APPLICATION OF HADAMARD TRANSFORM FOR FAST VQ IMAGE CODING

Jeng-Shyang Pan*, Zheng-Ming Lu**, Tsong-Yi Chen* and Sheng-He Sun**

*Department of Electronic Engineering, Kaohsiung University of Applied Sciences, Taiwan, R.O.C.
**Department of Automatic Test and Control, Harbin Institute of Technology, Harbin, P. R. C.
Email:jspan@cc.kuas.edu.tw

ABSTRACT

Image compression using vector quantization has been proved to be an efficient approach. The speed of codeword search is the key problem for real-time applications of vector quantization. In this paper, a new inequality based on the Hadamard transform is derived. An efficient VQ codeword search algorithm using this inequality is proposed for VQ image coding. Experimental results show that the algorithm performs better than the PDS, ENNS, IENNS, and WTPDS algorithms.

1. INTRODUCTION

As an effective technique for data compression, VQ [1][2] has been successfully used for various applications such as speech coding and image transmission. The codeword search problem in VQ is to assign one codeword $\vec{y}^j=(y_1^j, y_2^j,\ldots,y_k^j)$ in the codebook $C=\{\vec{y}^1,\vec{y}^2,\ldots,\vec{y}^N\}$ to the input vector $\vec{x}=(x_1,x_2,\ldots,x_k)$ such that the distortion between this codeword and the input vector is the smallest among all codewords. The distortion of representing the input vector $\vec{x}$ by the the codeword $\vec{y}^j$ can be measured by the squared Euclidean distance, i.e.,

$$D(\vec{x},\vec{y}^j)=\sum_{i=1}^{k}(x_i-y_i^j)^2 \qquad (1)$$

Thus the full search VQ needs to perform kN multiplications, $(2k\text{-}1)N$ addtions and N-1 comparisons for encoding each input vector. In order to release the computation load of the full search algorithm, a lot of efficient algorithms have been presented for fast codeword search. The PDS algorithm [3] is a simple and efficient algorithm, which allows early termination of the distortion calculation between a training vector and a codeword by introducing a premature exit condition in the search process. The hypercube approach [4] is a well know premature method which is efficient if the difference for any coefficient is generally larger than the difference of the other coefficient.

Vidal proposed the approximating and elimination search algorithm (AESA) [5] whose computation time is approximately constant for a codeword search in a large codebook size. The high correlation characteristics between data vectors of adjacent speech frames and the triangular inequality elimination (TIE) criterion were utilized to VQ-based recognition of isolated words [6]. A similar idea was also employed for VQ image coding by Huang and Chen [7].

The bound for Minkowski metric and quadratic metric was derived and applied to VQ codeword search [8]. The partial distortion search algorithm (PDS) [3], hypercube approach [4] and absolute error inequality criterion (AEI) [9] are all special cases in the bound for Minkowski metric. For the squared Euclidean distortion measure, the improved absolute error inequality criterion (IAEI) [10] can be obtained by setting the parameters from the bound for Minkowski metric. The Manhattan metric was modified to match the Euclidean distortion measure using a suitable training procedure so that the number of multiplication operations can be drastically reduced [11].

A fast algorithm [12] was proposed for image coding based on the assumptions that the distortion is measured by the squared Euclidean distance and the vector dimension is $2^n \times 2^n$. The algorithm uses the mean pyramids of codewords to reject many unmatched codewords. A more efficient algorithm based on the same assumptions as the mean pyramid algorithm was presented. The mean-variance pyramid data structure [13] was used to reject many more unmatched codewords than the mean pyramid algorithm.

The ENNS algorithm [14] uses the mean of an input vector to cancel the unlikely codeword. In the improved ENNS algorithm (IENNS) [15], a vector is separated into two sub-vectors, and two inequalities based on the sums of its two sub-vectors are used to reject unlikely codewords. Although these algorithms do not arouse performance degradation, they are spatial-domain based and not efficient enough. A new more generalized algorithm based on the sub-vector techniques is also proposed [16]. Recently, a new codeword search algorithm based on wavelet transforms has been presented [17], which is denoted by WTPDS here. In this paper, we present a new fast codeword search algorithm based on Hadamard transform (HT), which is more efficient than the PDS, ENNS, IENNS, and WTPDS algorithms.

