Automatic Segmentation of Brain Parenchyma and Tumor in MRI

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ABSTRACT

Orthogonal subspace projection (OSP) approach has shown success in Magnetic Resonance image classification. The proposed approach of OSP can be divided into two main steps, removal of the unwanted signature an undesired signature annihilator followed by the use of matched filter. An undesired signature annihilator is used to separate the desired signature from the unwanted signatures so that the unwanted signatures can be eliminated via orthogonal subspace projection. Therefore, it requires a complete knowledge of the desired signature and the unwanted signatures present in images. In this paper, an unsupervised orthogonal subspace projection (UOSP) approach is proposed where the only knowledge of the desired signature to be classified is required. UOSP comprises two processes. Target Generation Process (TGP) and Target Classification Process (TCP). The objective of TGP is to generate a set of potential target signatures from an unknown background, which will be subsequently classified by TCP. As a result, UOSP can be used to search for a specific target in unknown scenes. Finally, the effectiveness of UOSP in target detection and classification is evaluated by several MRI experiments. All experiments were under supervision of the expert radiologist. Results show that the cerebral tissue was segmented accurately into four images, tumor, gray matter, white matter and cerebral spinal fluid indicating the possible usefulness of this method.

Index Terms - Classification, Detection, Magnetic Resonance Imaging (MRI), Classification. Brain images, Tumor, Orthogonal subspace projection (OSP), Unsupervised OSP (UOSP), Target Generation Process (TGP), Target Classification Process (TCP).

1. INTRODUCTION

Magnetic Resonance Imaging (MRI) has become a useful modality since it provides unparallel capability of revealing soft tissue contrast as well as 3-D visualization. It produces a sequence of multiple spectral images of issues with a variety of contrasts using three magnetic resonance parameters, spin-lattice (T1), spin-spin (T2) and dual echo-echo proton density (PD). One potential application of MRI in clinical practice is the brain parenchyma classification and segmentation of normal and pathological tissue. It is the first step to address a wide range of clinical problems. By means of the volume, shapes and region distribution of the brain tissue, one can find the abnormalities that are commonly related to the conditions of heterotopias, lissencephaly, brain atrophy, and cerebral infarction. Over the past years many computer-assisted methods have been reported [1]-[11] such as neural networks [5]-[9], hybrid methods [10], knowledge-based techniques [11], etc. For example, neural networks have demonstrated their superior performance in segmentation of brain tissue to classical maximum likelihood methods; hybrid methods have shown a promise by combining imaging processing and model-based techniques in segmentation [10]; knowledge-based techniques allows one to make more intelligent classification and segmentation decisions. [11].

In image classification it generally requires a priori knowledge about the objects to be classified. Although it can be done in an unsupervised fashion, the results are generally not so good as supervised methods. It will be even worse if the objects are relatively small or the image background varies with pixel-by-pixel. Obtaining such prior information is not realistic in many practical applications. In this paper, we present an unsupervised orthogonal subspace projection (UOSP) approach that is based on spectral feature correlation of MR images. UOSP is derived from the concept of Orthogonal Subspace Projection (OSP), which has shown success in Magnetic Resonance image classification. [12-13] OSP can be divided into two main steps, removal of the unwanted signature an undesired signature annihilator followed by the use of matched filter. An undesired signature

