以類神經網路探討不確定殊異系統之穩定性

A Neural-Network Approach to Investigate Stability of Uncertain Singular Systems

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摘要

本研究應用 Lyapunov 穩定理論，探討不確定殊異系統的穩定性條件。文中證明，此類系統的穩定性可經由一廣義代數 Riccati 矩陣方程式的解加以決定。本文亦提出一迴歸式類神經網路，以解此代數 Riccati 矩陣方程式。

關鍵詞：殊異系統，穩定性，迴歸式神經網路，Riccati矩陣方程式

Abstract

Lyapunov stability theory for conventional systems is extended in a natural way to investigate stability conditions for a class of uncertain singular (descriptor) systems with or without external inputs. It is shown that the perturbed system stability can be determined via the solution of a generalized algebraic Riccati matrix equation. The algebraic Riccati matrix equation is solved using a recurrent neural network.

Keywords: Singular system, Stability, Recurrent neural network, Riccati matrix equation

1. Introduction

Recently, specific characteristics for singular (descriptor) systems, such as stability, regularity, impulse observerability and controllability, normalizability, stabilization, have drawn the considerable attention of many researchers due to extensive applications of these systems in large scale systems, singular perturbation theory, and in particular, constraint mechanical systems [1, 2]. There have also been publications on robust stability for perturbed linear or nonlinear singular systems [3-5]. However, these researches focus only on the worst possible case of perturbations, i.e. the unstructured perturbation. Since structural properties about uncertainties have not been used, it is unavoidable that these results may be conservative for some cases.

In this article, we first study the stability and estimate the state response bound of a class of singular systems with uncertain variations. Lyapunov stability theory for conventional systems is extended in a natural way to investigate stability conditions. The presented approach involves the bound as well as the structure information of the uncertain variations. Our result includes perturbations of the unstructured type as a special case. It is shown that stability of perturbed systems can be determined by solving a generalized algebraic Riccati matrix equation.

Traditionally, Lyapunov and Riccati equations are usually solved using a variety of numerical algorithms [6,7]. Much commercial computer software, such as MATLAB, is also available for solving these equations. However, most algorithms are not appropriate for solving our proposed generalized algebraic Riccati matrix equations. Specially, they cannot be used for on-line operation. We extend here the recurrent neural network given in [8] to find solutions of generalized algebraic Riccati matrix equations. Because of the parallel-distributed nature, the neural networks proposed here could be available computational models for promptly obtaining the desired solutions. Finally, an example is given to demonstrate the proposed results.

2. Uncertain singular systems

Consider the following linear time-invariant singular systems with parameter uncertainties

$$E\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) \qquad (1)$$

where

$$\Delta A = R_1 \Delta\tilde{A}(\sigma) R_2, \Delta B = S_1 \Delta\tilde{B}(\sigma) S_2$$

and $x(t) \in \Re^n$; $u(t) \in \Re^m$ describes the external input vector, it may be unknown or uncertain but with $u(t) \in L_2[0, T)$ for some $T \in [0, \infty)$, that is $\int_0^T \|u(t)\|^2 dt < \infty$ where $\|u(t)\| = \sqrt{u^T(t)u(t)}$; The uncertain vector $\sigma \in \Omega$, $\Omega \subseteq \Re^p$ is a prescribed subset of $\Re^p$. In the above, R_1, R_2 and S_1, S_2 represent the structures of the interconnection between the nominal and uncertain parts. Assume that $\{E, A\}$ is regular, i.e. $\det(sE - A) \neq 0$ and the uncertainties satisfy

$$\Delta\tilde{A}^T(\sigma)\Delta\tilde{A}(\sigma) \le I, \Delta\tilde{B}^T(\sigma)\Delta\tilde{B}(\sigma) \le I \quad (2)$$

3. Stability analysis

Lemma 1.

