On the cycle embedding of pancake graphs *

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abstract

The ring structure is important for distributed computing, and it is useful to construct a hamiltonian cycle or rings of various length in the network. Kanevsky and Feng [3] proved that all cycles of length l where $6 \leq l \leq n!-2$ or $l = n!$ can be embedded in the pancake graphs G_n. Later, Senoussi and Lavault [9] presented the embedding of ring of length l, $3 \leq l \leq n!$, with dilation 2 in the pancake graphs G_n. These results prompt us to explore the possibility of embedding a cycle of length $n! - 1$ into G_n, and to establish some topological properties of the pancake graphs. In this paper, we prove that there exists a hamiltonian path joining any two nodes of the pancake graph G_n. And we show that the pancake graph still has a hamiltonian cycle in the presence of one faulty node. As a consequence, a cycle of length $n! - 1$ can be embedded in G_n. And we expand Kanevsky and Feng's result as follows: A cycle of length l can be embedded in the pancake graph G_n, $n \geq 4$, if and only if $6 \leq l \leq n!$.

Keywords: *pancake graph, star graph, fault tolerant, hamiltonian, hamiltonian connected.*

1. Introduction

Since there are a rapid growing need for large scale computation and an ever increasing density of low cost VLSI circuit, a number of architectures have been studied. Most of the well accepted parallel topologies stem from Cayley graphs. Because these topologies can be recursively decomposed, they provide a simple way for the application of recursive algorithms.

Among hierarchical Cayley graphs, other than the binary hypercube, both the star and pancake interconnection networks are attractive alternatives to the hypercube in several aspects [1, 2]. For example, both n-star and n-pancake interconnection networks has $n!$ nodes, and both their degree and diameter are $O(n)$, that is, sublogarithmic in the number of nodes, while a hypercube with $n!$ nodes has degree and diameter of $O(\log n!) = O(n \log n)$, i.e., logarithmic in the number of nodes [7].

The n-dimensional pancake network, denoted by G_n, has several attractive properties. It is vertex symmetric, which implies that the congestion problems for transmission are minimized since the load will be distributed uniformly through all the vertices. Moreover, the pancake network has a very simple routing algorithm because it is built using algebraic groups (Cayley groups). Other attractive properties include that the pancake graphs are strongly hierarchical, maximally fault tolerant, hamiltonian, have a small diameter which is smaller than hypercubes [1, 3, 5, 6, 8].

The ring structure is important for distributed computing, it allows communication with low cost because the number of edges of the ring is low, it is free of branching, and it is often used in local area networks, for example, Token Ring [10]. Hence it is useful to construct a hamiltonian cycle or ring structure in the network. In [3], Kanevsky and Feng proved that all cycles of length l where $6 \leq l \leq n! - 2$ or $l = n!$ can be embedded in the pancake graphs G_n. In [9], Senoussi and Lavault presented the embedding of ring of length l, $3 \leq l \leq n!$, with dilation 2 into the pancake graph G_n. These results prompt us to explore the possibility of embedding a cycle of length $n! - 1$ into G_n. For example, we can find a cycle of length $4! - 1$ in G_4, as shown in Fig 4.

In this paper, we study some intriguing topological properties of the pancake networks G_n. First, we prove that there exists a hamiltonian path between any two nodes of the pancake networks. Based on the existence of hamiltonian paths between every pair of nodes, we then show that there exists a hamiltonian cycle in the pancake networks with the occurring of one faulty node. As a consequence, a cycle of length $n! - 1$ can be embedded into G_n for any $n \geq 4$. We then expand Kanevsky and Feng's result as follows: A cycle of length l can be embedded in the pancake graphs G_n, $n \geq 4$, if and only if $6 \leq l \leq n!$.

The paper is organized as follows. In Section 2, we describe the definitions and terminologies used in this paper. Section 3 is devoted to the Hamiltonian properties and the embedding of rings in the pancake networks. Section 4 summarizes the result of this paper.

