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Towards an Integration of Vague and Uncertain Information

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Abstract

In this paper, we have proposed truth-qualified fuzzy propositions as the representation of uncertain vague information where the fuzzy sets embody the intended meaning of imprecise information and the fuzzy truth values serve as the representation of uncertainty. We have developed an inference mechanism for fuzzy propositions with fuzzy truth values. There are three steps involved. First, the fuzzy rules and fuzzy facts with fuzzy truth values are transformed into a set of uncertain classical propositions with necessity and possibility measures by means of λ-cut. Second, our proposed inference called possibilistic entailment is performed on the set of uncertain classical propositions. Third, we reverse the process in the first step to synthesize all the λ-level-sets obtained in the second step into a fuzzy set, and to compose necessity and possibility pairs to form a fuzzy truth value. Compared with the existing work, our approach does not impose any restriction on the inference mechanism, that is, the intended meaning is not required to be unchanged; meanwhile, the confidence level can be partially certain. The proposed approach is not only a generalization of Zadeh's generalized modus ponens but also an uncertain reasoning for classical propositions with necessity and possibility pairs.

1 Introduction

Considerable expert systems have been developed in recent years[8][9][10]. Two of the most important components in rule-based expert systems are: knowledge base and inference engine, which serve the purpose of inferring a useful conclusion from established rules by experts and users' observed facts. However, certain and precise knowledge are not always available for human experts to establish knowledge base; furthermore, users' observations are sometimes uncertain and imprecise. Therefore, an adequate management of uncertainty and imprecision pervading in the rule base and the data base of expert systems has become a significant issue[2].

The distinction between imprecise and uncertain information can be best explained by the canonical form representation (i.e. a quadruple of attribute, object, value, confidence) proposed by Dubois and Prade[4]. Imprecision implies the absence of a sharp boundary of the value component of the quadruple; whereas, uncertainty is related to the confidence component of the quadruple which is an indication of our reliance about the information. Information is labeled as being imperfect if the imprecision and uncertainty simultaneously occur.

In order to perform reasoning for both imprecise and uncertain information, two important issues should be addressed. First, any improvement of the confidence level for a piece of information can only be achieved at the expense of the specificity of the information; and vise verse[14]. Second, the matching between a fact and the premise of a rule is not exact, but only partial[15]. We have roughly classified the existing approaches in dealing with both imprecise and uncertain information into three categories based on their treatments for the two issues.

- An uncertainty-qualified fuzzy proposition is translated into a proposition whose confidence level is certain but with less specific information, while partial matching is to modify the intended meaning of conclusions. This approach was advocated by Zadeh[15]. Zadeh proposed three uncertainty-qualifications for fuzzy propositions: probability-, possibility- and truth-qualifiers.

- The degree of partial matching is to influence the confidence level of conclusions (i.e. a truncation will occur), which was adopted by researchers such as Ogawa et al.[13], Martin-Clouair et al.[11]. Ogawa combined certainty factors and fuzzy sets to represent uncertain and imprecise information in an expert system SPERIL-2. Martin-Clouair attached possibility and necessity degrees to fuzzy propositions.

- No partial matching is allowed in Godo et al.[7]. Godo used the fuzzy truth value as uncertainty-qualifier of fuzzy propositions.

Notice that the first kind of research results in a completely certain conclusion whose intended meaning has been changed. On the other hand, the second one produces a new confidence level for a conclusion without modifying its intended meaning. The third one can be viewed as a special case of the second one.

It is obvious that these inference strategies are somewhat limited due to the fact that either the intended meaning is required to be unchanged, or the confidence level has to be completely certain.

In this paper, we propose the use of fuzzy truth values and fuzzy sets for representing uncertain and imprecise information, respectively. The fuzzy truth value is adopted for its capability to express the possibility of the degree of truth of a fuzzy proposition. We have developed an inference mechanism for fuzzy propositions with fuzzy truth values. There are three steps involved. First, the fuzzy rules and fuzzy facts with fuzzy truth values are transformed into a set of uncertain classical propositions with necessity and possibility measures by means of λ-cut. Second, an our proposed inference called possibilistic entailment is performed on the set of uncertain classical propositions. Third, we reverse the process in the first step to synthesize all the λ-level-sets obtained in the second step into a fuzzy set, and to compose necessity and possibility pairs to form a fuzzy truth value.