Let x and u be vectors, D and E be matrices with compatible dimensions. The following inequalities hold for any real constant $\rho > 0$:

$$\text{(i)} 2x^T DE^T u \le \rho x^T DD^T x + \frac{1}{\rho} u^T EE^T u \quad (3a)$$

$$\text{(ii)} 2x^T DE^T x \le \rho x^T DD^T x + \frac{1}{\rho} x^T EE^T x \quad (3b)$$

Lemma 2.

Given any real constant $\rho > 0$ and matrices D, E, F with compatible dimensions such that $F^T F \le I$ then

$$\text{(i)} 2x^T PDFEu \le \rho x^T PDD^T Px + \frac{1}{\rho} u^T E^T Eu \quad (4a)$$

$$\text{(ii)} 2x^T PDFEx \le \rho x^T PDD^T Px + \frac{1}{\rho} x^T E^T Ex \quad (4b)$$

In the above, $\rho = 1$ is a typical selection for most applications of (3) and (4).

With reference to the class of perturbations (2), it is possible to derive the following theorem for stability of the uncertain singular system (1) with or without external inputs.

Theorem 1.

Suppose the uncertainties appeared in (1) satisfy (2). If, for a given $Q = Q^T > 0$, there exist real constants $\varepsilon, \rho_1, \rho_2 > 0$ and a solution $P = P^T > 0$ for the algebraic matrix equation

$$A^T PE + E^T PA + E^T P(\frac{1}{\varepsilon^2} BB^T + \rho_1 R_1 R_1^T + \rho_2 S_1 S_1^T)PE + \frac{1}{\rho_1} R_2^T R_2 + E^T QE = 0 \quad (5)$$

then the system state response is bounded by

$$\left[\int_0^T \|Ex\|_Q^2 dt\right]^{1/2} \le [\|Ex(0)\|_P^2]^{1/2} + \left[\int_0^T \|u\|_R^2 dt\right]^{1/2} \quad (6)$$

where the norm $\|y\|_W \equiv (y^T W y)^{1/2}$ with y being a vector, W, a symmetric matrix , and

$$R = \varepsilon^2 I + \frac{1}{\rho_2} S_2^T S_2$$

Furthermore, if the external input $u(t) = 0$, the perturbed system would be asymptotically stable provided that $P = P^T > 0$ is a solution of

$$A^T PE + E^T PA + \rho_1 E^T PR_1 R_1^T PE + \frac{1}{\rho_1} R_2^T R_2 + E^T QE = 0 \quad (7)$$

Proof: It is known that stability of a singular system is mainly dependent on the dynamic part and is independent of the static part. According to the solution structure, the stability is correlated to the generalized state Ex. Thus, we first construct a generalized Lyapunov function as

$$V(Ex) = \frac{1}{2} x^T E^T PEx$$

where $P = P^T > 0$, $P \in \Re^{n \times n}$. It follows that

$$\frac{dV(Ex)}{dt} = \frac{1}{2} \dot{x}^T E^T PEx + \frac{1}{2} x^T E^T PE\dot{x}$$

Introducing (1) gives

$$\frac{dV(Ex)}{dt} = \frac{1}{2} x^T (A^T PE + E^T PA)x + x^T E^T PR_1 \Delta\tilde{A}(\sigma) R_2 x + \frac{1}{2}(x^T E^T PBu + u^T B^T PEx) + x^T E^T PS_1 \Delta\tilde{B}(\sigma) S_2 u \quad (8)$$

Applying Lemma 2, we have

$$2x^T E^T PR_1 \Delta\tilde{A}(\sigma) R_2 x \le \rho_1 x^T E^T PR_1 R_1^T PEx + \frac{1}{\rho_1} x^T R_2^T R_2 x$$

$$2x^T E^T PS_1 \Delta\tilde{B}(\sigma) S_2 u \le \rho_2 x^T E^T PS_1 S_1^T PEx + \frac{1}{\rho_2} u^T S_2^T S_2 u$$

where ρ_1 and ρ_2 are positive constants. Combining these with (8) yields

$$\frac{dV(Ex)}{dt} \le \frac{1}{2} x^T (A^T PE + E^T PA + \rho_1 E^T PR_1 R_1^T PE + \rho_2 E^T PS_1 S_1^T PE + \frac{1}{\rho_1} R_2^T R_2)x + x^T E^T PBu + \frac{1}{2\rho_2} u^T S_2^T S_2 u$$

