

## 二步搜尋的多階區塊截取編碼法

陳文儉

國立成功大學電機系  
中州工商專科學校電子科  
cwjel@dragon.ccc.edu.tw

戴顯權

國立成功大學電機系  
台南市701大學路1號  
sctai@mail.ncku.edu.tw

### 摘要

本文提出一個新的多階區塊截取編碼法，最佳的量化值可由全搜尋獲得，但是全搜尋相當耗時間且不具實用性。一個二步搜尋的方法用來降低計算複雜度；第一步，以臨界初值為中心，與左右二側固定距離的另外二點，共三點來搜尋，以最小絕對誤差的點為新的中心。相同的方式，可以在第二步搜尋找到較佳的結果。比較發現，本方法可得到相當接近全搜尋的最佳值，而計算量可大大減少。

關鍵詞：區塊截取編碼法，多階區塊截取編碼法，二步搜尋，臨界值，影像編碼。

### 1 Introduction

Block truncation coding (BTC) [1] is a simple and effective coding technique. In the original form, the image is first divided into a series of nonoverlapping blocks of pixels, and a two-level quantizer is independently designed for each block. Both the quantizer threshold and the two reconstruction levels are varied in response to the local statistics of a block.

A more efficient algorithm, the absolute moment BTC (AMBTC) [2] has been extensively used in signal compression because of its simple computation and better mean squared error (MSE) performance. Several different BTC approaches have been proposed during the past 15 years. [3]

BTC has the advantages of preserving edges and having low computational complexity. However, due to limited quantization levels, staircase artifact and ragged edge will appear. Thus it is necessary to develop multilevel BTC to improve the quality of the image. For the original two-level BTC, it is easy to search all the possible threshold candidates exhaustively to obtain the optimal results. However, it requires an enormous amount of computation and is thus impractical while we take an exhaustive search for the multilevel BTC. Recently, many multilevel BTC schemes have been developed including three-level BTC [4,5,6] and four-level BTC [7,8,9]. Ronson and DeWitte [4] exploit the output of 2-level AMBTC to generate the two thresholds for 3-level BTC. N. Efrati et al [5] employed a simple single iteration two-level and three-level BTC coding. They got the new threshold from the two outputs of the first pass. Therefore, the results of this method are not closed to the optimal. The other improved scheme spent a great cost in computation to increase the image quality. [6] L. Hui [7] proposed a Minimum MSE quantizer to design multilevel

BTC. He developed an iterative algorithm to determine the optimal threshold based on MMSE criteria. Wu and Coll [8] have given a simple non-uniform n-level quantizer, where each of the quantization levels is iteratively determined aiming at minimal MAE. However, it does not improve image quality. Kuo et al [9] described an iterative technique, in which they segmented a block into n regions to find n codewords based on the reduction in mean absolute errors (MAE) at each iteration. The performance of their algorithm is better than other multilevel BTC algorithms that have been proposed. This paper proposes a new multilevel BTC that has better both MSE and MAE performance. This improvement is achieved by exploiting light weighted searching to search the better thresholds in the block.

The remainder of this paper is organized as follows. Section 2 gives a brief description of block truncation coding. Section 3 describes related works on multilevel BTC. The new searching multilevel BTC scheme is proposed in Sec. 4. Simulation results are given in Sec. 5, and conclusions are drawn in Sec. 6.

### 2 Block Truncation Coding

The image compression technique Block truncation coding (BTC) was proposed by Edward J. Delp and O. Robert Mitchell on 1979. It uses a two-level nonparametric quantizer that adapts to local properties of the image. The quantizer that shows great promise is one that preserves the local sample moments. The main advantages of BTC are less data storage and low computation.

In Block Truncation Coding, an image is divided into  $n \times n$  (typically,  $4 \times 4$ ) nonoverlapping blocks of pixels, and each block needs a two-level quantizer independently. Both the quantizer threshold and the two reconstruction levels are varied in response to the local statistics of a block. Therefore, encoding is the generating of the two reconstruction levels and a block consists of an  $n \times n$  bit map indicating the reconstruction level associated with each pixel for one image. Decoding is a process that reconstructs the image by placing the corresponding appropriate reconstruction value at the pixel location as the bit map representation. Fig. 1 shows the diagram of the basic BTC scheme.