annihilator is used to separate the desired signature from the unwanted signatures so that the unwanted signatures can be eliminated via orthogonal subspace projection. Therefore, it requires a complete knowledge of the desired signature and the unwanted signatures present in images. UOSP does not require knowing the number of signatures nor the undesired signatures where the only knowledge of the desired signature to be classified is required. UOSP makes use of two processes, Target Generation Process (TGP) and Target Classification Process (TCP) to accomplish the task. TGP is first used to automatically generate a set of potential targets from an unknown background. Then these generated targets are subsequently classified by TCP. The objective of TGP is to produce potential targets to be used for TCP. These targets may include interferers as well as unwanted objects. If we assume that the initial target is $\mathbf{T}_0$, TGP then projects all image pixel vectors into the space orthogonal to $\mathbf{T}_0$. A pixel vector with the maximum projection in the orthogonal projection space will be selected as a first target signature, denoted by $\mathbf{T}_1$ other than $\mathbf{T}_0$. Since $\mathbf{T}_1$, produces the largest magnitude of projection in the orthogonal complement of $\mathbf{T}_0$, these two signatures $\mathbf{T}_0$ and $\mathbf{T}_1$, must have most distinct spectral features in the sense of orthogonal projection. The same procedure is then repeated again to generate one target at a time. The procedure converges when a stopping criterion is satisfied. At this point, the set of the generated targets will be used for target classification carried out by TCP. The classification implemented in TCP is a little bit different from the class labeling used in remote sensing [14] and pattern classification [15]. It is based on the abundance estimated for a particular target signature resident in each pixel vector. As a result, the classification results are presented by gray-scaled images where the gray scale of each pixel is represented by estimated target abundance contained in that pixel. The criterion to terminate TGP is based on the orthogonal projection correlation between a target and a set of targets, called orthogonal projection correlation index (OPCI). OPCI is gradually reduced as the number of generated targets grows. So, OPCI is monotonically decreasing and converges to zero. Using OPCI as a stopping rule, TGP will terminate its process when OPCI is sufficiently small.

In order to demonstrate the performance of OSP and UOSP in MR image classification, a series of experiments using brain MR images are conducted. The experimental results show that the cerebral tissue was classified accurately and show that UOSP significantly improves the classification performance of OSP.

The remainder of this paper is organized as follows. Section II formulates the MR image pixel classification as a linear mixing problem. Section III describes the OSP approach. Section IV describes the UOSP approach. Section V conducts a set of experiments to evaluate the performance of OSP and UOSP in MR classification. Section VI concludes some comments.

2. LINEAR SPECTRAL MIXTURE MODEL

Linear spectral unmixing is a widely used method in remote sensing community to classify and quantify multicomponent constituents. It views a multi/hyperspectral image as an image cube where each pixel is considered to be a column vector and modeled as a linear spectral mixture of substances resident in the pixel. More precisely, assume that there are p spectrally distinct substances $\{\mathbf{m}_1, \mathbf{m}_2, \cdots \mathbf{m}_p\}$ in the image and $\mathbf{r}$ is an image pixel represented by an $l \times 1$ column vector where l is the number of spectral bands. Let $\mathbf{M}$ be an $l \times p$ signature matrix, denoted by $[\mathbf{s}_1 \mathbf{s}_2 \cdots \mathbf{s}_p]$ where $\mathbf{s}_j$ is an $l \times 1$ column vector represented by the spectral signature of the j-th substance $\mathbf{m}_j$ in the pixel vector r. It further assumes that a is a $p \times 1$ abundance column vector associated with $\mathbf{M}$ given by $\mathrm{a} = (\alpha_1\ \alpha_2\ \ldots \alpha_p)^T$ where α_j denotes the fraction of the j-th signature $\mathbf{s}_j$ in the pixel vector $\mathbf{r}$. Then a linear spectral mixture model for r is given by

$$\mathbf{r} = \mathbf{M}\mathbf{a} + \mathbf{n}. \tag{1}$$

The $\mathbf{n}$ in Eq. (1) is an $l \times 1$ column vector and can be interpreted as either additive noise or measurement error. The linear spectral unmixing is to develop a method that finds or unmixes the abundance vector a from the pixel vector r through Eq. (1).