Using Lemma 1, we further have

$$2x^T E^T PBu \le \frac{1}{\varepsilon^2} x^T E^T PBB^T PEx + \varepsilon^2 u^T u$$

Combining the above inequalities gives

$$\frac{dV(Ex)}{dt} \le \frac{1}{2} x^T [A^T PE + E^T PA + E^T P(\frac{1}{\varepsilon^2} BB^T + \rho_1 R_1 R_1^T + \rho_2 S_1 S_1^T)PE + \frac{1}{\rho_1} R_2^T R_2]x + \frac{1}{2} u^T (\varepsilon^2 I + \frac{1}{\rho_2} S_2^T S_2)u \quad (9)$$

For the given $Q > 0$, if there exists a solution $P = P^T > 0$ of (5), (9) reduces to

$$\frac{dV(Ex)}{dt} \le -\frac{1}{2} x^T E^T QEx + \frac{1}{2} u^T Ru \quad (10)$$

where the symmetric, positive definite matrix R is defined as in (6). Integrating both sides of this from $t = 0$ to $T > 0$ gives

$$V(Ex(T)) - V(Ex(0)) \le -\frac{1}{2}\int_0^T x^T E^T QExdt + \frac{1}{2}\int_0^T u^T Rudt$$

Since $V(Ex(t)) \ge 0$ for all $t \ge 0$, then

$$\frac{1}{2}\int_0^T x^T E^T QExdt \le \frac{1}{2}(Ex(0))^T PEx(0) + \frac{1}{2}\int_0^T u^T Rudt$$

The state response bound can then be estimated as

$$\int_0^T \|Ex\|_Q^2 dt \le \|Ex(0)\|_P^2 + \int_0^T \|u\|_R^2 dt \le \left\{[\|Ex(0)\|_P^2]^{1/2} + \left[\int_0^T \|u\|_R^2 dt\right]^{1/2}\right\}^2$$

While in the absence of external inputs, the stability requirement can be obtained from (9) by letting $u, B, S_1, S_2 = 0$. This gives rise to (7). The Lyapunov derivative is

$$\frac{dV(Ex)}{dt} \le -\frac{1}{2} x^T E^T QEx$$

That is the generalized state $Ex \to 0$ as $t \to \infty$.

In (6), P, Q and R can be viewed as the weighting matrices where Q is the weight of the generalized sate. If the system is initially at rest, the state response can be estimated as

$$\sqrt{\int_0^T \|Ex\|_Q^2 dt} \le \sqrt{\int_0^T \|u\|_R^2 dt} \tag{11}$$

That is the system will be bounded-input bounded-output stable. For systems without uncertainties, it follows that $R_i, S_i = 0, i = 1,2$. It turns out that the stability requires

$$A^T PE + E^T PA + \frac{1}{\varepsilon^2} E^T PBB^T PE + E^T QE = 0 \tag{12}$$

The system response bound can be estimated as

$$\left[\int_0^T \|Ex\|_Q^2 dt\right]^{1/2} \le [\|Ex(0)\|_P^2]^{1/2} + \varepsilon\left[\int_0^T \|u\|^2 dt\right]^{1/2} \tag{13}$$

The response of Ex will be attenuated with respect to $u(t)$ by a factor ε.

Dropping out the input matrix B, (12) reduces to

$$A^T PE + E^T PA + E^T QE = 0 \tag{14}$$

which is the standard Lyapunov equation for linear singular systems.