The organization of this paper is as follows. The representation of uncertain vague propositions and its semantics are defined in the next section. In section 3, an algorithm for inference strategy is developed. Finally, a summary of our approach and its potential benefits are given in the section 4.

2 Representation

A classical proposition is true in some possible worlds and false in the rest of possible worlds, while a fuzzy proposition $\tilde{p}$ is true with respect to a possible world to a degree[6]. We model our uncertainty about the actual world by defining a possibility distribution over all possible worlds to specify the possibility that the actual world is in each possible world. Esteva et al.[6] have extended Dubois and Prades' definition about the possibility and necessity measures of classical propositions to the case of fuzzy propositions through fuzzy truth values. Given a possibility distribution π on the set of possible worlds Ω, the membership function of a fuzzy truth value of $\tilde{p}$ is defined as[1]:

$$\mu_{\tau(\tilde{p}|\pi)}(t) = Sup_{\omega\in\Omega}\{\pi(\omega)|\mu_{\tilde{p}}(\omega) = t\}; \quad t \in [0,1] \quad (1)$$

where $\mu_{\tilde{p}}$ denote the fuzzy set of possible worlds of $\tilde{p}$ in Ω, ω is a possible world and t is the degree of truth. $\tau(\tilde{p}|\pi)$ can be viewed as the possibility measure of a set of possible worlds in which the truth degree of $\tilde{p}$ is equivalent to t, that is,[6]

$$\mu_{\tau(\tilde{p}|\pi)}(t) = \Pi_{\pi}\{\omega \in \Omega|\mu_{\tilde{p}}(\omega) = t\}; \quad t \in [0,1] \quad (2)$$

To represent uncertain vague information, we have chosen a fuzzy proposition with a fuzzy valuation, denoted as

$$(\tilde{p}, \tau)$$

where $\tilde{p}$ is a fuzzy proposition of the form "X *is* $\tilde{F}$" (i.e. X is a variable and $\tilde{F}$ is a fuzzy set in a universe U), and τ is a fuzzy valuation. It should be noted that, for every formula $(\tilde{p}, \tau)$ (called a truth-qualified fuzzy proposition), we assume $\tau \geq \tau(\tilde{p}|\pi)$,

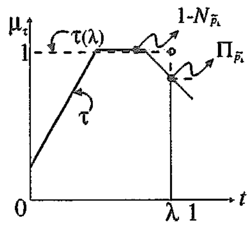

Figure 1: A fuzzy truth value

which means $\mu_\tau(t)$ is the upper bound of the possibility that $\tilde{p}$ is true to a degree t. The fuzzy set is to represent the intended meaning of imprecise information; while, the fuzzy truth value serves as the representation of uncertainty for its capability to express the possibility of the degree of truth.

To develop inference rules for truth-qualified fuzzy propositions, we have treated a truth-qualified fuzzy proposition as a set of weighted classical propositions, in which the weight is represented using necessity and possibility measures. For the purpose of explaining how a set of weighted classical propositions are induced from a truth-qualified fuzzy proposition, we first introduce the notion of l-cut.

Definition 1 *The crisp set of elements whose degree of membership in the fuzzy set $\tilde{F}$ are l is called the l-level-set:*

$$\tilde{F}_{(l)} \triangleq \{u \in U|\mu_{\tilde{F}}(u) = l\}$$

where U is the universe of discourse.

Based on definition 1, we can derive the following inequality:

$$\mu_\tau(t) \geq \Pi[\tilde{F}_{(t)}], \ t \in [0,1] \quad (3)$$

Thus, equation (3) can be interpreted as: The upper bound of the possibility measure of "X *is* $\tilde{F}_{(t)}$" is $\mu_\tau(t)$.

It is obvious that a λ-level-set $\tilde{F}_\lambda$ of $\tilde{F}$ is a union of some l-level-sets, i.e. $\tilde{F}_\lambda = \cup_l\{\tilde{F}_{(l)}, \forall l \geq \lambda\}$. Therefore, the upper bound $\Pi_{\tilde{F}_\lambda}$ of the possibility of "X *is* $\tilde{F}_\lambda$" (denoted as $\tilde{p}_\lambda$) is to take the maximum value among the membership degrees whose corresponding truth degrees are equal to or greater than λ; the lower bound $N_{\tilde{F}_\lambda}$ of the necessity of "X *is* $\tilde{F}_\lambda$" is equal to the lower bound of the duality of the possibility of "X *is not* $\tilde{F}_\lambda$", which is to take the difference between one and the maximum value among the membership degrees whose corresponding truth degrees are less than λ (see figure 1). These are formally defined below.