Eq. (1) can be used to model MR images. In this case, $\mathbf{r}$ is an MR image pixel, $\mathbf{M} = [\mathbf{s}_1\ \mathbf{s}_2 \cdots \mathbf{s}_p]$ is a signature matrix made up of the spectral signatures of tissue substances $\{\mathbf{m}_1\ \mathbf{m}_2 \cdots \mathbf{m}_p\}$ in MR images such as white matter, gray matter, cerebral spinal fluid, tumor etc. and the associated abundance vector a represents the abundance fractions of these p spectral signatures $\{\mathbf{s}_1 \mathbf{s}_2 \cdots \mathbf{s}_p\}$ present in the pixel vector $\mathbf{r}$.

3. ORTHOGONAL SUBSPACE PROJECTION-BASED APPROACHES

Many unmixing methods to solve Eq. (1) have been proposed in the past [20-22]. Of particular interest is the OSP approach that has been successfully applied to hyperspectral images. The idea of OSP is to divide the p substances $\{\mathbf{m}_1\ \mathbf{m}_2 \cdots \mathbf{m}_p\}$ into two classes, desired substance class and undesired substance class. Without loss of generality we assume that the desired substance class contains only one single substance $\mathbf{m}_p$ and undesired substance class consists of the remaining substances, denoted by $\{\mathbf{m}_1\ \mathbf{m}_2 \cdots \mathbf{m}_{p-1}\}$. Then we can rewrite Eq. (1) as

$$\mathbf{r} = \mathbf{d}\mathrm{a}_p + \mathbf{U}? + \mathbf{n}. \tag{2}$$

where $\mathbf{d} = \mathbf{s}_p$, $\mathbf{U} = (\mathbf{s}_1 \mathbf{s}_2 \cdots \mathbf{s}_{p-1})$, α_p is the abundance fraction of the desired spectral signature $\mathbf{d}$ and

$\mathbf{?} = (a_1 a_2 \ldots a_{p-1})$ is the abundance vector representing fractions of the undesired spectral signatures $\{\mathbf{s}_1 \mathbf{s}_2 \cdots \mathbf{s}_{p-1}\}$. Since the desired signature d is separated from the undesired signatures $\{\mathbf{s}_1 \mathbf{s}_2 \cdots \mathbf{s}_{p-1}\}$ in Eq. (2), we can design a subspace projection operator to eliminate $\{\mathbf{s}_1 \mathbf{s}_2 \cdots \mathbf{s}_{p-1}\}$ before extracting d. One such a projector is the least squares operator denoted by $P_{\mathbf{U}}^{\perp}$ given by

$$P_{\mathbf{U}}^{\perp} = \mathbf{I} - \mathbf{U}\mathbf{U}^{\#}. \tag{3}$$

where $\mathbf{U}^{\#} = (\mathbf{U}^T \mathbf{U})^{-1} \mathbf{U}^T$ is the pseudo-inverse of $\mathbf{U}$ and the notation $\mathbf{U}^T$ in $P_{\mathbf{U}}^{\perp}$ indicates that the projector $P_{\mathbf{U}}^{\perp}$ maps the observed pixel $\mathbf{r}$ into the space $< \mathbf{U} >^{\perp}$, the orthogonal complement of $< \mathbf{U} >$. Premultiplying Eq. (2) by $P_{\mathbf{U}}^{\perp}$ yields

$$P_{\mathbf{U}}^{\perp}\mathbf{r} = P_{\mathbf{U}}^{\perp}\mathbf{d}a_p + P_{\mathbf{U}}^{\perp}\mathbf{U}\mathbf{?} + P_{\mathbf{U}}^{\perp}\mathbf{n} = \mathbf{d}a_p + P_{\mathbf{U}}^{\perp}\mathbf{n}. \tag{4}$$

where the undesired substances $\{\mathbf{m}_1 \mathbf{m}_2 \cdots \mathbf{m}_{p-1}\}$ have been eliminated via $P_{\mathbf{U}}^{\perp}$ and the original noise is also suppressed to $P_{\mathbf{U}}^{\perp}\mathbf{n}$ by $P_{\mathbf{U}}^{\perp}$. As a result, Eq. (4) represents a standard signal detection problem. If the optimal criterion for the signal detection specified by (4) is chosen to maximize the signal-to-noise ratio (*SNR*)