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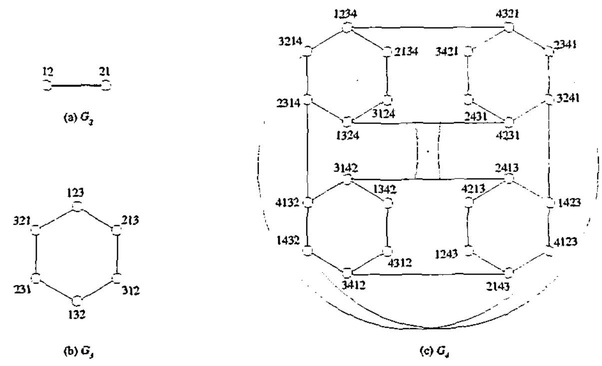


Fig 1. Examples of pancake graphs.

2. Definitions and preliminaries

An interconnection network is usually represented by a graph. Most of the graph definitions used in this paper are standard (see [4]). Let $G = (V, E)$ be a graph where V denotes the vertex/node set and E denotes the edge set of G. A cycle that traverses every vertex of the graph G exactly once is called a *hamiltonian cycle.* A graph G is *hamiltonian* if it contains a hamiltonian cycle. A *hamiltonian path* in graph G is a path that visits every vertex exactly once. A graph G is *hamiltonian connected* if every two vertices of G are connected by a hamiltonian path. A graph G is called *1-node fault-tolerant hamiltonian,* or simply *1-node hamiltonian,* if it remains hamiltonian after removing any single node.

Let $\langle n\rangle = \{1, 2, \ldots, n\}$, $p = (p_1p_2\ldots p_n)$ be a permutation such that $p_i \in \langle n\rangle$ and $p_i \neq p_j$ for $i \neq j$. An n-dimensional pancake graph $G_n = (P_n, E_n)$ of dimension n is defined as follows: $P_n = \{(p_1p_2\ldots p_n) \mid p_i \in \langle n\rangle,\ p_i \neq p_j \text{ for } i \neq j\}$ and $E_n = \{((p_1p_2\ldots p_jp_{j+1}\ldots p_n), (p_jp_{j-1}\ldots p_2p_1p_{j+1}\ldots p_n)) \mid (p_1p_2\ldots p_n) \in P_n \text{ and } 2 \leq j \leq n\}$. In other words, the set of P_n of all permutations form the vertices of G_n. Two nodes u and v are adjacent if and only if the permutation corresponding to node v can be obtained from that of u by flipping the objects in positions 1 through j. For each permutation, we can flip any number of objects from 1st to jth positions with $2 \leq j \leq n$, thus G_n is regular with degree $n-1$, $|P_n| = n!$, and $|E_n| = n!(n-1)/2$. Examples of G_n, for $2 \leq n \leq 4$, are given in Fig. 1.

Let $p = (p_1p_2\ldots p_n)$ be any permutation in P_n. We define *Head*(p) to be p_1, which is the object of the leftmost position; and define *Tail*(p) to be p_n, which is the object of the rightmost position. Moreover, we define $Flip_i(p)$ to be $(p_ip_{i-1}\ldots p_1p_{i+1}p_{i+2}\ldots p_n)$, which *is obtained by flipping the objects of p between positions* 1 through i for $2 \leq i \leq n$. Let $P_n[k]$ denote the set of all permutations p with $Tail(p) = k$. And let $G_n[k]$ be the subgraph induced by $P_n[k]$. $G_n[k]$ is called the nth *projection* corresponding to the kth symbol. The following lemma follows directly from the definition of pancake networks.

Lemma 1 *$G_n[k]$ is isomorphic to a $(n-1)$-dimensional pancake graph G_{n-1}.*

The pancake graph can also be defined recursively: G_n is constructed from n copies of $(n-1)$-dimensional pancake graphs $G_n[k]$ for $1 \leq k \leq n$. $G_n[i]$ and $G_n[j]$, $i \neq j$, are connected by $(n-2)!$ edges of the form $((j\ldots i), (i\ldots j))$. We consider each $G_n[k]$ to be a *super node.* The $(n-2)!$ edges connecting $G_n[i]$ and $G_n[j]$, $i \neq j$, are called *external edges,* while the edges joining a pair of nodes in the same $G_n[k]$ are called *internal edges.* We denote those $(n-2)!$ external edges collectively to be a *super edge* between super nodes $G_n[i]$ and $G_n[j]$. Let $G_n^s = (P_n^s, E_n^s)$ where P_n^s is the set of super nodes $G_n[k]$, $1 \leq k \leq n$, and E_n^s is the set of super edges between these super nodes. Obviously the number of super nodes of G_n^s is $|P_n^s| = n$, and the number of super edges of G_n^s is $|E_n^s| = n(n-1)/2$.

