A Compensated Fuzzy c-Means Applied on Image Compression

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Abstract

Due to the internet being widespread, the growing rate of data in computer network is increasing each year. It forces scientists to develop efficient algorithms to compress huge data using least code to represent source data in order to avoid overflowing storage capacity, consuming transmission time, and expending broadband. In this paper, a Compensated Fuzzy c-Means (CFCM) for vector quantization in image compression is proposed. Vector quantization is one of techniques in the lossy data compression. The purpose of vector quantization is to create a codebook for which the average distortion generated by approximating a training vector and a codeword in codebook is minimized. Such a method results in massive reduction of the image information in image transmission. The CFCM, modified from penalized fuzzy c-means algorithm (PFCM), is to speed up the convergence rate for clustering procedure. From the experiment results, the CFCM algorithm shows promising clustering results in comparison with FCM and PFCM algorithms. And in the image compression application, the proposed CFCM algorithm to design codebook for vector quantization produces better PSNR than the generalized Lloyd algorithm about 2dBs.

Key words: *FCM, PFCM, CFCM, clustering algorithm, vector quantization, image compression.*

1 Introduction

Vector quantization is a popular method in image compression. A number of vector quatization algorithms in image compression have been proposed in the past [1-4]. Clustering or codebook design is an essential process in image compression based on vector quatization. The purpose of vector quatization is to create a codebook for which the average distortion generated by approximating a training vector and a codevector in codebook is minimized. The minimization of the average distortion measure is widely performed by a gradient descent based iteration algorithm that is known as Generalized Lloyd algorithm (GLA) [1]. In accordance with the cluster center in previous iteration and nearest neighbor rule, the GLA performs a positive improvement to update the codebook at every iteration.

The clustering process is also referred to as the codebook design. Given a codebook, each block of the image can be represented by the binary address of its closest codebook vector. Such a strategy results in significant reduction of the information including image transmission and memory storage. The image is reconstructed by replacing each image block by its closest codebook vector. As a result, the quality of the reconstructed image strongly depends on the codebook design.

In this paper, a codebook design approach based on the compensated fuzzy c-means for image compression is presented. The whole image is divided into n blocks (a block represents a training vector which occupies $\lambda \times \lambda$ components) and the codebook design can be regarded as a minimization of a criterion defined as a function of the least square Euclidean distance between training vector and a codevector in a codebook. The CFCM, modified from penalized fuzzy c-means algorithm, is to speed up the convergence rate for clustering procedure.

In a simulated study, the CFCM algorithm shows promising clustering results in comparison with fuzzy c-means and penalized fuzzy c-means algorithms. Furthermore the proposed CFCM algorithm is demonstrated to have the capability for vector quantization in image compression and shows the promising results in comparison with the GLA method.

2 Fuzzy Clustering Algorithms

Clustering is a process for classifying the objects or patterns in such a way that samples within a cluster are more similar to one another than samples belonging to different clusters. The work presented in this paper is a compensated fuzzy reasoning, which can be viewed as a hierarchical partition method for vector quantization in image compression. The relative fuzzy clustering techniques are reviewed as follows.

2.1 Fuzzy c-Means algorithm (FCM)

The fuzzy c-means (FCM) clustering algorithm was first introduced by Dunn [5], and the related formulation and algorithm was extended by Bezdek [6]. The purpose of the FCM approach is to minimize the criteria in the least squared error sense. For $c \geq 2$ and m any real number greater than 1, the algorithm chooses $u_i : X \rightarrow [0,1]$ so that $\sum_i u_i = 1$ and $w_j \in R^d$ for $j = 1,2,\ldots,c$ to minimize the objective function

$$J_{FCM} = \frac{1}{2} \sum_{j=1}^{c} \sum_{i=1}^{n} (u_{i,j})^m |x_i - w_j|^2 , \tag{1}$$

where $u_{i,j}$ is the value of *jth* membership grade on *ith* sample x_i. The vectors $w_1,\ldots,w_j,\ldots,w_c$, are called cluster centroids. For the purpose of minimizing the objective function, the cluster centroids and membership grades are chosen so that a high degree of membership occurs for samples close to the corresponding cluster centroids. The FCM algorithm is reviewed as follows:

Step 1: Initialize the cluster centroids $w_j (2 \leq j \leq c)$, fuzzification parameter m ($1 \leq m < \infty$), and the value $\varepsilon > 0$.