Definition 2 *A truth-qualified fuzzy proposition $(\tilde{p}, \tau)$ is equivalent to a set of classical propositions with necessity and possibility pairs*

$$\{(\tilde{p}_\lambda, (N_{\tilde{F}_\lambda}, \Pi_{\tilde{F}_\lambda})), \lambda \in (0,1]\}$$

where $N_{\tilde{F}_\lambda}$ denotes the lower bound of the necessity measure; whereas, $\Pi_{\tilde{F}_\lambda}$ denotes the upper bound of the possibility measure, defined as

$$\begin{aligned} N_{\tilde{F}_\lambda} &= 1 - max_t\{\mu_\tau(t) | t \in [0, \lambda)\} \\ \Pi_{\tilde{F}_\lambda} &= max_t\{\mu_\tau(t) | t \in [\lambda, 1]\} \end{aligned} \quad (4)$$

The membership function of $\tilde{F}$ can be reconstructed in terms of the characteristic functions $\mu_{\tilde{F}_\lambda}$ of its λ-level-sets $\tilde{F}_\lambda$ derived based on equation (4), that is,

$$\mu_{\tilde{F}}(u) = Sup_\lambda min\{\lambda, \mu_{\tilde{F}_\lambda}(u)\} \quad u \in U \quad (5)$$

Reconstruction of τ from the set of $(N_{\tilde{F}_\lambda}, \Pi_{\tilde{F}_\lambda})$ pairs in equation (4) is through the use of the principle of minimum specificity[1]. The principle states that the least arbitrary choice among those candidates of τ, satisfying equation (4) for each pair of $(N_{\tilde{F}_\lambda}, \Pi_{\tilde{F}_\lambda})$, is the least specific solution $\tau(\lambda)$ (see figure 1), that is, for each λ,

$$\mu_{\tau(\lambda)}(t) = \begin{cases} \Pi_{\tilde{F}_\lambda} & if\ t \geq \lambda \\ 1 - N_{\tilde{F}_\lambda} & if\ t < \lambda \end{cases} \quad (6)$$

Thus, τ can be reconstructed by

$$\mu_\tau(t) = min_\lambda \mu_{\tau(\lambda)}(t) \quad t \in [0, 1] \quad (7)$$

3 Inference

The formulation of the proposed inference rule for truth-qualified fuzzy propositions is:

$$\frac{\begin{array}{l} \tilde{p} \rightarrow \tilde{q},\ \tau_1 \\ \tilde{p}' \quad\ ,\ \tau_2 \end{array}}{\tilde{q}',\ \tau_3} \quad (8)$$

where $\tilde{p}$, $\tilde{q}$, $\tilde{p}'$, and $\tilde{q}'$ are fuzzy propositions and characterized by "X *is* $\tilde{F}$", "Y *is* $\tilde{G}$", "X *is* $\tilde{F}'$" and "Y *is* $\tilde{G}'$", respectively; τ_1, τ_2, and τ_3 are fuzzy valuation for truth values and defined by $\mu_{\tau_1}(t)$, $\mu_{\tau_2}(t)$, and $\mu_{\tau_3}(t)$, respectively. $\tilde{F}$ and $\tilde{F}'$ are the subsets of U, while $\tilde{G}$ and $\tilde{G}'$ are the subsets of V.

There are three major steps for deriving $\tilde{q}'$ and τ_3 of equation (8). First, the fuzzy rules and fuzzy facts with fuzzy truth values are transformed into a set of uncertain classical propositions with necessity and possibility measures by means of λ-cut. Second, inference is performed on the set of uncertain classical propositions. Third, we reverse the process in the first step to synthesize all the λ-level-sets obtained in the second step into a fuzzy set, and to compose necessity and possibility pairs to form a fuzzy truth value.

[1] Of importance in possibility theory is the principle of minimum specificity[3], which says that, given a set of constraints restricting the value of a variable, the possibility distribution of the variable should be defined so as to allocate the maximal degree of possibility to each value, in accordance with the constraints.

