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Histogram Matching by Moment Normalization *

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Abstract

A moment-based method for histogram matching is proposed in this paper to estimate the quasi-linear illumination change between two images which are composed of affine-transformed objects. For simplicity, the illumination change is modeled as a linear model with two unknown parameters to be estimated. Since histogram is invariant to object's translation, rotation, and scaling simultaneously, it can be utilized to estimate the illumination relation between two images. By means of moment normalization, the moments of the two histograms can be normalized, by which the desired parameters can be estimated via the procedure of normalization. It has been shown that the proposed method can yield satisfactory estimations for an adequate range of image illumination change.

Key words: *histogram, moment normalization, affine transform*

1. Introduction

A new histogram matching method is proposed in this paper. By using this method, the histogram of an input image can match that of another image (*e.g.*, a reference or desired image) for the purpose of image restoration or enhancement. Murase [1] proposed a novel approach to the illumination planning for robust object recognition. It has been shown that the illumination planner can enhance the performance of an object recognition system. However, the recognition system will fail if the illumination changes. Bidasaria proposed the histogram matching method [2] to acquire an almost exact histogram matching. Zhang [3] proposed a method to improve the accuracy of the conventional direct histogram specification method for image enhancement. However, the dynamic range of the desired histogram must be smaller than that of the original histogram. In addition to the application of the histogram matching, *e.g.*, enhancement or restoration, our proposed method can also be employed to find the gray level mapping of the input image to the desired image where some useful gray level intervals have been designated to evaluate the centroids of thresholded regions as feature points [4].

The least squares method is the most popular and simple way to estimate parameters without any probability distribution known *a priori*. Let $g(x)$, $x = 1, \cdots, N^2$, be the function to describe the gray level of an image (dimension $N \times N$) and g' be the gray level function of the image whose brightness has changed. It is assumed that the gray level modification can be modeled as

$$g'(x) = \alpha g(x) + \beta, x = 1, \cdots, N^2 \tag{1}$$

where α and β are the parameters to be estimated. Eq. (1) can be expressed as matrix multiplication.

$$\begin{pmatrix} g'(1) \cdots g'(N^2) \end{pmatrix} = \begin{pmatrix} \alpha & \beta \end{pmatrix} \begin{pmatrix} g(1) \cdots g(N^2) \\ 1 \cdots 1 \end{pmatrix} \tag{2}$$

$$A = MB \tag{3}$$

Via the evaluation of the least squares estimation of α and β, the parameter matrix M can be easily derived as

$$M = AB^T(BB^T)^{-1}. \tag{4}$$

The above equation can lead to a very accurate estimation. However, this estimation is merely suitable for those images without any scaling, rotation, and translation (SRT). If affine transform relation exists between two images, there is no longer pairwise correspondence between counterparts of $g'(x)$ and $g(x)$, *i.e.*, Eq. (1) becomes invalid.

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In order to acquire the histogram matching parameters α and β, we propose a novel approach by means of the moment normalization method [5] whose goal is to generalize the derivations of all of the existent types of moment invariants. By using the four steps of normalization, specific moment invariants can be derived constructively; besides, each step is independent of the moment invariants. The main advantage of this method is that it can yield the so-called "standard position" of an object from which any extracted features are invariant to the pre-defined group of geometric transformations for the purpose of invariant pattern recognition. However, the evaluation of the moment invariants is not our major goal, and the normalization is merely an intermediate process to obtain the desired histogram matching parameters. The details of the four steps of normalization will be described in Sec. 2.1.

The organization of the paper is as follows. Sec. 2 describes the moment normalization method and the proposed histogram matching algorithm. Sec 3. demonstrates the experimental results. Sec. 4 gives the conclusions.

2. The illumination change estimation by moment normalization of histogram

2.1. The moment normalization procedure

In order to estimate the brightness mapping between two images in which objects are related by SRT, the image histogram is utilized to estimate the parameters α and β as shown in Eq. (1) owing to the property of histogram — the shape of a histogram is capable of being invariant to translation, rotation, and scaling when background is not taken into consideration.

