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Directional Property of Fractal —— Isotropic or Anisotropic

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Abstract

The fractal concept has been widely applied to analyze the image with different textural pattern. This is based on that the value of fractal dimension of an image with homogenous textural pattern should keep consistent. However several studies show identical values of fractal dimension of the images which are conceived as different textural pattern by human vision. It was found that the fractal dimension could not distinguish the texture with or without directional preference. With the question of the necessity of assuming isotropy of fractal, we examine the directional property of fractal dimension as a function of angle, F(α), of the image with 3 different textures. The results show a different pattern of F(α) with each image. In addition, this pattern is independent to the image orientation. This may suggest a new feature to represent a textural pattern.

1. Introduction

The fractal concept is a mathematical model to characterize the geometry of nature, whose irregularity and complexity are far beyond simulation by traditional geometry [7]. Basically the generation of a fractal set can be considered as the path (P) of a particle exhibiting fractional Brownian motion and which obeys a power law function in terms of

$$P(\lambda) \propto \lambda^F$$

where F is defined as a fractal dimension and λ is a scaling factor. The fractal dimension of an object can be interpreted physically as the dimension with which the object effectively fills the Euclidean space of dimension E, therefore F is fractional and smaller than E. Theoretically F is constant and independent of scale.

The Fractal concept has been applied in many investigations involving image analysis and image segmentation based on the consistency in a single parameter, the fractal dimension of an image[1, 3, 9]. Several studies [4, 6, 8] have shown that the fractal dimension of a natural object is a statistical constant rather than a fixed value, that is, the variation of the fractal dimension at different locations is confined to an acceptable level. In addition, Keller (1989) derived the fractal dimensions of 8 different texture types and found that these fractal dimensions were similar even though some of these textures showed a directional preference. However, few studies discussed the directional property of a fractal. This is due to the assumption that the development of fractal is a random procedure, and a random procedure is isotropic, therefore it is natural to assume that fractal is an isotropic object, that is, the fractal dimension is constant in any direction. Pentland (1984) has pointed out that the assumption of isotropy is a serious shortcoming of this technique of estimation of fractal dimensions, and, when the assumption is not valid, the estimates may not be truly representative of the textural pattern. This prediction has been verified in the study of Peleg et al (1984) where they found a different fractal signature at different directions in an image with directional preference. These findings highlight the limitation of the application of this single parameter to classify the natural textural pattern, since its distribution may be nonhomogenous and exhibits orientation preference.

In this pilot study, we firstly aim to examine the directional property of fractal behavior of the texture with directional preference. If the fractal behavior is anisotropic, then a metric should be developed to takes into account the directional properties of the image features.

2. Algorithm

Imaging techniques with fractal models have been applied to characterize 3-dimensional natural surfaces. This is based on the assumption that if a 3-D surface is of a spatially isotropic fractional Brownian shape, then its fractal dimension can be derived from the intensity surface of the image [10]. Because we attempt to inspect the isotropy of the fractal property beyond the single global fractal dimension, fractal dimension is derived in frequency domain. This is based on two properties of the 2-dimensional Fourier transform, which are linearity and rotation [2]. Linearity suggests that the Fourier spectrum (FS) of an image is the sum of the FS's of the individual image components. Rotation implies that the components on the FS at a particular angle α is contributed by the intensity changes in the orthogonal direction (= α +90°) on the image.

The power-law relation mentioned in previous section can be defined in terms of the rate at which the Fourier power spectrum of an image falls off with increasing spatial frequency [9]

$$|A|^2 \approx f^{-2H-1} \qquad (1)$$

therefore

$$2\log|A| \approx -(2H+1)\log f \qquad (2)$$

where $|A|$ is the amplitude of the Fourier spectrum (FS), f is spatial frequency and H is Hurst coefficient related to the slope of the logarithm relation between $|A|$ and f [7]. The fractal dimension) of a fractional Brownian surface is a function of H as follows:

$$F = 3 - H \qquad (3)$$

Based on the definition above, the fractal dimension of the image can be obtained from the slope determined by linear regression analysis on the log-log plot of the amplitudes $|A|$ as a function of frequency, f.

Therefore the two properties of the Fourier spectrum enable inspection of the fractal behavior along different directions on the image by determining the fractal dimensions of the radial components at various angle (α) in the FS as follows:

A 128×128 mask was placed randomly on the 512×512 image to select 9 locations for Fast Fourier transformation. The corresponding Fourier spectrum (FS) of each mask was obtained by equation (4):

$$|A| = |F(f,\alpha)| = \left[R^2(f,\alpha)+I^2(f,\alpha)\right]^{1/2} \qquad (4)$$

where $R(f,\alpha)$ and $I(f,\alpha)$ are the real and imaginary components of $F(f,\alpha)$, respectively.