Step 2: Calculate the membership matrix $U = [u_{i,j}]$ using Eq. (2) as below.

$$u_{i,j} = \frac{\left(\frac{1}{(d_{i,j})^2} \right)^{1/(m-1)}}{\sum_{l=1}^{c} \left(\frac{1}{(d_{l,i})^2} \right)^{1/(m-1)}} , \tag{2}$$

where $d_{i,j}$ is the Euclidean distance between the training sample x_i and the class centroid w_j.

Step 3: Update the class centroids

$$w_j = \frac{1}{\sum_{i=1}^{n} (u_{i,j})^m} \sum_{i=1}^{n} (u_{i,j})^m x_i \tag{3}$$

Step 4: Compute $\Delta = \max\left(\left| U^{(t+1)} - U^{(t)} \right| \right)$. If $\Delta > \varepsilon$, then go to step 2 and set t = t+1 ; otherwise stop the process.

2.2 Penalized fuzzy c-means algorithm

Another strategy of the fuzzy clustering method, called penalized fuzy c-means (PFCM) algorithm with the addition of a penalty term, was shown by Yang[7,8] that the PFCM algorithm is more meaningful and effective than the FCM method. The PFCM objective function is reviewed as follows:

$$J_{PFCM} = \frac{1}{2}\sum_{j=1}^{c}\sum_{i=1}^{n} u_{i,j}^{m}\left|x_i - w_j\right|^2 - \frac{1}{2}v\sum_{j=1}^{c}\sum_{i=1}^{n} u_{i,j}^{m}\ln\alpha_j$$

$$= J_{FCM} - \frac{1}{2}v\sum_{j=1}^{c}\sum_{i=1}^{n} u_{i,j}^{m}\ln\alpha_j\ , \qquad (4)$$

where α_j is a proportional constant,of class j and v (≥ 0) is a constant, When $v = 0$, J_{PFCM} equals to J_{FCM}, and α_j, w_j, and $u_{i,j}$ are defined as

$$\alpha_j = \frac{\sum_{i=1}^{n} u_{i,j}^{m}}{\sum_{j=1}^{c}\sum_{i=1}^{n} u_{i,j}^{m}}; j = 1,2,\ldots,c \qquad (5)$$

$$w_j = \frac{\sum_{i=1}^{n} u_{i,j}^{m} x_i}{\sum_{i=1}^{n} u_{i,j}^{m}}, \qquad (6)$$

and

$$u_{i,j} = \left(\sum_{\lambda=1}^{c} \frac{\left(\left|x_i - w_j\right|^2 - v\ln\alpha_j\right)^{1/(m-1)}}{\left(\left|x_i - w_\lambda\right|^2 - v\ln\alpha_\lambda\right)^{1/(m-1)}} \right)^{-1} ;i = 1,2,\ldots,n; j = 1,2,\ldots,c. \qquad (7)$$

Then the PFCM algorithm is presented as follows:

Step 1: Randomly set cluster centroids $w_j(2 \leq j \leq c)$, constant v $(v > 0)$, fuzzification Parameter $m(1 \leq m \leq \infty)$, and the value $\varepsilon > 0$. Give a fuzzy c-partition $U^{(0)}$.

Step 2: Compute the $\alpha_j^{(t)}, w_j^{(t)}$ with $U^{(t-1)}$ using Eqs. (5) and (6). Calculate the membership matrix $U = [u_{i,j}]$ with $\alpha_j^{(t)}, w_j^{(t)}$ using Eq. (7).

Step 3: Compute $\Delta = \max\left(\left|U^{(t+1)} - U^{(t)}\right|\right)$. If $\Delta > \varepsilon$, then go to step 2 and set t = t+1; otherwise stop the process.

2.3 Compensated fuzzy c-means algorithm

Yang has proved the convergent characteristic of J_{PFCM} in reference [7]. But the penalty degree is too heavy to rapidly converge. The penalty term $-\frac{1}{2}v\sum_{j=1}^{c}\sum_{i=1}^{n}\mu_{i,j}^{m}\ln\alpha_j$ in PFCM is replaced by a compensated term $+\frac{1}{2}v\sum_{j=1}^{c}\sum_{i=1}^{n}\mu_{i,j}^{m}\tanh(\alpha_j)$ then the objection function and membership function in a Compensated Fuzzy C-Means (CFCM) algorithm is defined as

$$J_{CFCM} = \frac{1}{2}\sum_{j=1}^{c}\sum_{i=1}^{n}\mu_{i,j}^{m}\left| x_i - w_j \right|^2 + \frac{1}{2}v\sum_{j=1}^{c}\sum_{i=1}^{n}\mu_{i,j}^{m}\tanh\alpha_j$$