Assuming that the histogram of the original image is $f(\phi)$ where ϕ denotes the gray level, the modified image (or the intensity-normalized image) has the histogram $f'(\phi')$ expressed as

$$f'(\phi') = \frac{1}{\alpha}s^2 f(\phi) \tag{5}$$

where

$$\phi' = \alpha\phi + \beta. \tag{6}$$

Note that the histograms in Eq. (5) do not include backgrounds. Considering that a histogram provides the distribution of gray levels and the area under the histogram should be conserved, the contrast modification factor "α" is expected to affect $f'(\phi')$ with the term $1/\alpha$ as shown in Eq. (5). "s" is the scaling factor between two objects in these two images.

By using the moment normalization method [5], the moments of two related histograms can be normalized such that the unknown parameters can be estimated. The four important steps in [5] are now described as follows.

Step 1: *Description of the function* (*e.g.*, the histogram function $f(\phi)$). In this step, it is necessary to choose feature functions to uniquely describe the function $f(\phi)$. For example, the general moment m_p of order p can be chosen to describe $f(\phi)$ as shown in Eq. (7). It is noted that the number of the feature functions must be greater than that of the unknown parameters.

$$m_p = \int \phi^p f(\phi)d\phi \tag{7}$$

Step 2: *Transformation.* It is required to derive the relationship between the original moment m_p and the moment m'_p of the histogram f' which is the histogram of a modified image. In our case, the transformation is derived as follows.

$$\begin{aligned} m'_p &= \int \phi'^p f'(\phi')d\phi' \\ &= s^2 \int (\alpha\phi + \beta)^p \frac{1}{\alpha} f(\phi)\alpha d\phi \\ &= s^2\alpha^p \int (\phi + \frac{\beta}{\alpha})^p f(\phi)d\phi \\ &= s^2\alpha^p \sum_{i=0}^{p} \binom{p}{i} (\frac{\beta}{\alpha})^{p-i} m_i \end{aligned} \tag{8}$$

Usually, to find a transformation could be the main problem in the normalization method in case of complicated illumination change.

Step 3: *Presentation of the transformation parameters.* If there are n unknown parameters to be estimated for the histogram matching, n equations must be solved, *i.e.*, n moments have to be designated and derived. In our case, there are three parameters to be estimated: the scaling factor s of the object in the scene, the image contrast parameter α, and the illumination offset β. Letting $p = 0$, we have

$$m'_0 = s^2 m_0 = c_0. \tag{9}$$

If $c_0 = 1$, we have

$$s = \frac{1}{\sqrt{m_0}}. \tag{10}$$

m'_1 is evaluated as

$$m'_1 = s^2\alpha \sum_{i=0}^{1} \binom{1}{i} (\frac{\beta}{\alpha})^{1-i} m_i$$

$$= s^2(\beta m_0 + \alpha m_1) = c_1 \quad (11)$$

$$\beta = \frac{c_1/s^2 - \alpha m_1}{m_0}. \quad (12)$$

c_1 is set to zero and the parameter β can be evaluated as

$$\beta = \frac{-\alpha m_1}{m_0}. \quad (13)$$

Letting $p = 2$, we have

$$m'_2 = s^2\alpha^2 \sum_{i=0}^{2} \binom{2}{i} (\frac{\beta}{\alpha})^{2-i} m_i \quad (14)$$

$$m'_2 = s^2(\beta^2 m_0 + 2\alpha\beta m_1 + \alpha^2 m_2) = c_2. \quad (15)$$

Letting $c_2 = 1$ and substituting β into Eq. (15), the parameter α can be evaluated as

$$\alpha = \frac{m_0}{\sqrt{m_2 m_0 - m_1^2}}. \quad (16)$$

Step 4: *Calculation of the invariants.* By substituting the estimated parameters into Eq. (8), the new moments will become "moment invariants" which are invariant to the pre-defined transformation used in the process of derivation. However, this step is not required in our proposed method.