This paper is organized as follows: Section 2 derives a new inequality based on the property of Hadamard transform. A novel fast codeword search algorithm based on this inequality is proposed in Section 3. Experimental results for the comparison of several algorithms are presented in Section 4. The conclusions are made in the final section.

2.Derivation Of New Inequality

Let $\boldsymbol{H}_n$ be the $2^n \times 2^n$ Hadamard square matrix with elements in the set {1,-1}. Assume all of the following vectors are k-dimensional vectors and $k = 2^n$ (n>0), some basic definitions and facts can be shown as follows:

(i) *Definition 1*:

$$\boldsymbol{H}_1 = \begin{Bmatrix} 1 & 1 \\ 1 & -1 \end{Bmatrix} \text{ and } \boldsymbol{H}_{n+1} = \begin{Bmatrix} \boldsymbol{H}_n & \boldsymbol{H}_n \\ \boldsymbol{H}_n & -\boldsymbol{H}_n \end{Bmatrix}.$$

(ii) *Theorem 1*: $\boldsymbol{H}_n \boldsymbol{H}_n = k\boldsymbol{I}_k$, where $\boldsymbol{I}_k$ is the unit identity matrix of order k.

(iii) *Definition 2*: The Hadamard-transformed vector $\vec{X}$ of the vector $\vec{x}$ can be defined as $\vec{X} = \boldsymbol{H}_n \vec{x}$.

(iv) *Definition 3*: The sum of the vector $\vec{x}$ is defined as $S_x = \sum_{i=1}^{k} x_i$ and the energy of the vector $\vec{x}$ is defined as $\|\vec{x}\| = \sum_{i=0}^{k} x_i^2$

(v) *Theorem 2*: X_0=S_x. Where X_0 is the first element of $\vec{X}$. This equation can be derived from the fact that each element in the first line of $\boldsymbol{H}_n$ is with the same value '1'.

(vi) *Theorem 3*: $\|\vec{X}\| = k\|\vec{x}\|$.

Proof.
$$\|\vec{X}\| = \vec{X}^T \vec{X} = (H_n \vec{x})^T (H_n \vec{x}) = \vec{x}^T H_n H_n \vec{x} = \vec{x}^T k\boldsymbol{I}_k \vec{x} = k\vec{x}^T \vec{x} = k\|\vec{x}\|.$$

Where T denotes the transposition operation.

(vi) *Theorem 4*: $D(\vec{X}, \vec{Y}^j) = kD(\vec{x}, \vec{y}^j)$. Where $\vec{y}^j$ is a codeword and $\vec{x}$ is the input vector.

Proof. Let $\vec{x} - \vec{y}^j = \vec{z}$ and $\vec{Z} = H_n \vec{z}$. Then $\|\vec{z}\| = D(\vec{x}, \vec{y}^j)$ and $\|\vec{Z}\| = D(\vec{X}, \vec{Y}^j)$. Therefore, it follows from Theorem 3 that $D(\vec{X}, \vec{Y}^j) = \|\vec{Z}\| = k\|\vec{z}\| = kD(\vec{x}, \vec{y}^j)$.

(vii) *Corollary 1*:

$$\left| X_0 - Y_0^j \right| < \sqrt{D(\vec{X}, \vec{Y}^j)} = \sqrt{kD(\vec{x}, \vec{y}^j)}$$

3. Proposed Algorithm

Now, suppose there are N codewords in the codebook C: $\vec{y}^1$, $\vec{y}^2, \ldots,$ $\vec{y}^N$, each one with dimension $k = 2^n$. Let $\vec{x}$ be the input vector with the same dimension as these codewords.

From Theorem 4, we know that the codeword that is closest to the input vector in the spatial domain is also closest to the input vector in the HT domain. Therefore we can find the corresponding best codeword in the spatial domain by searching the best codeword in the HT domain. Let $D^m(\vec{X},\vec{Y}^j) = \sum_{i=1}^{m}(X_i - Y_i^j)^2$ denote the partial distance between $\vec{X}$ and $\vec{Y}^j$, where $1 \le m \le k$ Before the search process, the HT is performed on all codewords in the codebook, and then the transformed codewords are sort in the ascending order of their first elements. Note that no multiplication is required for the HT.