$$SNR = \frac{(\mathbf{x}^T P_{\mathbf{U}}^{\perp}\mathbf{d})a_p (\mathbf{d}^T P_{\mathbf{U}}^{\perp}\mathbf{x})}{\mathbf{x}^T P_{\mathbf{U}}^{\perp} E[\mathbf{n}\mathbf{n}^T] P_{\mathbf{U}}^{\perp}\mathbf{x}}, \text{ over } \mathbf{x} \tag{5}$$

the maximum *SNR* of Eq. (5) can be obtained by a matched filter, denoted by $\boldsymbol{MF}_{\mathbf{d}}$ with $\mathbf{d}$ being the desired matched signal.

Based on Eqs. (4)-(5), a mixed pixel classification can be carried out by a two-stage process, i.e., an undesired signature annihilator $P_{\mathbf{U}}^{\perp}$ followed by matched filter, $\boldsymbol{MF}_{\mathbf{d}}$. More precisely, if we want to classify a desired signature d in a mixed pixel described by Eq. (1), we first apply $P_{\mathbf{U}}^{\perp}$ to Eq. (2) to eliminate $\mathbf{U}$, then use the matched filter $\boldsymbol{MF}_{\mathbf{d}}$ to extract $\mathbf{d}$ from Eq. (4). The operator coupling $P_{\mathbf{U}}^{\perp}$ with $\boldsymbol{MF}_{\mathbf{d}}$ is called an orthogonal subspace classifier, P_{OSP} the one derived in [18] and denoted by

$$P_{OSP} = \boldsymbol{MF}_{\mathbf{d}} P_{\mathbf{U}}^{\perp} = \mathbf{d}^T P_{\mathbf{U}}^{\perp}. \tag{6}$$

4. UNSUPERVISED SUPERVISED ORTHOGONAL SUBSPACE PROJECTION (UOSP)

As shown in Eq.(6), P_{OSP} requires a complete knowledge of the undesired signatures in $\mathbf{U}$. In many practical applications, it is difficult to determine $\mathbf{U}$. In this section, we present an unsupervised OSP (UOSP) approach to target detection and classification which requires no prior knowledge about the number of signatures in $\mathbf{U}$. It automatically and successively generates unwanted signatures, which also include interferers and background clutter.

In the target generation process, we suppose that there is a desired target of interest to be classified, $\mathbf{T}_0$. This target can be either directly extracted from the image scene or obtained from spectral library. Other than $\mathbf{T}_0$ there is no prior knowledge required for information about how many signatures within an image. UOSP begins with using the desired target signature $\mathbf{T}_0$ as an initial target. Then an orthogonal subspace projector $P_{\mathbf{T}_0}^{\perp}$ is employed to project all image pixel vectors into the orthogonal complement space of $< \mathbf{T}_0 >$, denoted by $< \mathbf{T}_0 >^{\perp}$. The maximum length of the pixel vector in $< \mathbf{T}_0 >^{\perp}$ corresponds to the maximum orthogonal projection with respect to $\mathbf{T}_0$. This pixel vector will be selected as a first target denoted by $\mathbf{T}_1$. The reason for this selection is that $\mathbf{T}_1$ will have the most distinct features from $\mathbf{T}_0$ because $\mathbf{T}_1$ in $< \mathbf{T}_0 >^{\perp}$ has the largest magnitude of the projection produced by $P_{\mathbf{T}_0}^{\perp}$. Then an OSP classifier $P_{OSP} = \mathbf{T}_0{}^T P_{\mathbf{U}}^{\perp}$ via Eq. (6) with $\mathbf{U} = \mathbf{T}_1$ is applied to the image. If the resulting images show the target $\mathbf{T}_0$ clearly, the $\mathbf{T}_0$ is declared to be detected and classified. Otherwise, another orthogonal subspace projector $P_{(\mathbf{T}_0 \mathbf{T}_1)}^{\perp}$ is applied again to the original image by projecting all image pixel vectors to the space orthogonal to $\mathbf{T}_0$ and $\mathbf{T}_1$. Once again, the pixel vector with maximum length in the space $< \mathbf{T}_0, \mathbf{T}_1 >^{\perp}$ will be selected for a second target denoted by $\mathbf{T}_2$. This is then followed by an OSP classifier $P_{OSP} = \mathbf{T}_0^T P_{(\mathbf{T}_1 \mathbf{T}_2)}^{\perp}$ with $\mathbf{U} = (\mathbf{T}_1 \mathbf{T}_2)$ applied to image. If the resulting image does not clearly show the target $\mathbf{T}_0$, the above procedure will be repeated again to find a third target $\mathbf{T}_3$, fourth target $\mathbf{T}_4$, etc. until the target $\mathbf{T}_0$ is detected. Since we do not know how many targets should be generated, OSP classifier must be applied every time as an new target is generated. Fortunately, this laborious TGP can be avoided provided that there is a reliable stopping rule to determine how many targets needed to be generated target is sufficient for target detection and classification. In the following, we will propose such a criterion on the basis of the correlation between the target $\mathbf{T}_0$ and the projection $P_{\mathbf{U}}^{\perp}$.