By the definition of the pancake graph, we have the following lemmas.

Lemma 2 *G_n^s is a complete graph.*

Lemma 3 *Let $p = (p_1p_2\ldots p_n)$ be a node in $G_n[p_n]$. Among the $n-1$ adjacent nodes of p, exactly one of them is not in $G_n[p_n]$, namely $Flip_n(p)$, and the other $n-2$ adjacent nodes are all in the same nth projection $G_n[p_n]$.*

In other words, each node $p = (p_1p_2\ldots p_n)$ in $G_n[p_n]$ has exactly one external edge $(p, Flip_n(p))$ incident to it, and has $n-2$ internal edges $(p, Flip_k(p))$ for $2 \leq k \leq n-1$ incident to it.

3. Hamiltonian properties and embedding of cycles

At the beginning of this section, we present the way how to connect any set of m nth projections $G_n[i_1]$, $G_n[i_2]$, $\ldots$, $G_n[i_m]$ by $m-1$ external edges. The remarks about the notations used in this paper are first explained. Considering each nth projection $G_n[i_j]$ as a super node for $1 \leq j \leq m$, the subgraph of G_n induced by $G_n[i_1]$, $G_n[i_2]$, $\ldots$, $G_n[i_m]$ is a complete graph on the m super nodes connected by the super edges. To simplify the notations, we relabel the nth projections $G_n[i_k]$ to be $G_n[k]$, $1 \leq k \leq m$. In the remainder of this paper, instead of writing $G_n[i_1]$, $G_n[i_2]$, $\ldots$, $G_n[i_m]$, we will write these nth projections as $G_n[1]$, $G_n[2]$, $\ldots$, $G_n[m]$. The notation $s \in G_n[i]$ signifies that s is a node in $G_n[i]$.

Lemma 4 *Let $\{G_n[1], G_n[2], \ldots, G_n[m]\}$ be a set of nth projections, $n \geq 4$. Let u be a node in $G_n[1]$ and v be a node in $G_n[m]$. Then $G_n[i]$ and $G_n[i+1]$ can be connected by an external edge (s_i, d_{i+1}) where $s_i \in G_n[i]$ and $d_{i+1} \in G_n[i+1]$, for $1 \leq k \leq m-1$, such that $s_1 \neq u$ and $d_m \neq v$.*

Proof. Consider the choice of the first edge (s_1, d_2). Because the number of nodes of the form $(2\ldots 1)$ in $G_n[1]$ is $(n-2)!$ and $n \geq 4$, we can always find a nodes s_1 other than u in $G_n[1]$ such that $Head(s_1) = 2$. Obviously $Flip_n(s_1)$ is a node in $G_n[2]$. Therefore, we

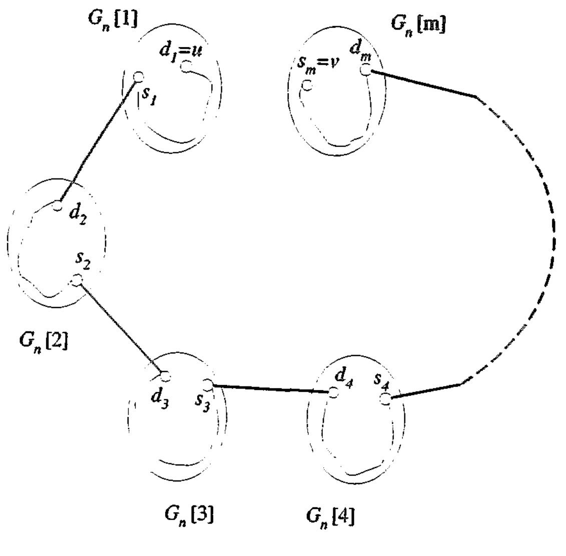


Fig 2. A pseudo path from $G_n[1]$ to $G_n[m]$

set d_2 to be $Flip_n(s_1)$. Then (s_1, d_2) is an external edge joining $G_n[1]$ to $G_n[2]$ such that $s_1 \neq u$.