To carry out the codeword search in the HT domain, we first perform the HT on the input vector $\vec{x}$ to obtain $\vec{X}$, and initialize the current closest codeword of $\vec{X}$ to be $\vec{Y}^p$, where $p = \arg\min_j |X_0 - Y_0^j|$, and set the current minimum distortion $D_{\min}$ to be $D(\vec{X},\vec{Y}^p)$. Then, we perform the codeword search in the order as shown in Fig.1, and set MINSUM=$X_0 - \sqrt{D_{\min}}$, MAXSUM=$X_0 + \sqrt{D_{\min}}$. For each codeword $\vec{Y}^j$ to be searched, if Y_0^j>MAXSUM or Y_0^j<MINSUM, then it follows from Corollary 1 that $|X_0 - Y_0^j| > \sqrt{D_{\min}}$. Hence $\vec{Y}^j$ is not the closest codeword to $\vec{X}$ and can be rejected. Otherwise, we perform the following PDS process. Starting from *m*=2, for each value of *m*, *m*=2,3,…*k*, we first evaluate $D^m(\vec{X},\vec{Y}^j)$. If $D^m(\vec{X},\vec{Y}^j) > D_{\min}$, then $\vec{Y}^j$ can be rejected. Otherwise, we go to next value of *m* and repeat the same process. This PDS process is repeated until $\vec{Y}^j$ is rejected or *m* reaches *k*. If *m*=*k*, then we compare $D(\vec{X},\vec{Y}^j)$ with $D_{\min}$. If $D(\vec{X},\vec{Y}^j) < D_{\min}$, then $D_{\min}$ is replaced by $D(\vec{X},\vec{Y}^j)$ and the current closest codeword of $\vec{X}$ is set to be $\vec{Y}^j$, MAXSUM and MINSUM are also recomputed. As shown in Fig.1, the search process can be stopped in the down direction once Y_0^j<MINSUM and stopped in the up direction once Y_0^j>MAXSUM. After the best codeword of $\vec{X}$ in the transformed domain is found, the corresponding codeword of $\vec{x}$ in the spatial domain is also found and $D_{\min}/k$ is the corresponding distance according to Theorem 4.

4. Experimental Results

We performed experiments on Pentium-II PC using four 512 × 512 monochrome images 'Lena', 'Sonic', 'Peppers' and 'F16' with 256 gray scales. Each image was divided into blocks with size 8× 8=64. The codebook was designed using LBG algorithm [2] with the Lena image as the training set. The other three images were used to test the effectiveness of the algorithms.

The proposed algorithm was compared to PDS, ENNS, IENNS and WTPDS algorithms in terms of the average CPU time per image and the arithmetic complexity (the average number of distance calculations per input vector) for different codebook sizes as shown in Table 1. For codebook size 512, the proposed algorithm needs only about 2% of distance calculations required by the full search algorithm.

5. Conclusions

The main contribution of this paper is to derive a new inequality based on the Hadamard transform. A fast VQ encoding algorithm based on this new inequality is applied for VQ codeword search. Preliminary experiments demonstrate that the proposed algorithm can dramatically reduce the computational complexity during the codeword search without sacrificing the coding quality. Experimental results confirm the effectiveness of the proposed algorithm.

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Table 1. Complexity comparisons of various fast search algorithms (Dimension for VQs is 8×8)

Codebook size	256		512	
	CPU time (s)	Complexity	CPU time (s)	Complexity
Full Search	11.42	256.00	22.78	512.00
PDS	2.20	35.74	4.18	65.26
ENNS	0.79	12.98	1.16	23.98
IENNS	0.77	12.45	1.03	20.09
WTPDS	0.53	8.21	0.75	13.95
Our algorithm	0.52	8.12	0.71	13.78

Fig. 1 Search order of the transformed codewords in the codebook
(Codewords are sorted in the ascending order of their first elements)