $W(M_0) > W(M_D)$a

]**Lemma 10**oLet H_1 and H_2 be a pair of PP-handles whose places are in a controlling S-invariant, then the number of t_d on H_1 and H_2 must be identical.

7. CONCLUSION & ACKNOWLEDGEMENTS

We can transform SNC with pure TP and PT into a GPN and apply existing theory to study its properties. The subclass of WMG is studied first and then extended to WSNC. Afterwards, we investigate the marking condition for WSNC. This work was sponsored under research grant number NSC88-2213-E-004-001.

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Appendix A: Procedure of Converting P-Semiflow ∈ in WSNC into S-invariant in ESNC

In WSNC, each P-component ∈ is both a minimal deadlock and an S-invariant. Each arc [t_i p_{i+1}] ([p_i t_i]) corresponds to a TP (PT) CFOS in the ESNC where each minimal deadlock and its controlled S-invariant correspond to a ∈ in the WSNC. Any minimal deadlock in the ESNC includes in $\in^e$ all TP CFOS and only one PT handle from each PT CFOS while the controlled S-invariant includes the minimal deadlock and |Z|-1 arcs from each RC for the circuit. The missing arc from a RC corresponds to the place with *l* satisfying ((*l*+*e*)%*k*)=0 in Eq. (v) of Appendix B. For an example, see Fig. 7. The procedure is summarized as follows:

1. Find y values for places in ∈.
2. For each place p_i with input arc weights a_o and $y_{i,}$, the y value for each place in each TP-handle of the corresponding TP CFOS equals $y_{i,}$,
3. For each place p_o with output arc weights a_i and $y_{o,}$ the y for each place in one PT-handle of the corresponding PT CFOS equals y_{o*} $a_{i,.}$ The y for all the places in the rest PT-handles of the PT CFOS equals 0. For each place p_j in the RC, $y_{j=}y_{o*}$ y'_j where y'_j is obtained based on the equation in Appendix B.

Appendix B: ↔ computation for RC with multiple tokens

Let RC_i be the regulation circuit and P_r^i={p_{ij} | p_{ij}∑ RC_i, j=1,2,..k} be the set of all places in $RC_{i;}$ that is RC_i=[p_{i1} t_{i1} p_{i2} t_{i2} p_{i3} t_{i3} .. p_{ik} t_{ik} p_{i1}] where k=$a_o(p_d)$; p_d is the drain place for RC. Based on Corollary 1 and the physical meaning of $\left|\frac{y_l^b}{y_d}\right|$, the maximum number of tokens sucked from D_m^b at p_d is

$$\leftrightarrow_i \bullet \max_{e=1tok} \subseteq_{b=1}^{k} . \subseteq_{l=1}^{k} (m_l * y_l{}^b) \bullet \max_{e=1tok} . \subseteq_{l=1}^{k} (m_l * ((l = e)\%k))$$

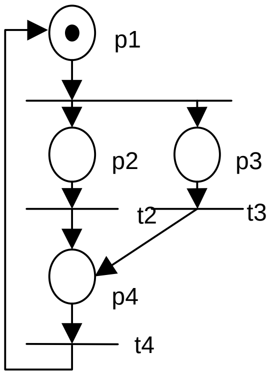

Fig. 5(a) An asymmetrical TP FOS.

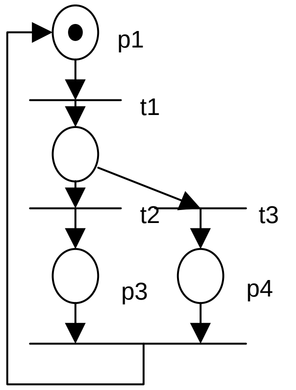

Fig. 5(b) An asymmetrical PT FOS

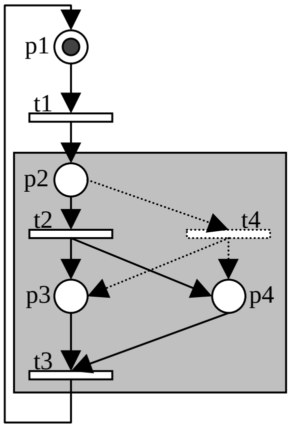

Fig. 1. An example of live & reversible SNC with no inconsistent pair.