Step 1: Transformation Based on definition 2, a truth-qualified fuzzy fact is equivalent to a set of classical propositions (the λ-level-sets of the fact) with necessity and possibility pairs. Similarly, a fuzzy rule with a fuzzy truth value can be viewed as a collection of classical implication relationships (the λ-level-sets of the fuzzy relation for this rule) with necessity and possibility pairs. Therefore, equation (8) can be transformed into

$$(\tilde{p} \rightarrow \tilde{q})_\lambda,\ (N_{(\tilde{F} \rightarrow \tilde{G})_\lambda},\ \Pi_{(\tilde{F} \rightarrow \tilde{G})_\lambda}) \quad (9)$$

$$\tilde{p}'_\lambda \quad\ ,\ (N_{\tilde{F}'_\lambda},\ \Pi_{\tilde{F}'_\lambda}) \quad (10)$$

$$\overline{\tilde{q}'_\lambda,\ (N_{\tilde{G}'_\lambda},\ \Pi_{\tilde{G}'_\lambda})} \quad (11)$$

where $\lambda \in (0, 1]$ and

$$\begin{aligned} N_{(\tilde{F} \rightarrow \tilde{G})_\lambda} &= 1 - max_t\{\mu_{\tau_1}(t) | t \in [0,\ \lambda)\} \\ \Pi_{(\tilde{F} \rightarrow \tilde{G})_\lambda} &= max_t\{\mu_{\tau_1}(t) | t \in [\lambda, 1]\} \\ N_{\tilde{F}'_\lambda} &= 1 - max_t\{\mu_{\tau_2}(t) | t \in [0,\ \lambda)\} \\ \Pi_{\tilde{F}'_\lambda} &= max_t\{\mu_{\tau_2}(t) | t \in [\lambda, 1]\} \end{aligned} \quad (12)$$

Step 2: Inference

Computing $\tilde{q}'_\lambda$ $\tilde{G}'_\lambda$ is computed through the use of compositional rule of inference, that is,

$$\tilde{G}'_\lambda = \tilde{F}'_\lambda \circ (\tilde{F} \rightarrow \tilde{G})_\lambda \quad (13)$$

where $\circ$ is a composition operator and $\rightarrow$ denotes an implication operator.

Computing $N_{\tilde{G}'_\lambda}$ and $\Pi_{\tilde{G}'_\lambda}$ Equation (9) is semantically equivalent to

$$\tilde{p}'_\lambda \rightarrow \tilde{q}'_\lambda,\ (N_{\tilde{F}'_\lambda \rightarrow \tilde{G}'_\lambda},\ \Pi_{\tilde{F}'_\lambda \rightarrow \tilde{G}'_\lambda}) \quad (14)$$

The $N_{\tilde{F}'_\lambda \rightarrow \tilde{G}'_\lambda}$ and $\Pi_{\tilde{F}'_\lambda \rightarrow \tilde{G}'_\lambda}$ are defined as

$$\begin{aligned} N_{\tilde{F}'_\lambda \rightarrow \tilde{G}'_\lambda} &= 1 - max\{\pi(u,v) |\ (u,v) \notin \sim \tilde{F}_\lambda \cup \tilde{G}_\lambda\} \\ \Pi_{\tilde{F}'_\lambda \rightarrow \tilde{G}'_\lambda} &= max\{\pi(u,v) |\ (u,v) \in \sim \tilde{F}_\lambda \cup \tilde{G}_\lambda\} \end{aligned} \quad (15)$$

where $\pi(u, v)$ denotes a possibility distribution over $U \times V$, derived by means of the principle of minimum specificity:

$$\pi(u,v) = \begin{cases} \Pi_{(\tilde{F} \rightarrow \tilde{G})_\lambda} & (u,v) \in (\tilde{F} \rightarrow \tilde{G})_\lambda \\ 1 - N_{(\tilde{F} \rightarrow \tilde{G})_\lambda} & (u,v) \notin (\tilde{F} \rightarrow \tilde{G})_\lambda \end{cases} \quad (16)$$