Let $\mathbf{U}_i = (\mathbf{T}_1 \mathbf{T}_2 \ldots \mathbf{T}_i)$ be the i^{th} target signature set used for OSP classifier in the *i*-th stage. We then define the orthogonal projection correlation index (OPCI) by

$$h_i = \mathbf{T}_0^T P_{U_i}^{\perp} \mathbf{T}_0 . \quad (7)$$

Since $\mathbf{U}_{i-1} \subset \mathbf{U}_i$, $h_i = \mathbf{T}_0^T P_{\mathbf{U}_i}^{\perp} \mathbf{T}_0 \le h_{i-1} = \mathbf{T}_0^T P_{\mathbf{U}_{i-1}}^{\perp} \mathbf{T}_0$ for all *i*'s. This implies that the sequence $\{\mathbf{T}_0^T P_{\mathbf{U}_i}^{\perp} \mathbf{T}_0\}$ is monotonically decreasing at *i*. In other words, OPCI sequence $\{h_i\}$ is monotonically decreasing at *i*. Using this property as a stopping criterion, the TGP can be given below.

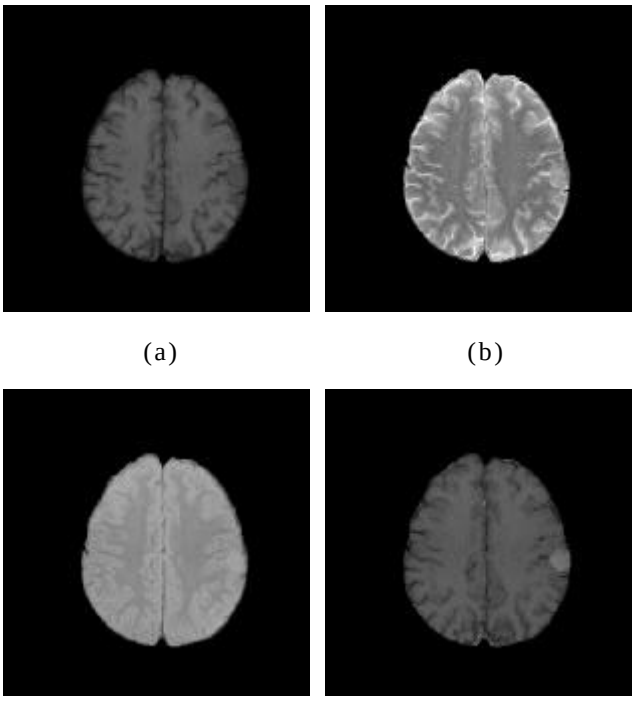
5. EXPERIMENTAL RESULTS

A set of multispectral MR brain images was used to evaluate the performance of OSP and UOSP. It consists of MR images acquired from a patient with abnormal physiology (shown in Fig. 1) using four bands with resolution 8-bit gray level and 256 by 256 pixels. T1-weighted and T2-weighted images were acquired for band one and two. PD-weighted and Gd-DTPA images were acquired for band three and four. The slice thickness of all MR images are 2mm and axial section were taken from Signa 1.5T SYS#GEMSOW. All experiments were under supervision of a neuroradiologist.

The radiance spectra of four cerebral tissues, gray matter (GM), white matter (WM), cerebral spinal fluid (CSF) and tumor used for OSP and UOSP are shown in Fig. 2. All spectra were extracted directly from the MR images and verified by experienced radiologists. Fig.3 show the classification results of OSP based on the four images in Fig. 1 where the images labeled by (a), (b), (c) and (d) were generated respectively by using GM, WM, CSF and tumor as desired signatures **d** in the classifiers while the other three signatures were made up to form the undesired signature matrix ***U***. Fig. 4 demonstrates the classification results of