We now consider the case of unstructured perturbations. If the uncertainties ΔA and ΔB are bounded by

$$\|\Delta A\| \le \alpha,\ \|\Delta B\| \le \beta \tag{15}$$

where $\|A\| \equiv [\lambda_{\max}(A^T A)]^{1/2}$, it is easily shown by the Rayleigh principle that

$$x^T(\Delta A^T \Delta A - \alpha^2 I)x \le 0,\ u^T(\Delta B^T \Delta B - \beta^2 I)u \le 0$$

Therefore (15) implies

$$\Delta A^T \Delta A \le \alpha^2 I, \Delta B^T \Delta B \le \beta^2 I$$

Comparing this to (1) and (2), this is equivalent to letting $R_1 = I$, $R_2 = \alpha I$, $S_1 = I$, $S_2 = \beta I$. The stability requirement can now be induced from (7):

$$A^T PE + E^T PA + \rho_1 E^T PPE + \frac{\alpha^2}{\rho_1} I + E^T QE = 0 \tag{16}$$

It is easily seen that the derived algebraic Riccati equation (6) encompasses various special cases of interest. And that Lyapunov stability theory for conventional systems is a particular case of the Lyapunov stability theory for singular systems. The result proposed in [5] for general singular systems is valid only for the worst possible case of ΔA, i.e. the unstructured perturbation. Since no more information other than the bound of ΔA is utilized, the result may be sometimes conservative .

4. Recurrent neural network solving for matrix equations

We start by considering the procedure for solving systems of linear matrix equations in terms of the recurrent neural networks. The first step is to formulate the objective matrix function

$$G(X) = 0 \tag{17}$$

where X is the solution matrix, $G \in \Re^{r \times s}$ is a matrix of functions of $X \in \Re^{p \times q}$. For example, to solve for Sylvester equation $XA - FX = C$ we define the objective matrix function $G(X) = XA - FX - C$. The second step is to construct an appropriate computation energy function

$$E_n[G(X)] = \sum_{i=1}^{r}\sum_{j=1}^{s} e_{ij}[g_{ij}(X)] \tag{18}$$

The derivation of the energy function enables us to transform the minimization problem into a set of ordinary differential equations on the basis of an artificial neural network with appropriate connection weights, input excitations, bias, and activation functions. A popular choice for the energy function is the following

$$e_{ij}(g_{ij}(X)) = \frac{1}{2} g_{ij}^2(X) \tag{19}$$

It should be emphasised that the choice of energy function is arbitrary. Other convex energy functions can also be chosen, for example, $e_{ij}(g_{ij}(X)) = |g_{ij}(X)|$, $e_{ij}(g_{ij}(X)) = \cosh(g_{ij}(X))$. The dynamic equation of the recurrent neural network for solving linear matrix equations is first proposed in [9]:

$$\frac{dx_{ij}}{dt} = -\eta \sum_{k=1}^{r}\sum_{l=1}^{s} \frac{\partial g_{kl}[X(t)]}{\partial x_{ij}} f_{kl}(g_{kl}[X(t)]), \tag{20}$$
$$i = 1,\dots,p,\ j = 1,\dots,q$$

where $\eta > 0$ is the update gain, $f_{kl}(g_{kl}) = \frac{\partial e_{kl}}{\partial g_{kl}}$ is the activation function. For the energy function defined by (19), the activation function is linear, i.e. $f_{kl}(g_{kl}) = g_{kl}$. If $e_{ij}(g_{ij}(X)) = |g_{ij}(X)|$ then $f_{kl}(g_{kl}) = \text{sgn}(g_{kl})$. If $e_{ij}(g_{ij}(X)) = \cosh(g_{ij}(X))$ then $f_{kl}(g_{kl}) = \sinh(g_{kl})$. The specific choice of η must ensure stability of the differential equations and an appropriate convergence speed to the stationary solution state. It is easy to show that the system of differential equations (20) is asymptotically stable and its steady state matrix represents the solution matrix.