Therefore, for each λ, we have

$$\frac{\begin{array}{l} \tilde{p}'_\lambda \rightarrow \tilde{q}'_\lambda,\ (N_{\tilde{F}'_\lambda \rightarrow \tilde{G}'_\lambda},\ \Pi_{\tilde{F}'_\lambda \rightarrow \tilde{G}'_\lambda}) \\ \tilde{p}'_\lambda \quad\ ,\ (N_{\tilde{F}'_\lambda},\ \Pi_{\tilde{F}'_\lambda}) \end{array}}{\tilde{q}'_\lambda,\ (N_{\tilde{G}'_\lambda},\ \Pi_{\tilde{G}'_\lambda})} \quad (17)$$

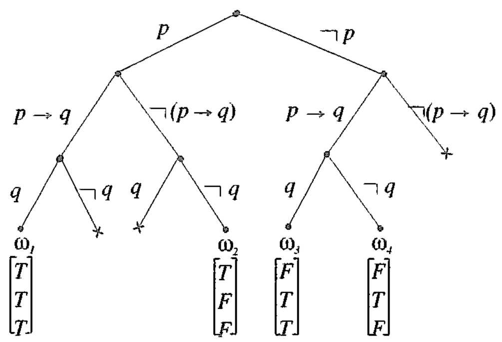

Figure 2: A semantic tree

To infer $N_{\tilde{G}'_\lambda}$ and $\Pi_{\tilde{G}'_\lambda}$, we propose a reasoning method called possibility entailment, inspired by Nilsson's probability entailment[12]. First, we determine the possible worlds based on the semantic tree for $\tilde{p}'_\lambda \rightarrow \tilde{q}'_\lambda$, $\tilde{p}'_\lambda$ and $\tilde{q}'_\lambda$ (see figure 2). In figure 2, inconsistent paths are indicated by an $\times$; T and F denote the truth values *true* and *false*, respectively. This semantic tree shows that there are four possible worlds:

	ω_1	ω_2	ω_3	ω_4
$\tilde{p}'_\lambda$	*true*	*true*	*false*	*false*
$\tilde{p}'_\lambda \rightarrow \tilde{q}'_\lambda$	*true*	*false*	*true*	*true*
$\tilde{q}'_\lambda$	*true*	*false*	*true*	*false*

We have mentioned that a possibility distribution over all possible worlds is used to model our uncertainty about the actual world, the possibility measure of any proposition $\tilde{p}$ is then reasonly taken to be the maximum of the possibilities of all possible worlds in which $\tilde{p}$ is true. Therefore, we can construct the relationship between the upper bound of the possibilities and the lower bound of the necessities of propositions and the possibilities of possible worlds:

$$\left\{\begin{array}{c} \Pi_{\tilde{p}'_\lambda} \\ 1-N_{\tilde{p}'_\lambda} \\ \Pi_{\tilde{p}'_\lambda \rightarrow \tilde{q}'_\lambda} \\ 1-N_{\tilde{p}'_\lambda \rightarrow \tilde{q}'_\lambda} \\ \Pi_{\tilde{q}'_\lambda} \\ 1-N_{\tilde{q}'_\lambda} \end{array}\right\} \geq \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right] \circ \left\{\begin{array}{c} \pi(\omega_1) \\ \pi(\omega_2) \\ \pi(\omega_3) \\ \pi(\omega_4) \end{array}\right\} \quad (18)$$

where $\circ$ indicates the composition operater "max-min" and $\pi(\omega_i)$ denotes the possibility that the actual world is in possible world ω_i. The first row of the above matrix gives truth values for $\tilde{p}'_\lambda$ in the four possible worlds; whereas, the second row gives opposite truth values for $\tilde{p}'_\lambda$ in the four possible worlds.

Based on equation (18) and the constraints "$max_i \pi(\omega_i) = 1$" and "$\pi(\omega_i) \leq 1$", we have

$$\begin{aligned} \pi(\omega_1) &\leq min\{\Pi_{\tilde{p}'_\lambda \rightarrow \tilde{q}'_\lambda}, \Pi_{\tilde{p}'_\lambda}\} \\ \pi(\omega_2) &\leq min\{1-N_{\tilde{p}'_\lambda \rightarrow \tilde{q}'_\lambda}, \Pi_{\tilde{p}'_\lambda}\} \\ \pi(\omega_3) &\leq min\{\Pi_{\tilde{p}'_\lambda \rightarrow \tilde{q}'_\lambda}, 1-N_{\tilde{p}'_\lambda}\} \\ \pi(\omega_4) &\leq min\{\Pi_{\tilde{p}'_\lambda \rightarrow \tilde{q}'_\lambda}, 1-N_{\tilde{p}'_\lambda}\} \end{aligned}$$