Then we choose the external edges (s_i, d_{i+1}) for $2 \leq i \leq m-2$ as follows. We set s_i to be any node of $G_n[i]$ with $Head(s_i) = i+1$. Then we set d_{i+1} to be $Flip_n(s_i)$. Because $Tail(d_{i+1})$ is $i+1$, d_{i+1} is a node in $G_n[i+1]$. Thus, (s_i, d_{i+1}) is an external edge joining $G_n[i]$ to $G_n[i+1]$.

Finally, we show the way how to choose the external edge (s_{m-1}, d_m) with $d_m \neq v$. First, we choose d_m other than v from $G_n[m]$ such that $Head(d_m) = m-1$. Because the number of nodes of the form $(m-1 \ldots m)$ in $G_n[m]$ is $(n-2)!$ and $n \geq 4$, there exists at least one node which satisfies our requirement. Then, we set s_{m-1} to be $Flip_n(d_m)$. Because $Tail(s_{m-1})$ is $m-1$, s_{m-1} is a node in $G_n[m-1]$. Thus, (s_{m-1}, d_m) is an external edge joining $G_n[m-1]$ to $G_n[m]$.

Therefore, the edges (s_1, d_2), (s_2, d_3), $\ldots$, (s_{m-1}, d_m) satisfy our requirement and this lemma is proved. □

In the previous lemma, the m nth projections are connected by $m-1$ external edges to form a "path-like" structure. We call this "path-like" structure a *pseudo path*. More precisely, a pseudo path denoted by $\langle u;\ G_n[1],\ G_n[2],\ \ldots,\ G_n[m];\ v\rangle$ where $u \in G_n[1]$ and $v \in G_n[m]$ consists of m nth projections $G_n[1], G_n[2], \ldots, G_n[m]$ and $m-1$ external edges (s_i, d_{i+1}) such that $s_i \in G_n[i]$, $d_{i+1} \in G_n[i+1]$, $s_1 \neq u$, and $d_m \neq v$, where $2 \leq m \leq n$ and $1 \leq i \leq m-1$. Let d_1 be u and s_m be v. By Lemma 3 it can be checked that $d_i \neq s_i$ for every i. Note that if there exists a hamiltonian path between d_i and s_i in each subgraph $G_n[i]$ for $1 \leq i \leq m$, then the pseudo path joining $G_n[1]$ to $G_n[m]$ can be extended to form a hamiltonian path from u to v in the subgraph of G_n induced by $G_n[1]$, $G_n[2]$, $\ldots$, $G_n[m]$. See the illustration of Fig 2. The following theorem is motivated by this idea.

Theorem 1 *The n-dimensional pancake graph G_n is hamiltonian connected for $n \geq 4$.*

Proof. We prove this theorem by induction.

For $n = 4$, it is easy to find all hamiltonian paths between one fixed node and all the other nodes. For simplicity, we omit these paths here.

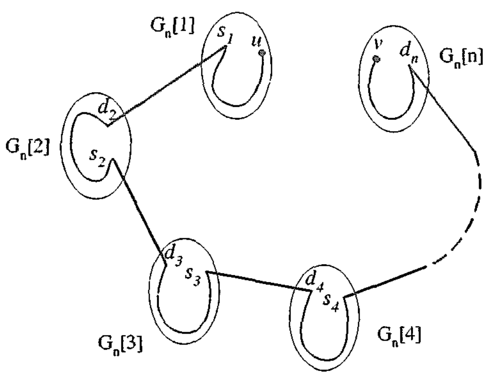
Assume that this theorem holds for $k \leq n-1$. That is, there exists a hamiltonian path between any two nodes in a $(n-1)$-dimensional pancake graph. Next, we show the way how to construct a hamiltonian path between any two nodes u and v in the n-dimensional pancake graph G_n. According to the locations of u and v, we discuss the following two cases:

1. **u and v are not in the same nth projection:** Since G_n^s is a complete graph, to simplify the notations, we may relabel all the nth projections and assume that $u \in G_n[1]$ and $v \in G_n[n]$. By Lemma 4, there exists a pseudo path $\langle u;\ G_n[1],\ G_n[2],\ \ldots,\ G_n[n];\ v\rangle$ such that $G_n[i]$ and $G_n[i+1]$ are connected by an external edge (s_i, d_{i+1}), $1 \leq i \leq n-1$, where $s_1 \neq u$ and $d_n \neq v$. Let $d_1 = u$ and $s_n = v$. By induction hypothesis, $G_n[i]$ is hamiltonian connected for each $1 \leq i \leq n$, so there exists a hamiltonian path between the node pair d_i and s_i in the subgraph $G_n[i]$. Combining these n hamiltonian paths of each nth projection with the pseudo path creates a hamiltonian path from u to v in G_n as illustrated in Fig 3(a).

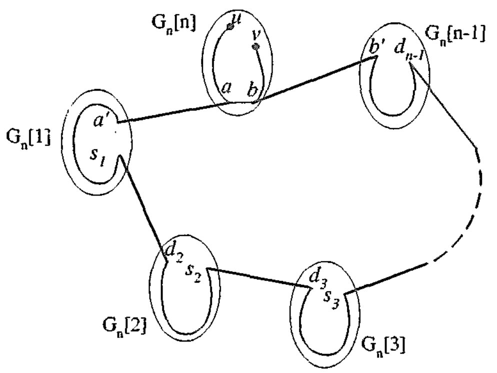
2. **u and v are in the same nth projection:** Without loss of generality, we assume that both u and v are nodes of $G_n[n]$. By induction hypothesis, there exists a hamiltonian path H_1 from u to v in the subgraph $G_n[n]$. Let (a, b) be an arbitrary edge of this path H_1. Let a' be $Flip_n(a)$ and let b' be $Flip_n(b)$. Because a and b are adjacent nodes, $Head(a) \neq Head(b)$. Thus, a' and b' are in different nth projections. Since G_n^s is a complete graph, to simplify the notations, we may relabel all the nth projections and assume that $a' \in G_n[1]$ and $b' \in G_n[n-1]$. By Lemma 4, there exists a pseudo path $\langle a';\ G_n[1],\ G_n[2],\ \ldots,\ G_n[n-1];\ b'\rangle$ such that $G_n[i]$ and $G_n[i+1]$ are connected by an external edge (s_i, d_{i+1}) for $1 \leq i \leq n-2$ where $s_1 \neq a'$ and $d_{n-1} \neq b'$. Let $d_1 = a'$ and $s_{n-1} = b'$. By induction hypothesis, $G_n[i]$ is hamiltonian connected for each $1 \leq i \leq n-1$, so there exists a hamiltonian path from d_i to s_i in the subgraph $G_n[i]$. Combining these $n-1$ hamiltonian paths of each nth projection and the pseudo path, we get a hamiltonian path H_2 from a' to b' in the subgraph of G_n induced by the $n-1$ nth projections $G_n[1]$, $G_n[2]$, $\ldots$, $G_n[n-1]$. Then, combining H_1 and H_2, adding two external edges (a, a') and (b, b'), and removing the edge (a, b) in H_1, we have a hamiltonian path from u to v in G_n as illustrated in Fig 3(b).

This completes the proof of the theorem. □

The following result follows directly from Theorem 1.



(a) u and v are not in the same nth projection



(b) u and v are in the same nth projection

Fig 3. Illustration of Theorem 1

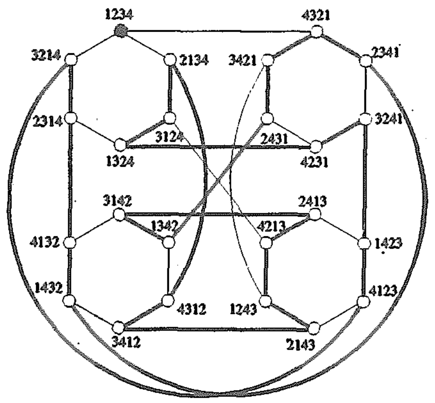


Fig 4. A hamiltonian cycle of G_4 with one faulty node (1234)

Corollary 1 *Given any edge (p,q) in the pancake graph G_n, $n \geq 4$, there exists a hamiltonian cycle containing the edge (p,q).*

In the following theorem, we show that the pancake graph still has a hamiltonian cycle in the presence of one faulty node.