$$= J_{FCM} + \frac{1}{2}v\sum_{j=1}^{c}\sum_{i=1}^{n}\mu_{i,j}^{m}\tanh(\alpha_j) \qquad (8)$$

and

$$\mu_{i,j} = \left(\sum_{\lambda=1}^{c} \frac{\left(\left| x_i - w_j \right|^2 + v\tanh(\alpha_j)\right)^{1/(m-1)}}{\left(\left| x_i - w_\lambda \right|^2 + v\tanh(\alpha_\lambda)\right)^{1/(m-1)}} \right)^{-1} \qquad (9)$$

$$i = 1,2,\ldots,n;\ \ j = 1,2,\ldots,c$$

where α_j and v are the same definition as Eq. (4). Eq. (8) can be rewrite as

$$J_{CFCM} = J_{FCM} + \frac{1}{2}v\sum_{j=1}^{c}\sum_{i=1}^{n}\mu_{i,j}^{m}\tanh(\alpha_j)$$
$$= J_{FCM} - \frac{1}{2}v\sum_{j=1}^{c}\sum_{i=1}^{n}\mu_{i,j}^{m}\tanh(-\alpha_j) \qquad (10)$$

Due to 0< α_j <1, from Fig. 1 we can find $\tanh(-\alpha_j) \subset \ln(\alpha_j)$ which implies that J_{CFCM} can also be convergent. Then the CFCM algorithm is presented as follows:

Step 1: Randomly set cluster centroids $w_j(2 \leq j \leq c)$, constant v $(v > 0)$, fuzzification Parameter $m(1 \leq m \leq \infty)$, and the value $\varepsilon > 0$. Give a fuzzy c-partition $U^{(0)}$.

Step 2: Compute the $\alpha_j^{(t)}, w_j^{(t)}$ with $U^{(t-1)}$ using Eqs. (5) and (6). Calculate the membership matrix $U = [u_{i,j}]$ with $\alpha_j^{(t)}, w_j^{(t)}$ using Eq. (9)

Step 3: Compute $\Delta = \max\left(\left|U^{(t+1)} - U^{(t)}\right|\right)$. If $\Delta > \varepsilon$, then go to step 2 and set t = t+1; otherwise stop the process.

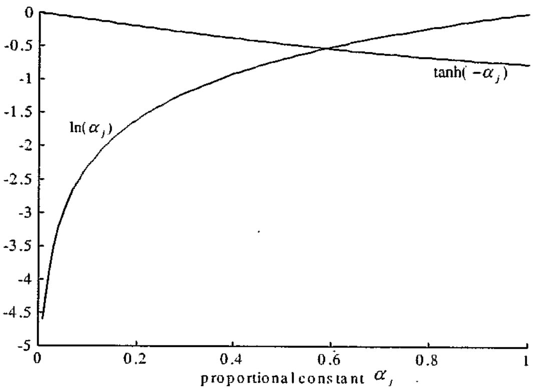

Fig 1. The curves of $\ln(\alpha_j)$ and $\tanh(-\alpha_j)$ within $0 \le \alpha_j \le 1$.

3 Vector Quantization and Compensated Fuzzy c-Means Algorithm

A vector quantizer is a technique that maps Euclidean $\lambda\times\lambda$ -dimensional space $\mathbf{R}^{\lambda\times\lambda}$ into a set $\{\mathbf{Y}_x, x=1,2,\ldots,n\}$ of points in $\mathbf{R}^{\lambda\times\lambda}$, called a codebook. A vector quantizer approximates a training vector from $\mathbf{R}^{\lambda\times\lambda}$ as little distortion as possible by one of the codevectors in the codebook. Suppose an image is divided into n blocks (vectors of pixels) and each block occupies $\lambda\times\lambda$ pixels. The performance of a system by an average distortion $E[d(\mathbf{X}_x,\mathbf{Y}_x)]$ between input sequence of training vectors $\{\mathbf{X}_x, x=1,2,\ldots,n\}$ and output sequence of codevectors is defined as

$$D_{x,y} = E[d(\mathbf{X}_x,\mathbf{Y}_x)] = \frac{1}{n}\sum_{x=1}^{n} d(\mathbf{X}_x,\mathbf{Y}_x) \tag{11}$$

The distortion measure $d(\mathbf{X}_x,\mathbf{Y}_x)$, the squared Euclidean distance between vectors, is defined as

$$d(\mathbf{X}_x,\mathbf{Y}_x) = |\mathbf{X}_x - \mathbf{Y}_x|^2 = \sum_{k=1}^{\lambda\times\lambda}(x_k - y_k)^2 \tag{12}$$

A vector quantizer is optimal if the average distortion is converged to a minimum value.