Thus, the upper bound of the possibility and the lower bound of the necessity of $\tilde{q}'_\lambda$ are derived:

$$\begin{aligned} \Pi_{\tilde{q}'_\lambda} &= max\{min[\Pi_{\tilde{p}'_\lambda \rightarrow \tilde{q}'_\lambda}, \Pi_{\tilde{p}'_\lambda}], \\ &\quad min[\Pi_{\tilde{p}'_\lambda \rightarrow \tilde{q}'_\lambda}, 1-N_{\tilde{p}'_\lambda}]\} \\ N_{\tilde{q}'_\lambda} &= min\{max[N_{\tilde{p}'_\lambda \rightarrow \tilde{q}'_\lambda}, 1-\Pi_{\tilde{p}'_\lambda}], \\ &\quad max[1-\Pi_{\tilde{p}'_\lambda \rightarrow \tilde{q}'_\lambda}, N_{\tilde{p}'_\lambda}]\} \end{aligned} \quad (19)$$

Step 3: Composition Based on equation (5), the construction of membership function of $\tilde{G}'$ is performed by

$$\mu_{\tilde{G}'}(v) = Sup_\lambda min\{\lambda, \mu_{\tilde{G}'_\lambda}(v)\} \quad v \in V \quad (20)$$

Meanwhile, the construction of τ_3 is calculated by equation (7):

$$\mu_{\tau_3}(t) = min_\lambda \mu_{\tau_3(\lambda)}(t) \quad t \in [0,1] \quad (21)$$

where

$$\mu_{\tau_3(\lambda)}(t) = \left\{\begin{array}{ll} \Pi_{\tilde{G}_\lambda} & if\, t \geq \lambda \\ 1-N_{\tilde{G}_\lambda} & if\, t < \lambda \end{array}\right. \quad (22)$$

Notice that if "Sup-min" is chosen as the composition operator for equation (13) and both τ_1 and τ_2 in equation (8) are "*true*" which is defined by its membership function, $\mu_{true}(t) = t$ for all $t \in [0,1]$, τ_3 is then "*true*". That is, our proposed inference rule reduces to

$$\begin{array}{l} \tilde{p} \rightarrow \tilde{q},\ true \\ \underline{\tilde{p}' \qquad ,\ true} \\ \quad \tilde{q}',\ true \end{array}$$

It should be also noted that if both $\tilde{p}$ and $\tilde{p}'$ are equal to a classical proposition p, and both $\tilde{q}$ and $\tilde{q}'$ are equal to a classical proposition q, and each τ_i (i=1-3) reduces to $\mu_{\tau_i}(0)$ which means the possibility of falsity (i.e. the duality of the necessity of truth) and $\mu_{\tau_i}(1)$ which means the possibility of truth, our inference rule becomes the form of equation (17)

$$\begin{array}{l} p \rightarrow q,\ (N_{p\rightarrow q}, \Pi_{p \rightarrow q}) \\ \underline{p \qquad ,\ (N_p, \Pi_p)} \\ \quad q,\ (N_q, \Pi_q) \end{array}$$

where N_i is the lower bound of the necessity measure and Π_i is the upper bound of the possibility measure, different from the lower bounds for both measures defined by Dubois and Prade[5]. To sum up, our proposed algorithm is not only a generalization of Zadeh's generalized modus ponens but also an uncertain reasoning for classical propositions with necessity and possibility pairs.

4 Conclusion

We have proposed truth-qualified fuzzy propositions as a representation of uncertain vague information, since a fuzzy truth-value is capable of expressing the possibility of the degree of truth of a fuzzy proposition. An uncertain vague proposition is interpreted as a set of classical propositions with necessity and possibility pairs. Based on the interpretation, we have also developed an algorithm for reasoning with such propositions.

There are two important features in the proposed algorithm.

- Compared with the existing work, our approach does not impose any restriction on the inference, that is, the intended meaning is not required to be unchanged; meanwhile, the confidence level can be partially certain.

- This algorithm is not only a generalization of Zadeh's generalized modus ponens but also an uncertain reasoning for classical propositions with necessity and possibility pairs.

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