A single regression line was fitted to the spectrum data at a specific angle α and the fractal dimension was derived from the slope of the line (using equation (2) to (3) and denoted as $F_i(\alpha)$, i indicating the sequence of the selected mask and equal to 1,2,...,9. Because of the symmetry of the power spectrum, $F_i(\alpha)$ was only calculated from 0° to 180° every 2 degrees. The mean value of 9 $F_i(\alpha)$, denoted as $F(\alpha)$ was calculated according to equation (5) and used to plot fractal dimension as a function of α.

$$F(\alpha) = \frac{1}{9}\sum_{i=1}^{9} F_i(\alpha) \qquad (5)$$

3. Results of Test Images

The procedure to obtain the $F(\alpha)$ described in section 2 was initially applied to three images with different textural patterns: (1) grass pattern in Fig. 1 shows no special directional preference, (2) the texture of brick wall in Fig. 2 is well organized in orthogonal directions, and (3) elastin network in Fig. 3 shows a mesh-like texture.

The fractal function, $F(\alpha)$, of these images have very different pattern. The $F(\alpha)$ of grass, a random fractal, tends to be a horizontal line with minimal discrete fluctuation (Fig. 4), For brick image it shows a periodic pattern with two main peaks, while for elastin network only one peak presents. This result indicates that the fractal dimension of a texture with directional preference is anisotropic.

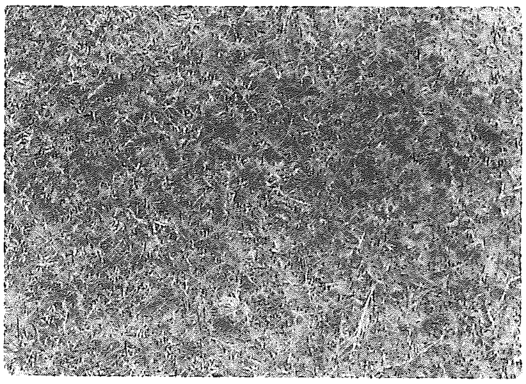

Fig 1 Grass image with a random texture pattern.

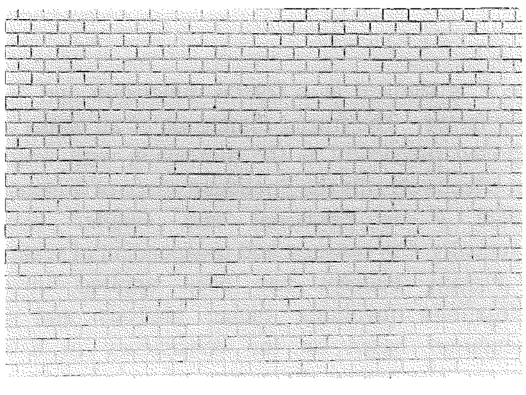

Fig 2 A brick structure with a preferentially orthogonal organization.

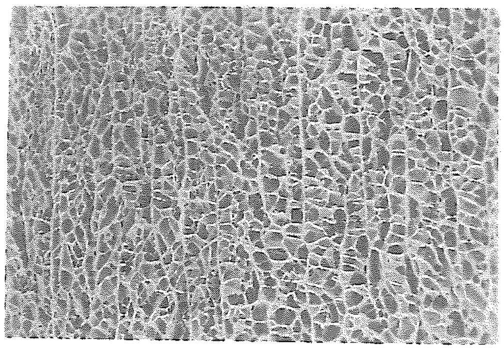

Fig 3 The pattern of elastin network shows a high directional preference.

The pattern of $F(\alpha)$ was also examined in different locations in the images. The results (Fig 7) show a consistent pattern of $F(\alpha)$ at different locations, even though the fractal dimension scatters at different locations. The periodic patterns of $F(\alpha)$ of textures is generally independent to the orientation of the image. The $F(\alpha)$ of brick corresponding to the image rotating 40° and 120° still consists of 2 dominant peaks with phase shift about 40° and 120° (Fig. 5). However a small degree of smearing of the curve of $F(\alpha)$ occurs in Fig 5 (middle) and Fig 5 (bottom). This may be induced by the quantization error of rotation of the vertical and horizontal line segments in brick image. The phase-shift of $F(\alpha)$ associated with the rotation of the image is more obvious in elastin texture (Fig. 6). The curve pattern of $F(\alpha)$ is more consistent in these images in comparison with Fig 5.