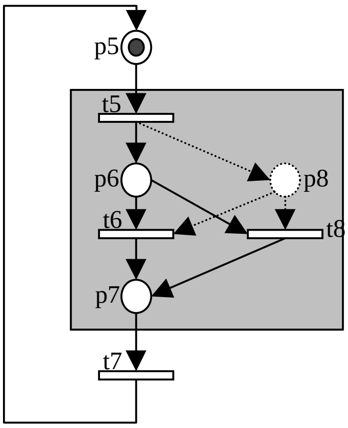

Fig.2 Dual of the net in Fig.1. This net is live and reversible without inconsistent

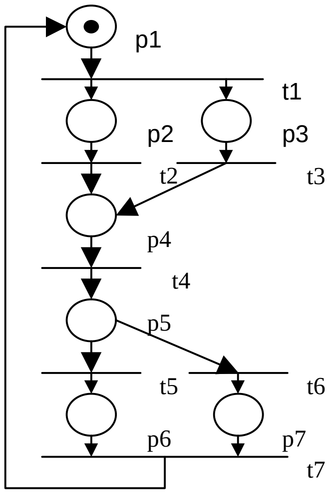

Fig. 5(c). Combining TP and PT CFOS.

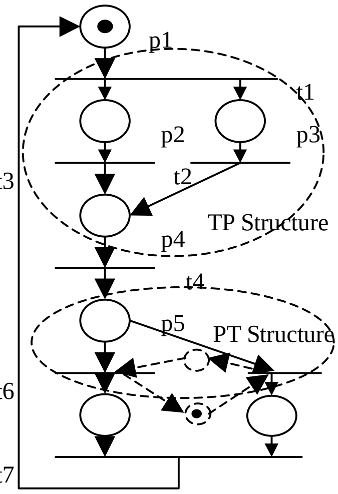

Fig. 5(d). Adding a RC in the PT CFOS.

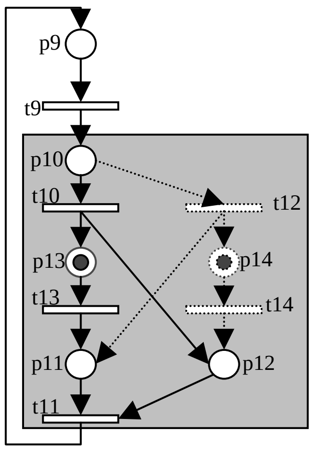

Fig. 3a. Irreversible SNC with PT-inconsistent pair (p13, p14)

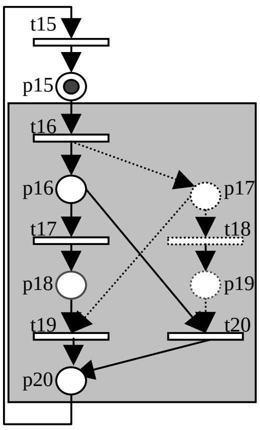

Fig.4. Dual of the net in Fig. 3a. The SNC is not live with TP-inconsistent pair (p18, p19)

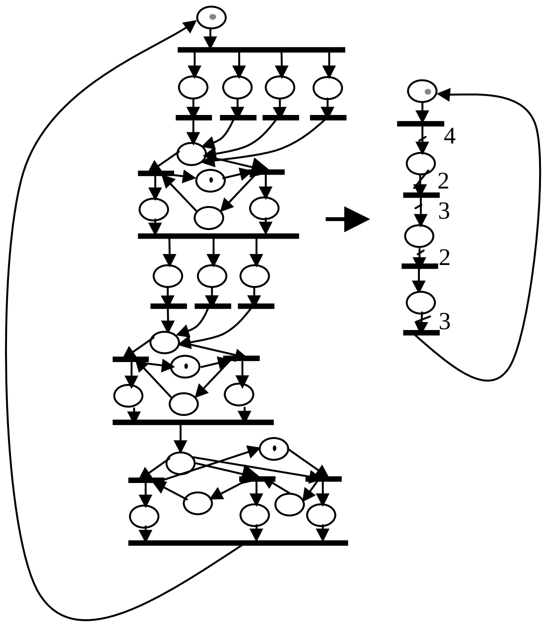

Fig.6. Extra tokens generated from one TP FOS are not totally consumed by one PT FOS. The net is SL&SB since the loop gain is one. The ESNC on the left is reduced to a WSNC on the right.