UOSP. Compared to Figs. 3, the unsupervised OSP-based classifiers performed better than OSP-based classifiers. As a final comment, in order to evaluate OSP and UOSP in all aspects only one representative MR image sequence was studied for experiments in this paper. In fact, more experiments were also conducted for performance evaluation. The results draw similar conclusions.

6. CONCLUSION

In this paper, we present an Unsupervised Orthogonal Subspace Projection (UOSP) approach to target detection and classification, which requires no prior knowledge about signatures (particularly, the number of signatures). UOSP implements a two-stage process, TGP and TCP. TGP extracts potential targets from unknown image scenes, which may include interferers, unwanted targets, natural background signatures and clutter. Then these generated targets are subsequently classified by TCP. UOSP is derived from the concept of Orthogonal Subspace Projection (OSP), which has shown success in Magnetic Resonance image classification. OSP views an MR image sequence as a multispectral image cube and models each pixel vector as a linear mixture of tissue substances resident in the MR pixel vector. Unlike the traditional image classification techniques, which are carried out on a pure pixel basis, OSP-based classifiers are mixed pixel classification techniques. They use the linear mixture model to generate a fractional image for each object required for classification. The advantages of mixed pixel classification have been demonstrated in the experiments. Brain MR images segmentation is the critical step in the analysis of brain pathology. Experimental results have shown that the UOSP technique was able to correctly classify the cerebral tissue into four-image gray matter, white matter, cerebral spinal fluid and tumor indicating the promising possibilities of this method.



(a) (b)

(c) (d)

Fig. 1. The MR images of the brain. Axial section. (a) T1-weighted image; (b) T2-weighted image; (c) Proton density image; (d) Gd-DTPA