Many reported BTC schemes make use of moment-preserving quantizers. These quantizers preserve a limited number of moments of a block, mean and variance. Let the image block be of  $m = n \times n$ , and  $X_1, X_2, \dots, X_m$  be the pixel values in the original picture block. Then the first and second sample moments and the

sample variance are

$$\bar{X} = \frac{1}{m} \sum_{i=1}^m X_i \quad (1)$$

$$\overline{X^2} = \frac{1}{m} \sum_{i=1}^m X_i^2 \quad (2)$$

$$\overline{\sigma^2} = \overline{X^2} - \bar{X}^2 \quad (3)$$

As a one-bit quantizer, there is a threshold,  $X_{th}$ , and two output levels,  $a$  and  $b$ , such that

$$\begin{cases} \text{if } X_i \geq X_{th} \text{ output} = b \\ \text{if } X_i < X_{th} \text{ output} = a \\ \text{for } i = 1, 2, \dots, m \end{cases} \quad (4)$$

Let  $q$  be the number of  $X_i$ 's that have values greater than  $X_{th}$  ( $= \bar{X}$ ). Then, to preserve  $\bar{X}$  and  $\overline{X^2}$ , we have

$$\begin{cases} m\bar{X} = (m-q)a + qb \\ m\overline{X^2} = (m-q)a^2 + qb^2 \end{cases} \quad (5)$$

Solving for  $a$  and  $b$ , we have

$$\begin{aligned} a &= \bar{X} - \bar{\sigma} \sqrt{\frac{q}{m-q}} \\ b &= \bar{X} + \bar{\sigma} \sqrt{\frac{m-q}{q}} \end{aligned} \quad (6)$$

The disadvantage of the quantizer is that it needs the squaring and square root operations. An approach with simple operation is absolute moment block truncation coding (AMBTC), in which the quantizer is designed to preserve the absolute moments. The absolute moment block truncation coding preserves the mean and the first absolute central moment of a  $n \times n$  ( $m = n \times n$ ) block.

The first absolute central moment is defined as

$$\alpha = \frac{1}{m} \sum_{i=1}^m |X_i - \bar{X}| \quad (7)$$

The reconstruction levels that preserve  $\bar{X}$  and  $\alpha$  are therefore

$$a = \bar{X} - \frac{m\alpha}{2(m-q)} \quad (8)$$

$$b = \bar{X} + \frac{m\alpha}{2q} \quad (9)$$

### 3 Related Works on Multilevel BTC

The AMBTC still is not the optimal two-level BTC quantizer. Let a block with  $n \times n$  be sorted into a sequence  $X$  that has  $m=n^2$  pixels. That is

$$X = \{x_0, x_1, \dots, x_{m-2}, x_{m-1}\}, \quad (10)$$

where  $x_0 \leq x_1 \leq \dots \leq x_{m-2} \leq x_{m-1}$ . Therefore, MSE optimal quantization can be obtained by selecting the quantization threshold so that it minimizes

$$MSE = \sum_{i=1}^{m-q-1} (x_i - a)^2 + \sum_{i=m-q}^m (x_i - b)^2 \quad (11)$$

An obvious method to find the right threshold, is an exhaustive search among the  $m$  candidate pixel values of a block. [3]. In the similar way, we can get the MAE optimal quantization by an exhaustive search among the  $m$  candidate to minimize:

$$MAE = \sum_{i=1}^{m-q-1} |x_i - a| + \sum_{i=m-q}^m |x_i - b| \quad (12)$$

However, it requires an enormous amount of computation and is thus impractical while we take an exhaustive search for the multilevel BTC. For example, the three-level BTC requires  $\sum_{i=1}^m i$  search points that each search point needs

to compute the Equation (11) or (12) for the MSE or MAE optimal quantization. The time complexity is  $O(m^3)$ .

Therefore, the 4-level optimal BTC needs an enormous amount of computation  $O(m^3)$  and is thus impractical.

However, a practical implementation limits the number of levels to four.