In this section, we will show that a two-dimensional image is divided into n blocks (a block represents a training vector that captures $\lambda\times\lambda$ pixels) can be clustered by a compensated fuzzy c-means algorithm. For an image divided into n training vectors and defined interesting classes c, these vectors were trained with the nearest neighbor condition and iteratively updating the membership function and class centroids same as the algorithm proposed by karayiannis [9]. Therefore, the CFCM-based vector quatizer is performed as follows:

Step 1: Randomly set cluster centroids $\mathbf{w}_j (2 \le j \le c)$, constant ν $(\nu > 0)$, fuzzification Parameter $m (1 \le m \le \infty)$, and the value $\varepsilon > 0$. Give a fuzzy c-partition $U^{(0)}$.

Step 2: Compute the $\alpha_j^{(t)}, \mathbf{w}_j^{(t)}$ with $U^{(t-1)}$ using Eqs. (5) and (6). Calculate the membership matrix $U = [u_{i,j}]$ with $\alpha_j^{(t)}, \mathbf{w}_j^{(t)}$ using Eq. (14).

Where

$$\mathbf{w}_j = \frac{\sum_{i=1}^{n} u_{i,j}^{\,m}\mathbf{x}_i}{\sum_{i=1}^{n} u_{i,j}^{\,m}} \tag{13}$$

$$u_{i,j} = \left(\sum_{i=1}^{c} \left(\frac{|\mathbf{x}_i - \mathbf{w}_j|^2 + \nu\tanh(\alpha_j)}{|\mathbf{x}_i - \mathbf{w}_\lambda|^2 + \nu\tanh(\alpha_\lambda)} \right)^{1/(m-1)} \right)^{-1} \tag{14}$$

$= 1,2,\ldots,n;\ j = 1,2,\ldots,c.$

Step 3: Compute $\Delta = \max\left(\left|U^{(t+1)} - U^{(t)}\right|\right)$. If $\Delta > \varepsilon$, then go to step 2 and set t = t+1; otherwise go to step 4.

Step 4: Update the codevectors using the centroids of c classes.

4 Experimental Results

To see the convergence rate of the FCM, PFCM, and the proposed algorithm CFCM for clustering procedure, the butterfly example [10,11] was used to classify into 2 classes for comparing the performance of different algorithms. The energy functions converge to a local minimum within 15 iterations averagely. The convergence results are shown in Fig. 2 and 3 with m=1.25, and 2.0 respectively. From the simulated study, the proposed approach is the fastest one in convergence rate with average iterations.

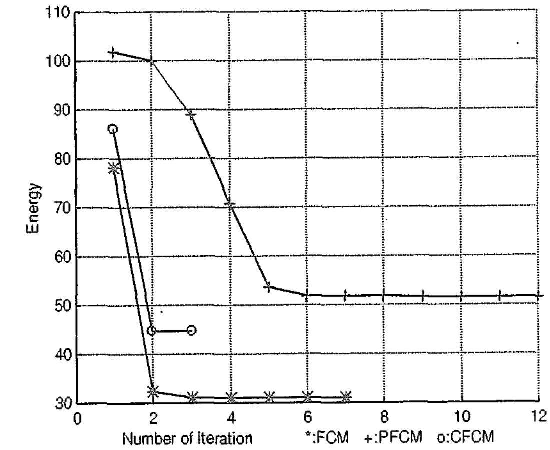

Fig. 2 The convergent curve with m=1.25 and c=2 in different algorithms.

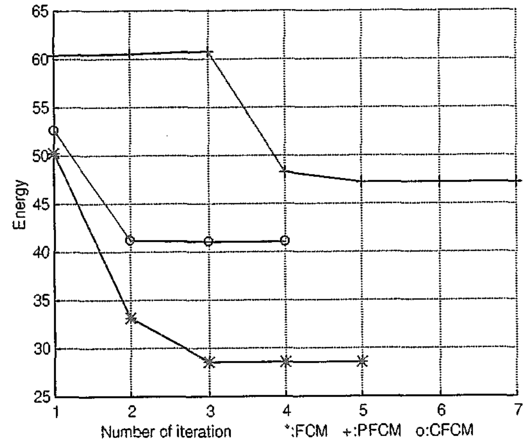

Fig.3 The convergent curve with m=2.0 and c=2 in different algorithms.