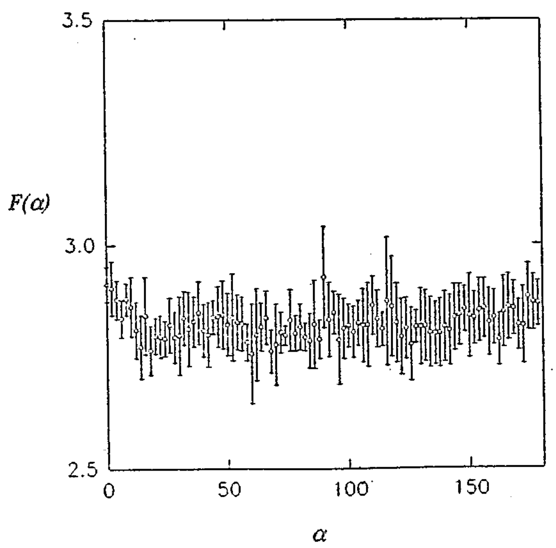

Fig 4 The fractal function $F(\alpha)$ of grass image shows minimal directional preference with no major fluctuations. The abscissa denotes an angle with an arc of 180 degrees. Each point represents the mean of $Fi(\alpha)$ form 9 masks($=F(\alpha)$) with the bar showing stardard deviation. . This applies to Fig 5 and Fig 6.

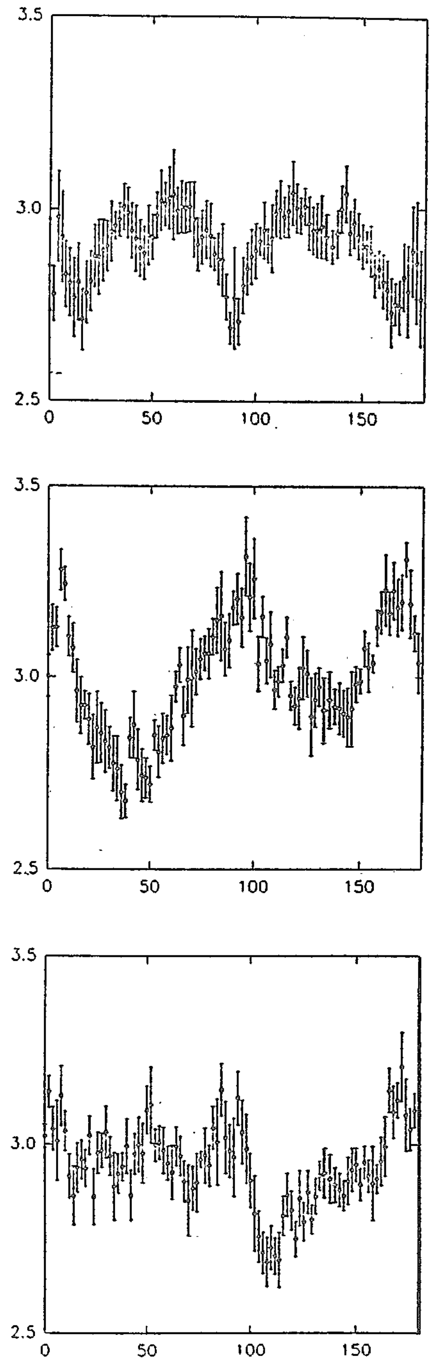

Fig 5 Top, the fractal function $F(\alpha)$ of the brick image shown in Fig 2. A first order representation by a sinusoidal wave show two

major peaks corresponding to the major orthogonal directions in the brick pattern. **Middle**, the fractal function $F(\alpha)$ corresponds to the image rotates 40° and **bottom**, 120°

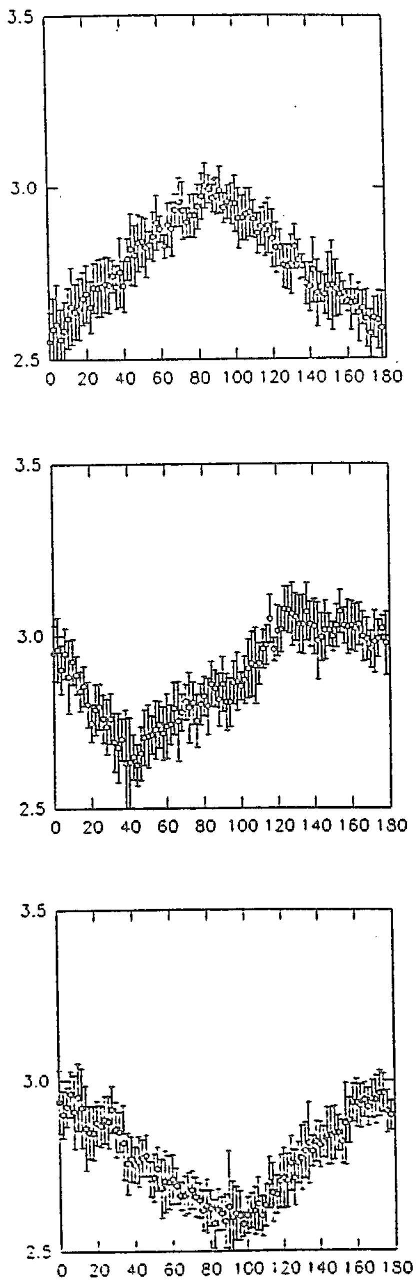

Fig 6 **Top**, the fractal function $F(\alpha)$ of the pattern of the elastin network shown in Fig 3. This shows a single peak corresponding to the major preferential direction of the elastin lamellae. **Middle**, the fractal function $F(\alpha)$ corresponds to the image rotates 40° and **bottom**, 90°