The formula described in (20) is, in fact, a standard result of the conventional gradient algorithm [8], i.e.

$$\frac{dX(t)}{dt} = -\eta \frac{\partial E_n}{\partial X} = -\eta\left\{\sum_{k=1}^{r}\sum_{l=1}^{s} \frac{\partial E_n}{\partial g_{kl}} \frac{\partial g_{kl}}{\partial x_{ij}}\right\}$$
$$= -\eta\left\{\sum_{k=1}^{r}\sum_{l=1}^{s} \frac{\partial g_{kl}[X(t)]}{\partial x_{ij}} f_{kl}(g_{kl}[X(t)])\right\}, \tag{21}$$
$$i = 1,\dots,p, j = 1,\dots,q$$

where $\{a_{ij}\} i=1,\ldots,p, j=1,\ldots,q$ indicates the $p\times q$ matrix with elements a_{ij}. If the energy function (19) is used, (21) simplifies to

$$\frac{dX(t)}{dt}=-\eta\left\{\sum_{k=1}^{r}\sum_{l=1}^{s}\frac{\partial g_{kl}[X(t)]}{\partial x_{ij}}g_{kl}[X(t)]\right\}, \quad (22)$$

$i=1,\ldots,p, j=1,\ldots,q$

is convenient to adopt the matrix expression (21) as a standard form, and use this to convert various control problems into solvable matrix equations. Before extending the result to synthesize a neural net-based LQ controller, we first present the following useful derivative operations of a scalar-valued function defined as (18) with respect to the solution matrix X. These equalities will be useful for putting the generalized algebraic Riccati matrix equations of (5) and (7) in the form of (21).

Lemma 3.

For the solution matrix $X\in\Re^{n\times n}$ and the matrix of non-decreasing activation functions $F=\{f_{ij}(g_{ij})\}=F^T$, $A\in\Re^{n\times n}$, $E\in\Re^{n\times n}$, the following derivatives hold

(i) if $G(X)=A^TXE$ then $\frac{\partial E_n}{\partial X}=AFE^T$ (23a)

(ii) if $G(X)=E^TXA$ then $\frac{\partial E_n}{\partial X}=EFA^T$ (23b)

(iii) if $G(X)=E^TXAXE$ then

$$\frac{\partial E_n}{\partial X}=A^TXEFE^T+EFE^TXA^T \quad (23c)$$

5. Neural network solving for generalized algebraic riccati matrix equations

Solving the generalized algebraic Riccati matrix equation (5) in terms of recurrent neural networks, we first introduce the following objective matrix function

$$G_1(P)=A^TPE+E^TPA+E^TP(\frac{1}{\varepsilon^2}BB^T+\rho_1R_1R_1^T+\rho_2S_1S_1^T)PE$$
$$+\frac{1}{\rho_1}R_2^TR_2+E^TQE$$
$$=[g_{1ij}(P)],\quad i,j=1,\ldots,n \quad (24a)$$

It is well known that a Riccati equation has many solutions. However, only one of them is symmetric positive definite. To avoid the solution obtained converges to the one which is not positive definite, we suggest to impose the following constraint

$$G_2(P,R_r)=P-R_rR_r^T=[g_{2ij}(P,R_r)], i,j=1,\ldots,n \quad (24b)$$

where R_r is some nonsingular matrix. By defining the computation energy function $E_n[P,R_r]=\sum_{i=1}^{n}\sum_{j=1}^{n}\{e_{1ij}[g_{1ij}(P)]+e_{2ij}[g_{2ij}(P,R_r)]\}$, using (21) to minimize the computation energy and making use of equalities (23), it is straightforward to derive the following dynamic equation of the neural network