2.2. The histogram matching algorithm

After the four fundamental steps of normalization procedure, the intensity of an image can be normalized by the parameters α and β. Suppose the histogram f_1 of image 1 has the gray level ϕ_1, and the histogram f_2 of image 2 has the gray level ϕ_2. ϕ_1 and ϕ_2 are related by

$$\phi_2 = \hat{\alpha}\phi_1 + \hat{\beta} \quad (17)$$

where $\hat{\alpha}$ and $\hat{\beta}$ are the parameters to be estimated. ϕ_1 and ϕ_2 can be normalized to gray ϕ' by the following equations.

$$\phi' = \alpha_1\phi_1 + \beta_1 \quad (18)$$

$$\phi' = \alpha_2\phi_2 + \beta_2. \quad (19)$$

α_1 and α_2 can be evaluated by Eq. (16). β_1 and β_2 are evaluated by Eq. (13). Equating Eqs. (18) and (19), we have

$$\alpha_2\phi_2 + \beta_2 = \alpha_1\phi_1 + \beta_1 \quad (20)$$

$$\phi_2 = \frac{\alpha_1}{\alpha_2}\phi_1 + \frac{\beta_1 - \beta_2}{\alpha_2} \quad (21)$$

By inspecting Eqs. (17) and (21), the parameters are estimated as

$$\hat{\alpha} = \frac{\alpha_1}{\alpha_2} \quad (22)$$

and

$$\hat{\beta} = \frac{\beta_1 - \beta_2}{\alpha_2}. \quad (23)$$

Since the illumination change will cause some gray levels to saturate to the boundary of the histogram, *e.g.*, the gray 0 or 255. Hence, in order to obtain $\hat{\alpha}$ and $\hat{\beta}$, it is required to develop an algorithm to find the scopes of two histograms where these two scopes are mapped to each other. Let ϕ_L denote the minimum gray level and ϕ_U denote the maximum gray level, *i.e.*, $\phi_L = 0$ and $\phi_U = 255$. Heavy illumination change can make some gray levels saturate to ϕ_L or ϕ_U. Hence, the statistical values at ϕ_L and ϕ_U can not be used to evaluate the moments because these two values are distorted due to the saturation effect. Thus, it is necessary to find the appropriate boundaries on histogram f_1 and f_2 when using the histogram data for estimating the unknown parameters.

It is assumed that 1) $f_1(\Phi_1)$ and $f_2(\Phi_2)$ are the peaks of the histograms, *i.e.*, Φ_1 and Φ_2 are the gray levels of backgrounds; 2) $\mathcal{V}(\phi)$ is the vicinity of ϕ; 3) image dimension is $N \times N$; 4) $\xi(\phi)$ denotes the pixel number of the background of image 2 whose gray level is mapped to ϕ. (ϕ is the vicinity of the gray level of the background in image 1.) $\xi(\phi)$ is expressed as

$$\xi(\phi) = N^2 - (N^2 - f_1(\phi))S^2 \quad \phi \in \mathcal{V}(\Phi_1) \quad (24)$$

where $N^2 - f_1(\phi)$ denotes the area of the object in image 1, S is the scaling factor between two objects in image 1 and image 2.

The initial boundaries of useful histogram are $\phi_L + 1$ and $\phi_U - 1$. Suppose B_1 and B_2 are the lower boundaries (or the upper boundaries) of f_1 and f_2 , respectively. The following algorithm is used to find an approximate B_1 for a fixed B_2. The lower boundary of f_2 can be found by setting $\phi = \phi_L$ and the gray level increment $\lambda = 1$. The upper boundary of f_2 can be found by setting $\phi = \phi_U$ and $\lambda = -1$. The proposed "boundary mapping algorithm" is described as follows:

step 1: Set the initial value for the gray level ϕ of f_1 and the gray level increment λ.

step 2: $B_1 = \phi + \lambda$, $B_2 = \phi + \lambda$. B_2 is fixed. B_1 can be found via the following steps.

step 3: $\eta = f_2(\phi)$.

step 4: If $\eta > 0$ continue; otherwise, go to step 8.

step 5: If $\phi \in \mathcal{V}(\Phi_1)$, $\eta = \eta - \xi(\phi)$, *i.e.*, subtract the statistical values from η where $\xi(\phi)$ denotes the pixel number of the background of image 2 whose gray level is mapped to ϕ; otherwise, $\eta =$

$\eta - S^2 f_1(\phi)$ where ϕ is associated with the gray level of object in image 1.

step 6: $\phi = \phi + \lambda$.

step 7: Go to step 4.

step 8: If $\phi \neq B_1 - \lambda$, $B_1 = \phi$.