Theorem 2 *The n-dimensional pancake graph G_n is 1-node hamiltonian for $n \geq 4$.*

Proof. We show this theorem by induction.

For $n = 4$, Fig 4 presents a fault-free hamiltonian cycle of G_4 with one faulty node (1234). The bold lines indicate a cycle of length $4! - 1 = 23$. Since pancake graph is node symmetric, this theorem holds for 4-dimensional pancake graph with any one faulty node.

Assume that this theorem holds for $k \leq n-1$. That is, there exists a fault-free hamiltonian cycle in a $(n-1)$-dimensional pancake graph G_{n-1} under any one faulty node occurring.

Now we show the way how to construct a hamiltonian cycle in the n-dimensional pancake graph G_n in the presence of one faulty node. Without loss of generality, we assume that the only faulty node, denoted by f, is in $G_n[n]$. By induction hypothesis, there exists a fault-free hamiltonian cycle H_1 in the subgraph $G_n[n] - f$. Let (a,b) be an arbitrary edge of this hamiltonian cycle, then $Head(a) \neq Head(b)$. Let a' be $Flip_n(a)$ and let b' be $Flip_n(b)$. So, a' and b' are in different nth projections. Using the similar argument in Theorem 1, we assume that $a' \in G_n[1]$ and $b' \in G_n[n-1]$. By Lemma 4, there exists a pseudo path $\langle a';\ G_n[1],\ G_n[2],\ \ldots,\ G_n[n-1];\ b' \rangle$ such that $G_n[i]$ and $G_n[i+1]$ are connected by the external edge (s_i, d_{i+1}) for $1 \leq i \leq n-2$ where $s_1 \neq a'$ and $d_{n-1} \neq b'$. Let $d_1 = a'$ and $s_{n-1} = b'$. Since each $G_n[i]$ is a $(n-1)$-dimensional pancake graph, by Theorem 1, there exists a hamiltonian path joining d_i to s_i in the subgraph $G_n[i]$, $1 \leq i \leq n-1$. Combining these $n-1$ hamiltonian paths of each nth projection and the pseudo path, we get a hamiltonian path H_2 from a' to b' in the subgraph of G_n induced by the

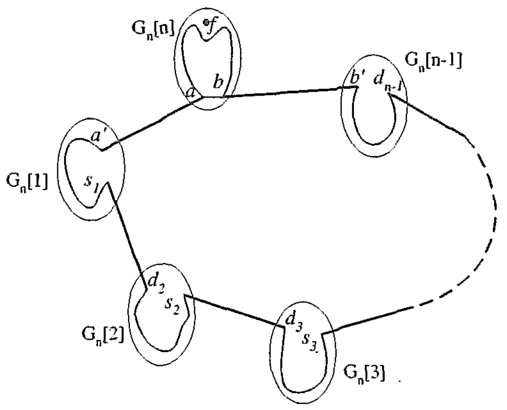


Fig 5. Illustration of Theorem 2

$n-1$ nth projections $G_n[i]$, $1 \le i \le n-1$. Finally, we combine H_1 and H_2 by adding two external edges (a, a') and (b, b') and removing the edge (a, b) in H_1. The resulting cycle is a fault-free hamiltonian cycle in $G_n - f$ as illustrated in Fig 5. Thus, this theorem holds. □

Therefore, deleting any one node from the pancake network, the resulting graph still has a hamiltonian cycle. Since a n-dimensional pancake graph has $n!$ nodes, the following result follows from Theorem 2.

Corollary 2 *A cycle of length $n!-1$ can be embedded into the n-dimensional pancake graph G_n, $n \ge 4$.*

The following theorem is proposed by Kanevsky and Feng in [3].

Theorem 3 *All cycles of length l where $6 \le l \le n!-2$, or $l = n!$ can be embedded in the pancake graph G_n.*

This theorem does not mention the case $l = n! - 1$, or $l < 6$. We have proven that for $l = n! - 1$ the cycle of length $l = n! - 1$ can also be embedded in pancake graph G_n. As for $l < 6$, the following lemma gives a negative answer.