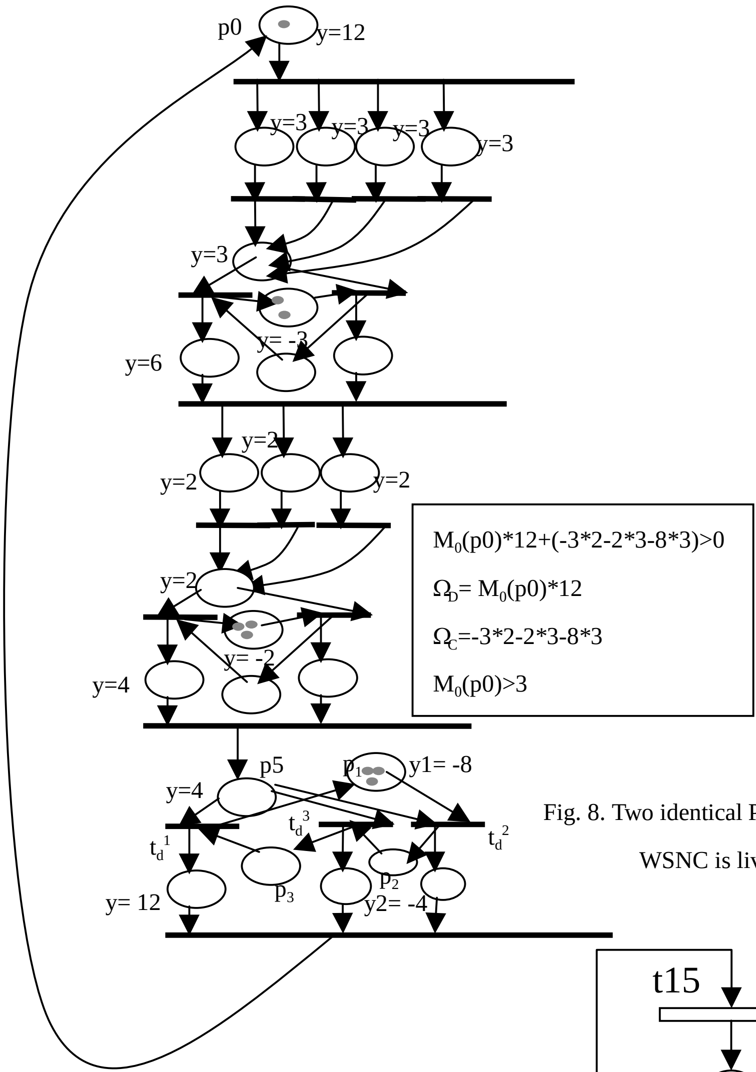

Fig. 7. A more complicated example with multiple tokens in the initial marking of all RC.

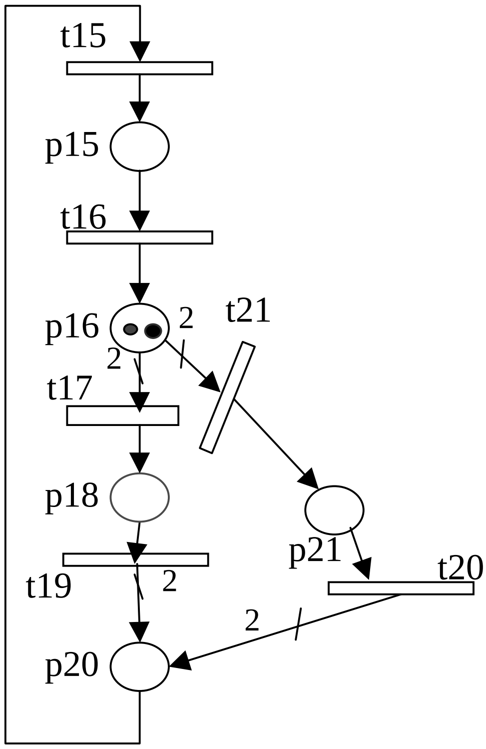

Fig. 8. Two identical PT CFOS sharing the same n_s and the WSNC is live but the ESNC is not live.

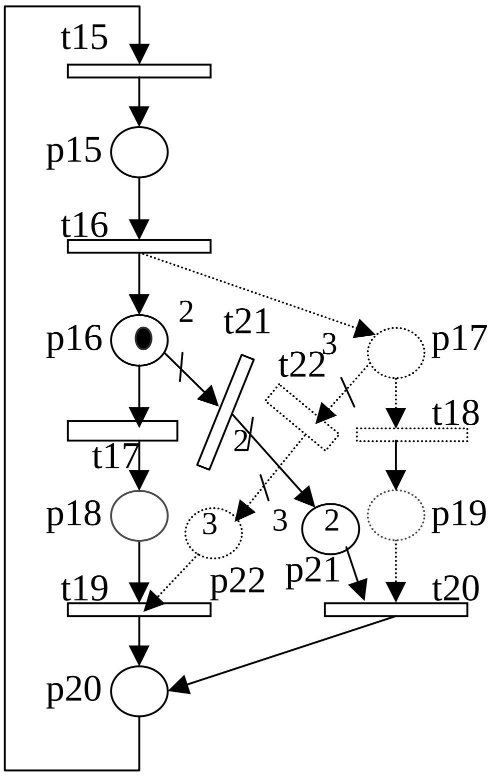

Fig. 9. Net with TP inconsistent pair of places. Multiple arc weights may not prevent a bad siphon from being emptied.