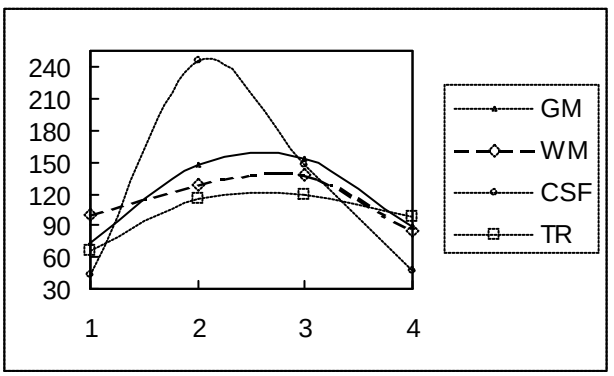
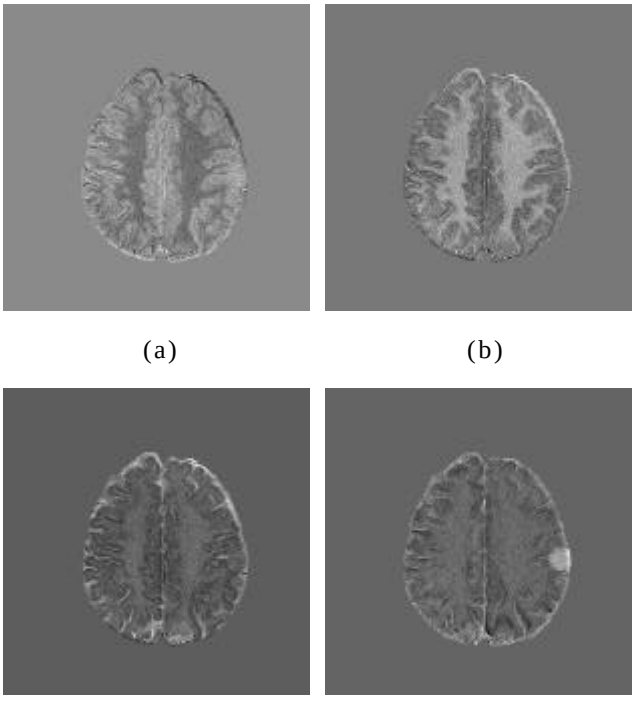


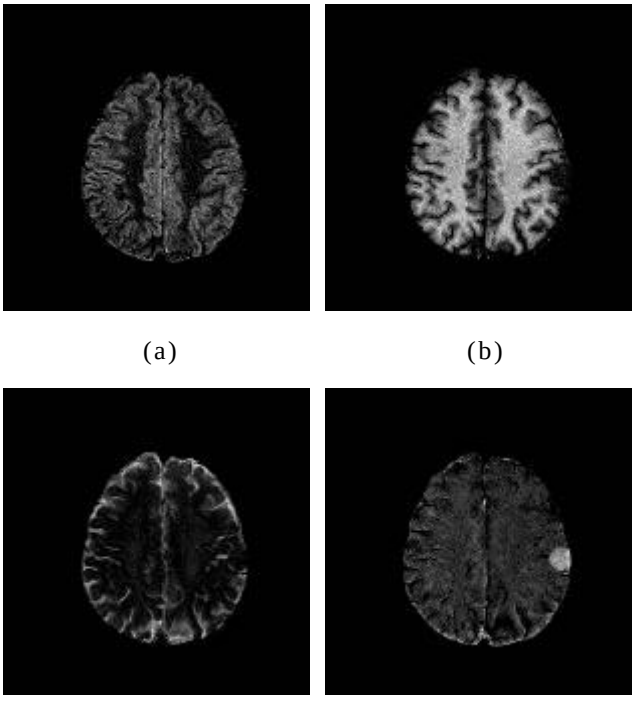
Fig. 2. Four bands radiance spectrum.



(a) (b)

(c) (d)

Fig. 3. Classification results of OSP. (a) gray matter; (b) white matter; (c) cerebral spinal fluid, (d) tumor.



(a) (b)

(c) (d)

Fig.4. Classification results of UOSP. (a) gray matter; (b) white matter; (c) cerebral spinal fluid, (d) tumor.

7. REFERENCES

[1] B. Johnston, M. S. Atkins, B. Mackiewich, and M. Anderson, "Segmentation of Multiple Sclerosis Lesions in Intensity Corrected Multispectral MRI," IEEE Transactions on Medical Imaging, vol. 15, no. 2, pp. 154-169, April 1996

[2] M. Stella Atkins and Blair T. Mackiewich, " Fully Automatic Segmentation of Brain in MRI," IEEE Transactions on Medical Imaging, vol. 17, no. 1, pp. 98-107, February 1998.