Ronson and DeWitte [4] exploit the output of 2-level AMBTC to generate the two thresholds for 3-level BTC. To start with, each block is coded with 2-level AMBTC algorithm. Then the two thresholds are generated by:

$$t_1 = (3b_2 + a_2)/4, \quad (13)$$

$$t_2 = (3a_2 + b_2)/4. \quad (14)$$

where  $a_2$  and  $b_2$  are the two outputs of 2-level AMBTC. The final outputs are  $a_3$ ,  $(a_3+b_3)/2$  and  $b_3$ , where

$$a_3 = \text{mean of pixels} \leq t_2, \quad (15)$$

$$b_3 = \text{mean of pixels} > t_1. \quad (16)$$

Only  $a_3$ ,  $b_3$  and the bitmap will be transmitted. The advantage of 3-level AMBTC is that the extra level does not send to the receiver. N. Efrati et al [5] employed a simple single iteration to improve the quality of three-level BTC coding. They got the new thresholds by repeating the encoding processing using  $(a_3+3b_3)/4$  for  $t_1$  and  $(3a_3+b_3)/4$  for  $t_2$ .

In 4-level BTC, Wu and Coll [8] have given a non-uniform  $n$ -level quantizer, which minimizes the maximum quantization error in each block. They have much better MSE performance than the Max quantizers because they are adaptive to the local statistics. Recently, Kuo and Chen [9] present a nearly optimum multilevel BTC with an iterative technique. Their method segments a block into  $n$  regions to find  $n$  codewords based on the reduction in MAE at each iteration. Their algorithm has the best results

among the other methods proposed in [5,6,7,8].

We have reviewed several related works that are expected to find the minimum MSE or MAE multilevel BTC quantizer. In the next section, the two -steps search for generate the nearly optimum multilevel BTC is proposed.

#### 4 A New Searching Multilevel BTC

By studying image statistics, many pixel distributions are found biased in some block. The AMBTC still is not the optimal two-level BTC quantizer. We assume that the optimal threshold(s) will also be biased, but lie around the mean of the block. We propose a two -steps searching method to achieve nearly optimal solution. In the step one, the three positions that contain the initial threshold(s) and the both sides of the initial threshold with a fixed length are searched. We can choose the position, which get minimum MAE (or MSE) from three positions to be a new center. Then, we can the final threshold(s) by the similar way as the step one. The initial threshold(s) set on the mean of the block for 2 -level BTC, the  $t_1$  and  $t_2$  as described in Equation (13), (14) for 3-level BTC, and the means of lower segment, the block, and the higher segment for 4-level BTC as shown in Fig. 1. The distance of the both sides and the center point is two for 2 -level BTC but one for 3-level or 4-level BTC. The algorithm is given as follows.

- (1). Sort the data of the block.
- (2). Calculate the initial threshold(s) of the block as a center searching point.
- (3). Compute the MAE between the original block and the reconstructed block by the candidate thresholds.
- (4). Find the minimal MAE threshold(s) as the center of the second searching point.
- (5). Repeat (3), and get the best threshold(s).

It needs only 3+2 searching points for 2-level BTC, 9+8 searching points for 3-level BTC and 27+26 searching points for 4-level BTC. By comparing to the exhaustive search, the proposed scheme can obtain nearly the same results with only much less computation.

#### 5 Simulation Results

The four tested 512x512 images including Lena, and Peppers, Airplane and Baboon are used to simulate the proposed multilevel BTC algorithm as shown in Fig.2. The block size for BTC is 4 x 4. Table 1 listed the results by the exhaustive search based on the minimum MAE. As a measure of reconstructed image quality, the PSNR (peak signal-to noise ratio) is also listed in Table 1, defined as follows

$$PSNR = 10 \log_{10} \left[ \frac{255^2}{MSE} \right] \quad (17)$$

where 255 is the maximum intensity and MSE is the mean

square error between the original image and the reconstructed image. It needs only 5/16 searching points (s. p.) for the proposed two -steps scheme in 2-level BTC, 17/256 in 3-level BTC, and 53/4096 in 4-level BTC. Therefore, the results of the proposed scheme are very close to the optimal quantization. The MAE difference between the optimal result and the proposed scheme is only 0.09 for 2-level, 0.32 for 3-level, and 0.16 for 4-level in average.