The codebook design is the primary problem in image compression based on vector quantization. In this paper, the quality of the image reconstructed from the designed codebooks was compared using the CFCM and GLA algorithms involving real image data. The training vectors were extracted from 256×256 real images, which were divided into 4×4 blocks to generate 4096 nonoverlapping 16-dimension vectors. Three codebooks of size 64, 128, and 256 were built using this training data. In this experiment the compression rate were 6/16=0.375, 7/16=0.438, and 8/16=0.5 bits per pixel respectively. The test images is 256×256 pixels with 8-bit gray levels, and the resulting images were evaluated subjectively by the

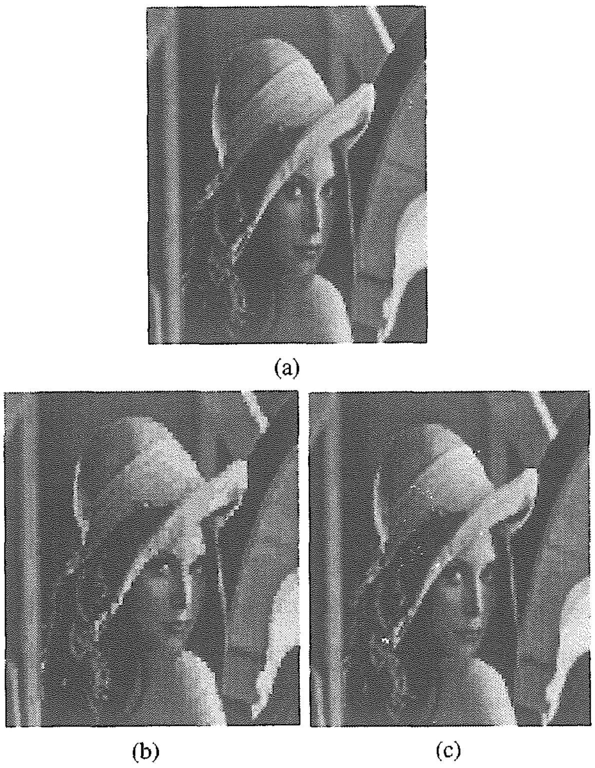

Fig. 4 Reconstructed images with codevectors c = 256; (a) original; (b) GLA; (c) CFCM.

peak signal to noise ratio (PSNR) that is defined for images of size N×N as

$$PSNR = 10\log_{10}\frac{255\times255}{\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(x_{i,j}-\hat{x}_{i,j}\right)^2} \tag{15}$$

where $x_{i,j}$ and $\hat{x}_{i,j}$ are the pixel gray levels from the original and reconstructed images and 255 is the peak gray level, respectively. Figure 4 shows the original "Lena" image with their reconstructed images from the codebook of size c=256 designed by the GLA algorithm and the proposed CFCM algorithm, respectively. Table 1 shows the PSNR of the various images reconstructed from three codebooks of size c=64, 128, and 256 designed in these experiments. From experimental results, the reconstructed images obtained from the CFCM are better than those obtained from the GLA algorithm, and in accordance with Table 1, the proposed CFCM algorithm produces better PSNR than those designed by the GLA algorithm about 2 dBs.

5 Discussion and Conclusions

This paper proposed a compensated fuzzy c-means algorithm to design better codebook for vector quantization in image compression. From a simulated study, the proposed approach is the fastest one in convergence rate with average iterations in comparison with FCM and PFCM algorithms for the butterfly example. And in the image compression application, It has been also demonstrated that the reconstructed images obtained from the CFCM algorithm produce better PSNR than those obtained from the GLA algorithm about 2 dBs in experimental results. Due to fuzzy reasoning converging to a local optimum solution, the availability to incorporate genetic algorithm to generate near optimum solution will be our research in the future.

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Table 1 PSNR of the images reconstructed from codebooks of various size designed by the GLA and proposed CFCM.

Image/Algorithm \ Codebook Size		64	128	256
Lena	GLA	25.256	26.374	27.064
	CFCM	26.590	27.795	29.128
Boygirl	GLA	28.264	29.403	31.198
	CFCM	30.206	31.593	33.103
Girl	GLA	27.683	28.512	29.691
	CFCM	29.434	30.523	31.711
Pepper	GLA	24.821	25.593	26.779
	CFCM	26.171	27.582	29.250
F16	GLA	24.105	25.291	26.335
	CFCM	25.640	26.820	28.353