[3] Atam P. Dhawan, Aleksandar Zavaljevski, Alok Sarwal, Walliam S. Ball, "A System for MR Brain Imae Segmentation," 18th Annual International Conference of the IEEE Engineering in Medicine and Biology Society, pp. 732-733 Amsterdam 1996.

[4] Laurent Verard, Jalal Fadili, Su Ruan, and Daniel Bloyet, "3D MRI Segmentation of Brain Structures," 18th Annual International Conference of the IEEE Engineering in Medicine and Biology Society, pp. 1081-1082 Amsterdam 1996.

[5] Wilburn E.Reddick, John O. Glass, Edwin N. Cook , T. David Elkin, and Russell J. Deaton, "Automated Segmentation and Classification of Multispectral Magnetic Resonance Images of Brain Using Artificial Neural Networks," IEEE Transactions on Medical Imaging, vol. 16, no. 6, pp. 911-918 December 1997.

[6] Javad Alirezaie, M. E. Jernigan and C. Nahmias "Automatic Segmentation of Cerebral MR Images using Artificial Neural Networks," IEEE Transactions on Nuclear Science, vol. 45, no. 4, pp. 2174-2182, August 1998.

[7] J.S. Lin, R.M. Chen, Y.M. Huang, "Medical image segmentation using mean field annealing network," IEEE International Conference on Image Processing, vol. 2, pp. 855-858, October 1997.

[8] J.S. Lin, K.S. Cheng, C.W. Mao, "Multispectral magnetic resonance images segmentation using fuzzy Hopfield neural network," International Journal of Biomedical Computing, pp. 205-214, August 1996.

[9] J.S. Lin, K.S. Cheng, C.W. Mao, "Modified Hopfield neural network with fuzzy c-means technique for multispectral MR image segmentation," IEEE International Conference on Image Processing, vol. 1, pp. 327-330, September 1996.

[10] M. Stella Atkins and Blair T. Mackiewich, " Fully Automatic Segmentation of Brain in MRI," IEEE Transactions on Medical Imaging, vol. 17, no. 1, pp. 98-107 February 1998.

[11] Matthew C. Clark, Lawrence O. Hall, Dmitry B. Goldgof, Robert Velthuizen, F. Reed Murtagh, and S. Silbiger, "Automatic Tumor Segmentation Using Knowledge-Based Techniques," IEEE Transactions on Medical Imaging, vol. 17, no. 2, pp.187-201 April 1998.

[12] C.M. Wang, P.C. Chung, C.C. Chen, and C.-I. Chang, " A Generalized Orthogonal Subspace Projection Approach to MR Images Classification," 13th IPPR Conference on Computer Vision, Graphics and Image Processing, pp. , Taipei, Taiwan, Aug. 2000.

[13] C.M. Wang, P.C. Chung, C.-I. Chang. C.W. Yang, and C.C. Chen, "An Orthogonal Subspace Projection to

MR Images Classification," Fourth Asian Conference on Computer Vision Taipei, Taiwan, Jan. 2000.
[14] R.A.Schowengerdt, Techniques for Image Processing and Classification in Remote Sensing, New York, Academic Press, 1983.
[15] R.O. Duda and P.E. Hart, Pattern Classification and Scene Analysis, NY: John Wiley & Sons. 1973.
[16] H. Ren and C.-I Chang, "A generalized orthogonal subspace projection approach to unsupervised multispectral image classification," SPIE Conf. On Image and Signal Processing for Remote Sensing IV, vol. 3500, Spain, pp. 42-53, 1998.
[17] J. B. Adams and M. O. Smith, "Spectral mixture modeling: A new analysis of rock and soil types at the Viking Lander 1 site," J. Geophys. Res., vol. 91, pp. 8098–8112, July 1986.
[18] J.C. Harsanyi and C.-I Chang, "Hyperspectral image classification and dimensionality reduction: an orthogonal subspace projection approach," IEEE Trans. Geoscience and Remote Sensing, vol. 32, no. 4, pp. 779-784, July 1994.