It is noted that too many saturated gray levels are expected to reduce the range of the useful histogram and result in poor estimation of parameters.

The procedure to estimate $\hat{\alpha}$ and $\hat{\beta}$ for histogram matching is elucidated as follows.

step 1: Detect the peak of f_1 at Φ_1 and the peak of f_2 at Φ_2.

step 2: Evaluate s_1 and s_2 by Eq. (10), $S = s_1/s_2$. Because the histograms of backgrounds don't have the same scaling effect as the objects, the evaluations of s_1 and s_2 cannot involve $f_i(\phi_i)$ when $\phi_i \in \mathcal{V}(\Phi_i)$, $i = 1, 2$.

step 3: Employ the boundary mapping algorithm to find the appropriate boundaries of histograms.

step 4: Replace $f_i(\phi)$, $\phi \in \mathcal{V}(\Phi_i)$ by the average of the values of the neighborhood.

step 5: Employ the moment normalization method to estimate α_1 and α_2 by Eq. (16) and estimate β_1 and β_2 by Eq. (13).

step 6: Evaluate $\hat{\alpha}$ and $\hat{\beta}$ by Eqs. (22) and (23), respectively.

If the gray level of background Φ_1 is saturated to ϕ_L or ϕ_U, the boundary mapping algorithm will result in a less accurate B_1. This is clarified as follows. For example, if ϕ_1 ($\phi_1 < \Phi_1$) in f_1 is mapped to ϕ_U in f_2, then we have

$$\eta_1 = S^2 \sum_{\phi=\phi_1}^{\phi_U} f_1(\phi) \quad (25)$$

$$\eta = f_2(\phi_U). \quad (26)$$

If $S > 1$, the object will be enlarged and the area of the background will decrease, which will lead to $f_2(\Phi_2) < f_1(\Phi_1)$. Hence, $\eta < \eta_1$ and $\hat{\phi}_1(> \phi_1)$ will be mapped to ϕ_U. If $S < 1$, the object will decrease and the area of the background will be enlarged, which will lead to $f_2(\Phi_2) > f_1(\Phi_1)$. Hence, $\eta > \eta_1$ and $\hat{\phi}_1(< \phi_1)$ will be mapped to ϕ_U. Due to this effect, the error of the boundary estimation may arise to some extent.

3. Experimental results

Fig. 1 is the reference image of tool 1. Its histogram is shown in Fig. 2. In our experiments, four images of tool 1 are tested. Tool A1 and B1 have the same geometric features as in the reference image. Tool C1 and D1 have scale=1.2 and rotation=$-135°$ relative to the reference image. Fig. 3 is the tool D1. Its histogram is shown in Fig. 4. The true parameters of tool A1 and C1 are ($\hat{\alpha}$=0.9, $\hat{\beta}$=5.0). The true parameters of tool B1 and D1 are ($\hat{\alpha}$=1.2, $\hat{\beta}$=-50.0).

Fig. 5 is the reference image of tool 2. Its histogram is shown in Fig. 6. Also, four images of tool 2 are tested. Tool A2 and B2 have the same geometric features as in the reference image. Tool C2 and D2 have scale=0.6 and rotation=90° relative to the reference image. Fig. 7 is the tool D2. Its histogram is shown in Fig. 8. The true parameters of tool A2 and C2 are ($\hat{\alpha}$=0.8, $\hat{\beta}$=10.0). The true parameters of tool B1 and D1 are ($\hat{\alpha}$=1.3, $\hat{\beta}$=-40.0).

Due to the possible extra large statistical values at gray levels 0 and 255, all histograms do not show the values at gray 0 and 255. In addition, the values at $f_i(\phi_i)$, $\Phi_i \in \mathcal{V}(\Phi_i)$, $i = 1, 2$, are replaced by the average of the values of the neighborhood on account that the statistical value of the background does not have the same scaling factor as the object. Thus, the computation of moments cannot be affected by the statistical values of backgrounds. Tables 1 and 2 show the estimation results. It is clear that the estimated parameters are accurate. The estimation results of tool B2 and D2 are involved with larger error because the common mapping areas of f_1 (the histogram of the reference image of tool 2) and f_2 (the histogram of tool B2 or D2) are less than other cases.