$$\frac{dP}{dt}=-\eta_p\frac{\partial E_n}{\partial P}$$
$$=-\eta_p\{AF_1(P)E^T+EF_1(P)A^T+EF_1(P)[(\frac{1}{\varepsilon^2}BB^T+$$
$$\rho_1R_1R_1^T+\rho_2S_1S_1^T)PE]^T+[E^TP(\frac{1}{\varepsilon^2}BB^T+\rho_1R_1R_1^T \quad (25a)$$
$$+\rho_2S_1S_1^T)]^TF_1(P)E^T+F_2(P,R_r)\}$$

$$\frac{dR_r}{dt}=-\eta_r\frac{\partial E_n}{\partial R_r}=\eta_rF_2(P,R_r)R_r \quad (25b)$$

where the update gains $\eta_p,\eta_r>0$ and

$$F_1(P)=F_1[A^TPE+E^TPA+E^TP(\frac{1}{\varepsilon^2}BB^T+\rho_1R_1R_1^T+$$
$$\rho_2S_1S_1^T)PE+\frac{1}{\rho_1}R_2^TR_2+E^TQE] \quad (26a)$$

$$F_2(P,R_r)=F_2[P-R_rR_r^T] \quad (26b)$$

The architecture of the proposed network for solution of the generalized algebraic Riccati matrix equation of order n consists of two bidirectionally connected layers and each layer consists of an $n\times n$ array of neurons. Equation (26) acts as the hidden layer, (25) as the output layer. The hidden layer performs a functional transformation, it calculates and propagates $G_{1,2}(P)$ through the matrices of activation functions $F_{1,2}(P,R_r)$. The matrix $\frac{1}{\rho_1}R_2^TR_2+E^TQE$ in (26a) acts as biasing threshold matrices adding to the hidden layer. There are no bias for the neurons in the output layer. Ther is an integral transformation in the output layer. Solutions $P_{ij}, i,j=1,2,\ldots,n$ of the generalized algebraic Riccati equation are present on the weights of the network. Since the matrix dynamic equation (25) possesses stable feature, if the update gains η_p are η_r chosen appropriately, $P(t)$ and $R_r(t)$ will approximately reach their steady values in finite time. Note that to ensure the steady value P being positive definite, we require that $R_r(t)$ converges faster than P. Therefore, we suggest to choose $\eta_p\ll\eta_r$.

Note that there are $n\times n$ dynamic equations needed to be solved in (25a). Since the bias $\frac{1}{\rho_1}R_2^TR_2+E^TQE$ is constrained to be symmetric, (25a) is symmetric. Thus, nearly half the computations are redundant. The recurrent network for solving (25a) can be reduced to $n\times(n+1)/2$ unique training pairs, which corresponds to the upper or lower triangular part of (25a). After solutions of these pairs are found, the remaining solutions of P can be obtained directly by copying the solved weights from the opposite elements along the diagonal. From this fact, neurons in (26a) are arranged into n blocks with the i-th block containing $n+1-i$ neurons. After calculation for the hidden layer is completed, elements of the upper triangular part of $F_1(P)$ are copied onto the opposite elements along the diagonal to form a complete $F_1(P)$ matrix. Calculation for the hidden layer of (26b) can also be reduced to $^{n\times(n+1)}/_2$ pairs. The neuron outputs of the output layer are multiplied with the update gain $-\eta_p$. After integration of $^{n\times(n+1)}/_2$ neuron outputs, the upper triangular part of the matrix P is generated. These elements are then copied to the opposite elements along the diagonal to form a complete solution matrix P.

Similarly, to solve for the generalized Riccati matrix equation (7) we define the objective function matrices

$$G_3(P)=A^TPE+E^TPA+\rho_1E^TPR_1R_1^TPE+\frac{1}{\rho_1}R_2^TR_2$$
$$+E^TQE=[g_{3ij}(P)],\quad i,j=1,\ldots,n \tag{27a}$$
$$G_4(P,R_r)=P-R_rR_r^T=[g_{4ij}(P,R_r)],i,j=1,\ldots,n \tag{27b}$$