Lemma 5 *The pancake graph G_n does not contain any cycle of length $l < 6$.*

Proof. We show this lemma by induction. Since a 2-dimensional pancake graph G_2 has only one edge, and a 3-dimensional pancake graph G_3 is a 6-cycle, obviously this lemma holds for $n = 2$ and $n = 3$.

Assume that the lemma is true for $n-1$. Thus, each cycle in a $(n-1)$-dimensional pancake graph G_{n-1} has length at least 6.

Now we show that each cycle in a n-dimensional pancake graph G_n has length at least 6. Let C be an arbitrary cycle in G_n. Suppose that C is totally within one nth projection. By induction, the length of C is at least 6.

Assume that C goes through more than three nth projections. Then C contains at least three external edges. By Lemma 3, no two external edges are incident to each other, so C has length at least 6.

Now suppose that C goes through exactly two nth projections $G_n[i]$ and $G_n[j]$. Then C contains at least two external edges $(a, Flip_n(a))$ and $(b, Flip_n(b))$ where a and b are two nodes in $G_n[i]$, and $Flip_n(a)$ and $Flip_n(b)$ are two nodes in $G_n[j]$. If (a, b) is an internal edge in $G_n[i]$, then $Head(a) \ne Head(b)$. So, $Flip_n(a)$ and $Flip_n(b)$ are in different nth projections. This is not the case. So a and b are not adjacent. Similarly, $Flip_n(a)$ and $Flip_n(b)$ are not adjacent either. Therefore, C has length at least 6. This proves the lemma. □

By Corollary 2, Theorem 3, and Lemma 5, we expand Kanevsky and Feng's result as follows.

Theorem 4 *A cycle of length l can be embedded in the pancake graph G_n, $n \ge 4$, if and only if $6 \le l \le n!$.*

4. Conclusion

The main purpose of this paper is to study some intriguing topological properties of the pancake networks G_n. We prove that there exists a hamiltonian path between any two nodes of G_n. This result is useful to construct a hamiltonian cycle in a faulty pancake network. Applying this result we show that there exists a hamiltonian cycle in G_n with the occurring of any one faulty node. As a consequence, a cycle of length $n! - 1$ can be embedded into G_n for any $n \ge 4$. We then expand Kanevsky and Feng's result as follows: A cycle of length l can be embedded in the pancake graph G_n, $n \ge 4$, if and only if $6 \le l \le n!$.

References

[1] S. B. Akers, and B. Krishnamurthy, A group theoretic model for symmetric interconnection networks, *IEEE Trans. Comput.* 4(c-38) (1989) 555-566.

[2] S. B. Akers, D. Harel, and B. Krishnamurthy, The star graph: An attractive alternative to the n-cube, *Proc. International Conference on Parallel Processing*, St. Charles, Illinois, (1987) 393-400.

[3] A. Kanevsky, and C. Feng, On the embedding of cycles in pancake graphs, *Parallel Computing*, 21 (1995) 923-936.

[4] F.T. Leighton, Introduction to parallel algorithms and architectures: arrays · tree · hypercubes, *Morgan Kaufmann Publisher, Inc*, 1992.

[5] A. Menn and A. K. Somani, An efficient sorting algorithm for the star graph interconnection, *Proc. 19th Int. Conf. on Parallel Processing* (1990) 1-8.

[6] K. Qiu, The star and pancake interconnection networks: properties and algorithms, PhD thesis, Queen's University, Kingston, Ontario, Canada, August 1992.

[7] K. Qiu, S. G. Akl, and H. Meijer, On some properties and algorithms for the star and pancake interconnection networks, *Journal of Parallel and Distributed Computing*, 22 (1991) 16-25.

[8] K. Qiu, S. G. Akl, and I. Stojmenovic, Data communication and computational geometry on the the star and pancake interconnection networks, *Proc. 3rd IEEE Symp. on Parallel and Distributed Processing* (1991) 415-422.

[9] H. Senoussi, and C. Lavault, Embeddings into the pancake interconnection network, *High Performance Computing on the Information Superhighway*, 1997, HPC Asia '97, 73-78.

[10] A. S. Tanenbaum, *Computer Networks*, (Prentice-Hall, 1988).