4. Conclusions

This paper proposes a histogram matching algorithm to estimate the illumination change between two images which contain affine-transformed objects. For simple implementation, the illumination change is modeled by a linear model. By using the moment normalization method in our algorithm, the model parameters can be easily found. On account of the saturation effect of the histograms at boundaries, the appropriate boundaries of useful histograms must be found *a priori*; hence, a boundary mapping algorithm is also proposed in this paper. From the results of the experiments , it is prominent that the estimated parameters are fairly accurate. Therefore, the illumination change detection (or the histogram matching) can be accomplished by this novel method for a reasonable variation of illumination.

References

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Table 1. The estimated parameters. The correct $(\hat{\alpha}, \hat{\beta})$ for tool A1 and C1 are (0.9, 5.0). The correct $(\hat{\alpha}, \hat{\beta})$ for tool B1 and D1 are (1.2, -50.0).

	$\hat{\alpha}$	$\hat{\beta}$
tool A1	0.91	4.69
tool B1	1.20	-51.00
tool C1	0.93	5.53
tool D1	1.23	-49.41

Table 2. The estimated parameters. The correct $(\hat{\alpha}, \hat{\beta})$ for tool A2 and C2 are (0.8, 10.0). The correct $(\hat{\alpha}, \hat{\beta})$ for tool B2 and D2 are (1.3, -40.0).

	$\hat{\alpha}$	$\hat{\beta}$
tool A2	0.80	9.43
tool B2	1.35	-47.67
tool C2	0.80	9.93
tool D2	1.34	-43.81

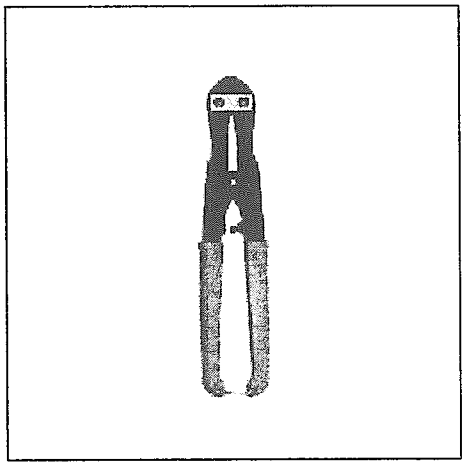

Fig. 1. Reference image of tool 1.

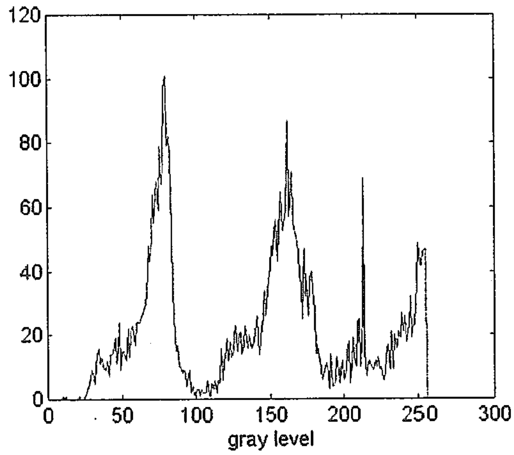

Fig. 2. Histogram of reference image of tool 1.

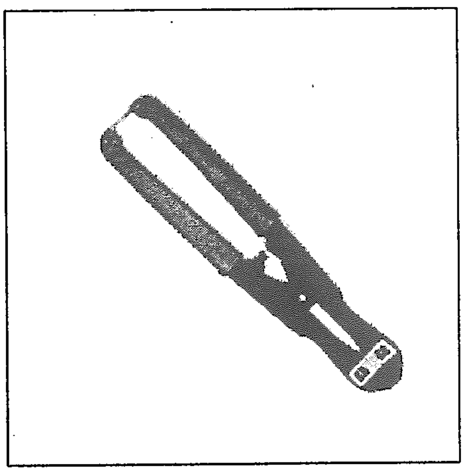

Fig. 3. Tool D1.