The neural dynamic equaions of the output layer solving for (7) are given by

$$\frac{dP}{dt}=-\eta_p\{AF_3(P)E^T+EF_3(P)A^T+\rho_1EF_3(P)(R_1R_1^TPE)^T$$
$$+\rho_1(E^TPR_1R_1^T)^TF_3(P)E^T+F_4(P,R_r)\} \tag{28a}$$
$$\frac{dR_r}{dt}=\eta_rF_4(P,R_r)R_r \tag{28b}$$

The hidden layer is constructed by

$$F_3(P)=F_3[A^TPE+E^TPA+\rho_1E^TPR_1R_1^TPE$$
$$+\frac{1}{\rho_1}R_2^TR_2+E^TQE] \tag{29a}$$
$$F_4(P,R_r)=F_4[P-R_rR_r^T] \tag{29b}$$

6. Simulation results

Example: Let's consider the following linear time-invariant singular system with parameter uncertainties. Let the system matrices be

$$E=\begin{bmatrix}0&0\\0&5\end{bmatrix},\ A=\begin{bmatrix}1&-1\\0&-1\end{bmatrix},\ B=\begin{bmatrix}0\\1\end{bmatrix}$$

Assume the initial state vector is $x=[0\ \ 0]^T$. The system is subjected to the following unceraintie

$$\Delta A=\begin{bmatrix}0&0.2\sigma_1\\0&0.3\sigma_1\end{bmatrix},\ \Delta B=\begin{bmatrix}0\\0.1\sigma_2\end{bmatrix}$$

where the perturbation σ_1 is uniform in the interval $-0.5\le\sigma_1\le0.5$ and σ_2 is uniform in $-1\le\sigma_2\le1$. The input is a unit step signal. For the above perturbation matrices, we can have the following decompositions: $R_1=\begin{bmatrix}0.2\\0.3\end{bmatrix}$, $R_2=[0\ \ 1]$, $\Delta\tilde{A}(\sigma)=\sigma_1$ and $S_1=\begin{bmatrix}0\\1\end{bmatrix}$, $S_2=1$, $\Delta\tilde{B}(\sigma)=0.1\sigma_2$.

Let's choose $Q=\begin{bmatrix}0.001&0\\0&0.0001\end{bmatrix}$, $\rho_1=1$, $\rho_2=0.1$ and $\varepsilon=1$. Update gains for the neural netwoerks are chosen as $\eta_p=5000$ and $\eta_r=20000$. We use the neural dynamics (25) to solve for the generalized algebraic Riccati matrix equation (5).

Transitions of the objective matrix functions G_1 and G_2 are, respectively, shown in Figs. 1 and 2 for 5×10^{-4} seconds of numerical integration. Transitions of the solution matrix P and the additional constraint R_r are, respectively, illustrated in Figs. 3 and 4. The steady state values of P and R_r are approximately obtained as

$$P=\begin{bmatrix}0.0093&5.8165\times10^{-7}\\5.8165\times10^{-7}&0.0629\end{bmatrix},$$
$$R_r=\begin{bmatrix}0.0963&-0.0009\\0.0025&0.2508\end{bmatrix}$$

Clealy matrix P is symmetric and positive definite. Therefore, the perturbed singular system would be stable with respect to the perturbations. Responses for the perturbed system states x_1 and x_2 are, respectively, shown in Figs. 5 and 6. It is also found that the state response is bounded by

$$\left[\int_0^{35}\|Ex\|_2^2dt\right]^{\frac{1}{2}}=0.262\le\left[\int_0^{35}\|u\|_2^2dt\right]^{\frac{1}{2}}=19.621$$

where $R=11$. This verifies inequality (6).

7. Conclusion

Quantitative stability conditions for singular systems with uncertainties are derived using the Lyapunov stability theory. The presented approach utilizes the possible bounds as well as the structure of the uncertain variation; the results apply for systems subject to perturbations of the unstructured or structured type. We show that the state response bound and stability are guaranteed by generalized algebraic Riccati matrix equations. A set of multilayer recurrent neural networks is also proposed to solve the Riccati equation. The nature of parallel and distributed processing renders the proposed neural networks possessing the computational advantages over the traditional sequential algorithms in solving Riccati equations.