#### 6 Conclusions

In this paper, we presented a two -steps searching method to achieve nearly optimal solution for multilevel BTC with less computational complexity. The performance of the proposed algorithm is significantly better than the other related works. The results of our proposed method do not achieve the global optimal solution yet. We will further study other schemes including the genetic algorithm, neural network to the design of multilevel quantizer.

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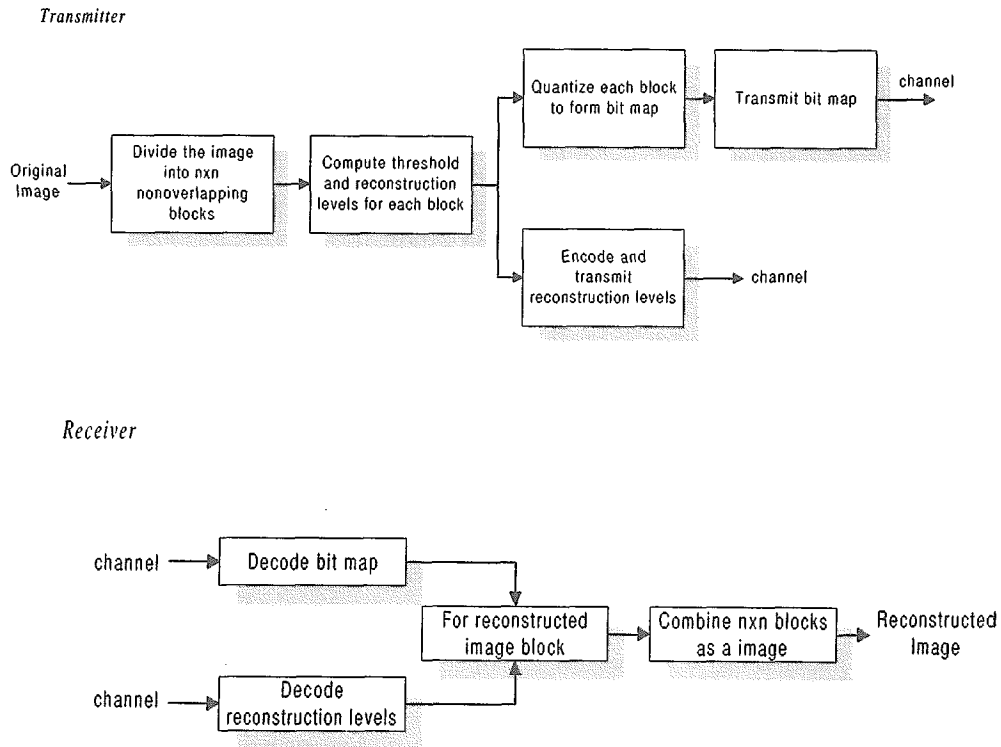
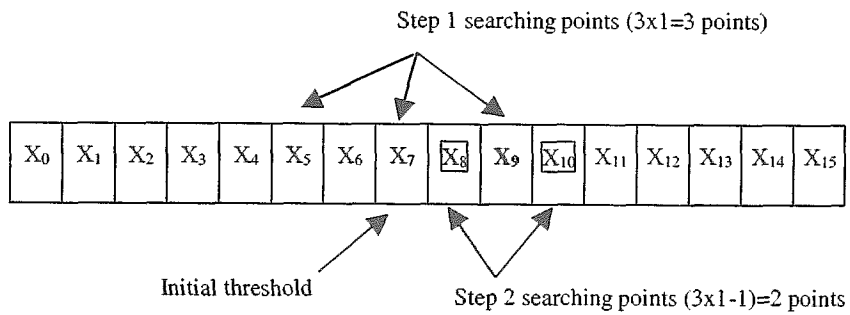
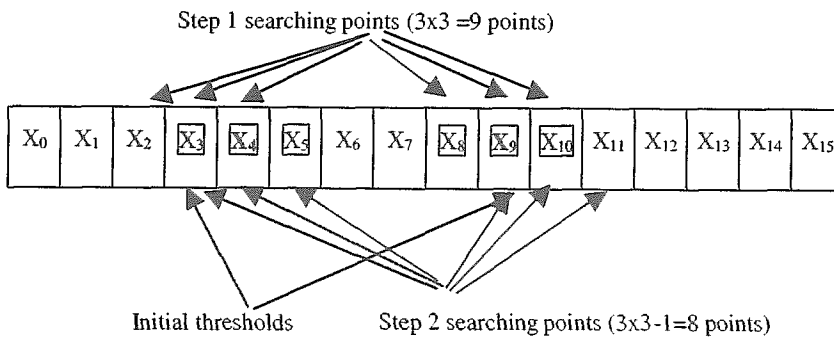