4. Discussion and Conclusion

The application of fractal feature to analyze the textural pattern is commonly based on the consistency of fractal dimension. Several investigations have proposed the limitation of using a single fractal dimension as an indicator of different textural pattern [Pentland 1984; Peleg *et al* 1984; Keller 1989; CHAN *et al*, 1995]. However little attention has been paid to the directional property of fractal behavior.

The fractal function, $F(\alpha)$ composed of a series of fractal dimensions, is developed in this present investigation to examine the fractal behavior of a natural texture within an arc of 180 degrees. It tends to be essentially invariant for a highly disorganized texture such as an image of grass, but a curve with sinusoidal fluctuations for texture organized in some directions such as images of brick and elastin network. The oscillatory pattern suggests that significant differences in fractal dimensions exist along different directions. The $F(\alpha)$ of brick shows a biphasic pattern over 180 degrees where the peaks are separated by intervals of approximately 90 degrees, corresponding to the repetition of the orthogonal directions of the brick structure. It would be expected that the major peaks should occur at intervals of exactly 90 degrees. For the images of elastin network, the $F(\alpha)$ shows essentially a monophasic pattern with marked differences in magnitude of fractal dimension over an arc of 180 degrees. The results indicate that the fractal behavior of natural texture is not necessary to be isotropic. In addition, the consistency of the $F(\alpha)$ pattern after the rotation of the image may suggest another feature to recognize the texture pattern.

It may be noticeable that the phase-shift of $F(\alpha)$ due to the rotation of image with angle α is identical to the phase-shift of the Fourier spectrum. One may infer this to the derivation of $F(\alpha)$ from Fourier spectrum. However, the physical information of $F(\alpha)$ is not the same as that of Fourier spectrum since the $F(\alpha)$ is the result of logarithmic transformation of Fourier spectrum. The relation between them in the aspect of mathematics is out of the scope in this paper but it may be worth further analysis.

This new fractal feature is successfully applied to characterize the aging process of the elastin texture in the aortic wall. [11]

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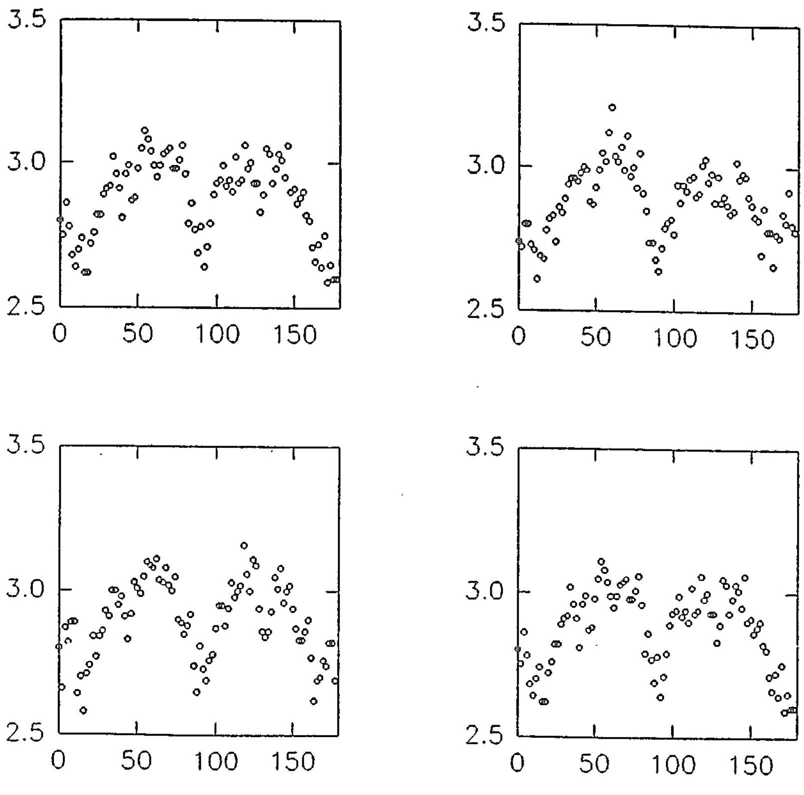

Fig 7 The fractal function $F(\alpha)$ of the brick image from 4 different locations. The consistent pattern of the curves shows that the $F(\alpha)$ is independent of location on the image.