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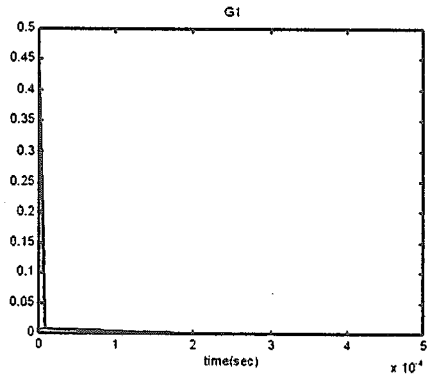

Fig. 1 Transient of G_1

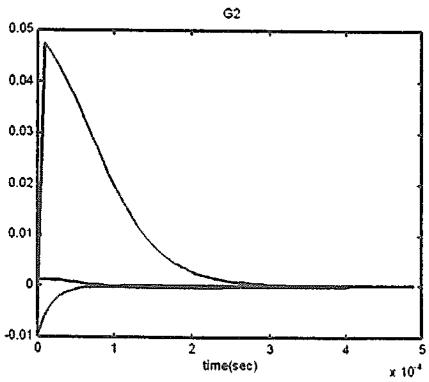

Fig. 2 Transient of G_2

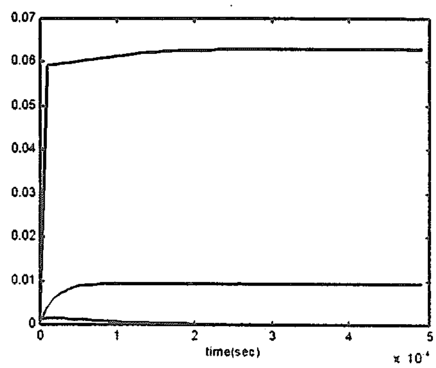

Fig. 3 Transient of P

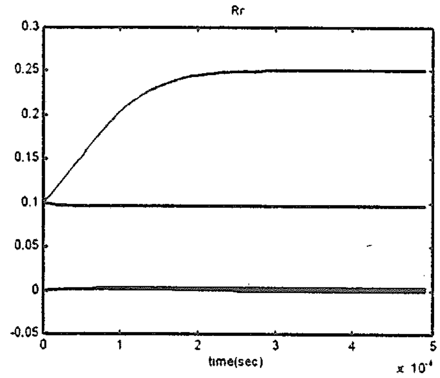

Fig. 4 Transient of R_r

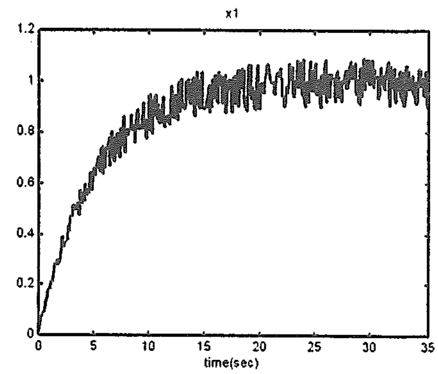

Fig. 5 Unit step input response of x_1

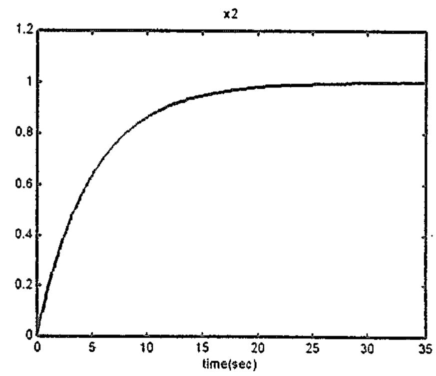

Fig. 6 Unit step input response of x_2