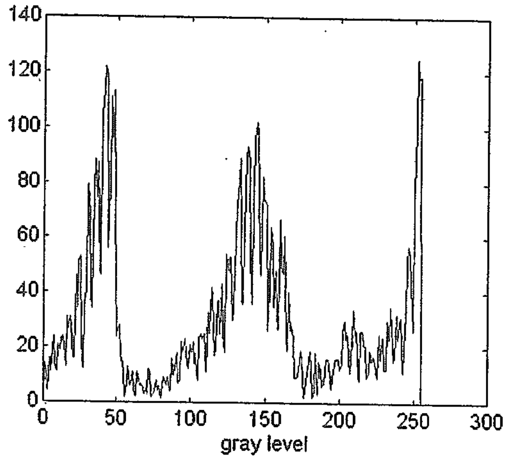

Fig. 4. Histogram of tool D1.

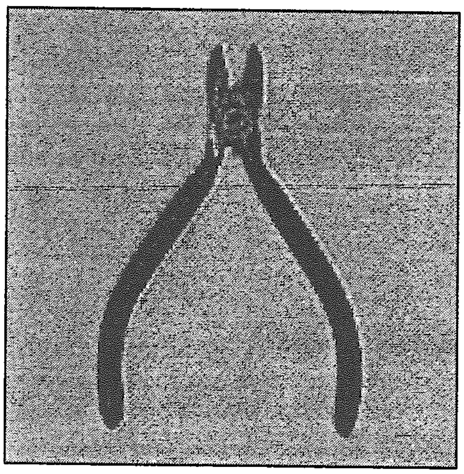

Fig. 5. Reference image of tool 2.

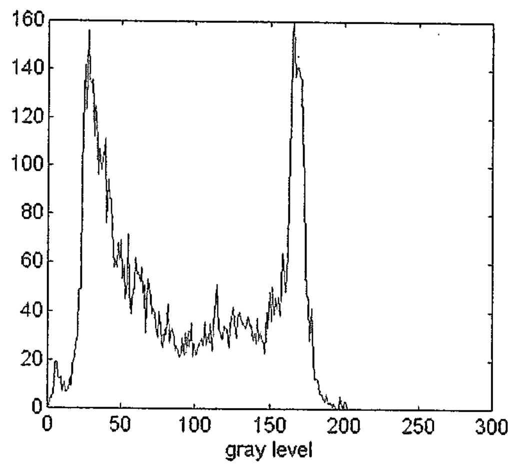

Fig. 6. Histogram of reference image of tool 2.

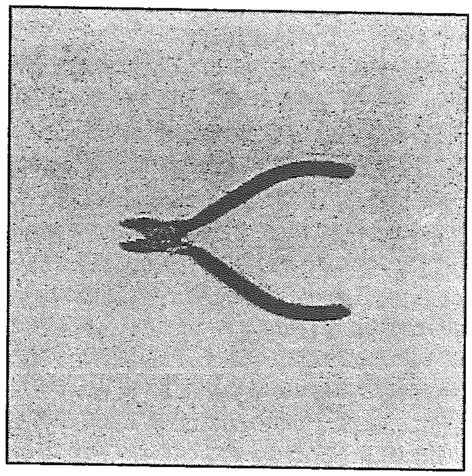

Fig. 7. Tool D2.

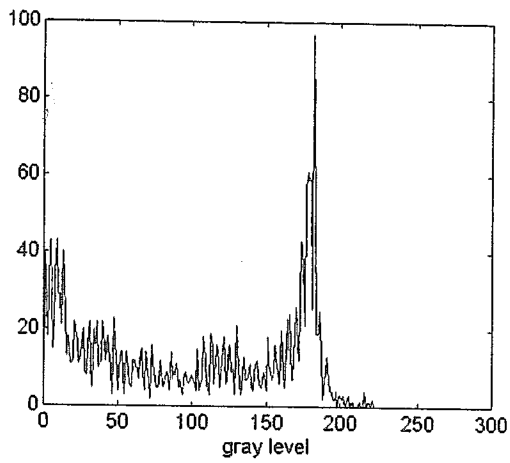

Fig. 8. Histogram of tool D2.