Figure 1 Block diagram of BTC

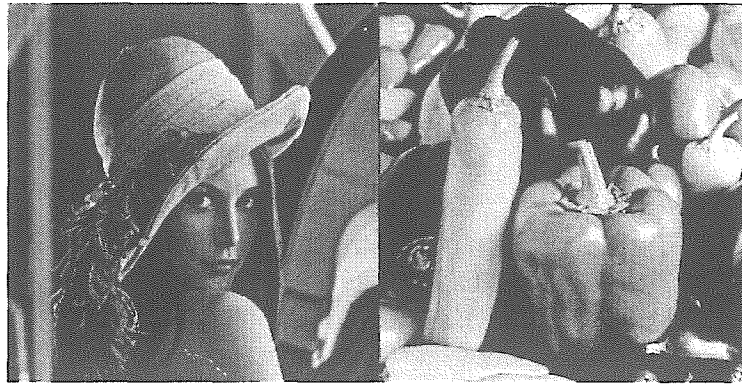


(a) 2-level BTC



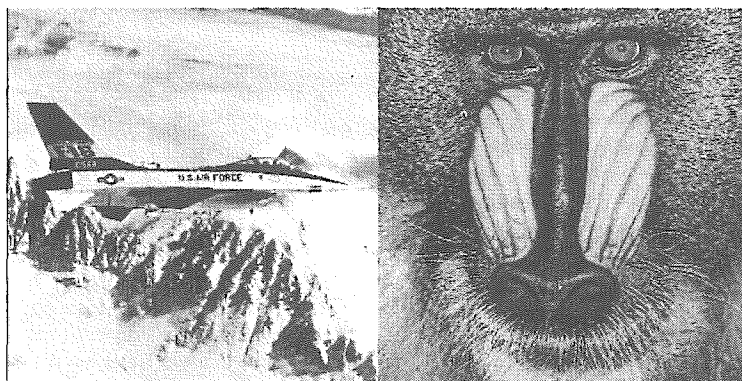
(b) 3-level BTC

Figure 2 Examples of two-steps searching method (a) 2-level BTC (b) 3-level BTC



(a)

(b)



(c)

(d)

Figure 3 Test images (a) lean, (b) pepper, (c) airplane, and (d) baboo.

Table 1. Optimal MAE results by exhaustive search with 4x4 block size.

	2-level				3-level				4-level			
	MAE	MSE	PSNR	S. P.	MAE	MSE	PSNR	S. P.	MAE	MSE	PSNR	S. P.
Lena	3.49	37.27	32.42	16	2.12	14.31	36.57	256	1.41	6.77	39.82	4096
Pepper	3.47	41.02	32.00	16	2.08	15.17	36.32	256	1.38	7.10	39.62	4096
Airplane	3.29	44.53	31.65	16	1.91	15.69	36.18	256	1.25	7.27	39.51	4096
Baboo	8.65	154.59	26.24	16	5.27	60.49	30.31	256	3.56	28.93	33.52	4096

Table 2. The results of the proposed two steps searching method with 4x4 block size.

	2-level				3-level				4-level			
	MAE	MSE	PSNR	S. P.	MAE	MSE	PSNR	S. P.	MAE	MSE	PSNR	S. P.
Lena	3.54	37.90	32.34	5	2.32	16.35	36.00	17	1.52	7.87	39.17	53
Pepper	3.53	42.15	31.88	5	2.39	17.72	35.65	17	1.50	8.53	38.82	53
Airplane	3.38	47.09	31.40	5	2.13	19.08	35.32	17	1.37	9.17	38.51	53
Baboo	8.79	158.54	26.13	5	5.82	71.70	29.58	17	3.86	34.47	32.76	53