

# SPECTRAL RENDERING OF GLOBAL ILLUMINATION MODELS WITH PRESENCE OF PARTICIPATING MEDIA

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## ABSTRACT

Many algorithms for synthesizing scenes with participating media assumed that absorption coefficients and scattering coefficients are all the same for every wavelength or every primaries of the RGB color model. In another word, the scattering coefficients and absorption coefficients are treated as wavelength independent. Hence, the color of participating media rendered is always white. Furthermore, since transmittances of participating media are nonlinear, it's hard to map the fraction of transmittances from spectra domain to RGB color domain. Thus a spectral rendering method is needed for synthesizing images with presence of participating media. In this paper, we present a spectral rendering method for synthesizing images containing participating media. The scattering coefficients and absorption coefficients are treated as wavelength dependent to meet the requirements of realistic world. Instead of using OPAW (One Path All Wavelength), the proposed method uses OPOW (One Path One Wavelength) and stochastic processes to render participating media even though they are colored.

*Keywords:* participating media, transmittance, important sampling, wavelength dependent, global illumination.

## 1. INTRODUCTION

The goal of rendering of global illumination models is to pursue synthesizing physically correct images. Physically correct rendering can be widely utilized to the applications of entertainment, indoor design, fire-fighting drill, military, aeronautics and educations[1]. Most works of realistic image synthesizing assuming that the environment of scenes is a vacuum, hence light color computation considers only light emitting and light reflecting. Apply the same color calculation method as a vacuum environment to the color calculation of environments with presence of participating media, significant color errors will occur. Thus rendering of scenes with presence of participating media is more complex than rendering of scenes in a vacuum.

Rushmeier *et al.* developed the first algorithm for rendering participating media--the zonal method[2]. The zonal method, which extended the classical radiosity method to render isotropically scattering participating media, computes light intensity by voxels in which the self-emitted and the scatter energy is stored. Another

stochastic method is based on Monte Carlo method also proposed by Rushmeier *et al.* later[3]. A 0.0~1.0 random number is picked to compare with the transmittance of participating media of point interacted with for deciding which light contribution should be computed. If the random number less than or equal to the transmittance, the contribution of surfaces is computed, otherwise the contribution of light interaction with participating media is computed. [4] extended his work of bidirectional path tracing[5] to render scenes including participating media. The bidirectional path tracing algorithm shoots two categories of rays from light sources and eyes, respectively. Shadow rays are the rays connecting points of the light paths and points of the eye paths. The contributions of all shadow rays are added together to contribute to the illumination of eyes. The rendering of participating media is picking a number under probability density function of

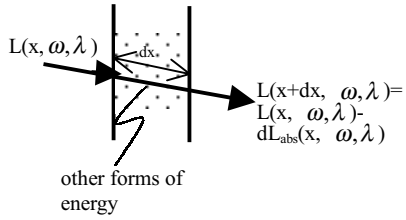
$\mathcal{K}e^{-\mathcal{K}s}$  ( $\mathcal{K}$  is the extinction coefficient of participating media), then compared with the distance  $d$  of the nearest surface along the ray. If  $d > s$  estimates the contribution of radiance at surfaces, otherwise, estimates the contribution of radiance within participating media. [6] was an extension of [7] which is a two pass algorithm where the first pass is the construction of two view-independent photon maps and the second pass is optimized rendering using these photon maps. As the radiance of scenes is stored in memory, the photon maps method consuming a great number of memories while the calculation is performed.

As the principles of light transport are based on the radiation and heat transfer theory[8], and discussions of radiation and heat transfer theory is wavelength dependent. Hence, the calculation of light color based on the spectral energy computation preserves the accuracy of results of calculation, otherwise, leads to errors of colors. Efforts had been devoted to the study of spectral color calculation such as [9][10][11][12][13]. But all of these works assuming that the rendering environments are vacuums do not meet the requirements of rendering participating media. Methods of rendering of participating media described in previous paragraph didn't consider that absorption coefficients and scattering coefficients of participating media are wavelength dependent[14][8][18]. Therefore the colors of participating media are always white, do not fit in with the requirements of realistic world.

In our work, we develop a method of rendering participating media by spectra which considers absorption coefficients and scattering coefficients of participating

media as wavelength dependent. In this paper, we will describe the method of rendering participating media by spectra to achieve the goal of properly rendering colored participating media.

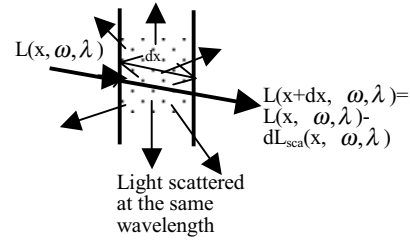
The remainders of this paper are organized as follows: section 2 discusses the basic concepts for rendering participating media and spectral rendering. The rendering method proposed in this paper is depicted in section 3. Results of our implementation are demonstrated in section 4. Conclusion and future works are given in section 5.



$$dL_{abs}(x, \omega, \lambda) = \sigma_a(x, \lambda)L_e(x, \omega, \lambda)$$

$\sigma_a(x, \lambda)$  : absorption coefficient

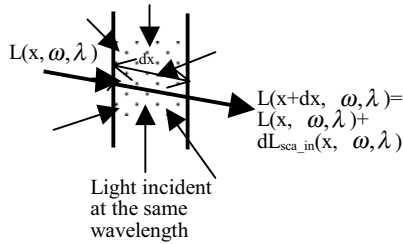
(a).Absorption in the participating media.



$$dL_{sca}(x, \omega, \lambda) = \sigma_s(x, \lambda)L_e(x, \omega, \lambda)$$

$\sigma_s(x, \lambda)$  : scattering coefficient

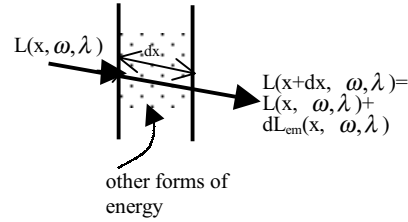
(b).Scattering out of the participating media.



$$dL_{sca\_in}(x, \omega, \lambda) = \sigma_s(x, \lambda)L_e(x, \omega, \lambda)$$

$\sigma_s(x, \lambda)$  : scattering coefficient

(c).Scattering into the participating media.



$$dL_{em}(x, \omega, \lambda) = \omega_a(x, \lambda)L_e(x, \omega, \lambda)$$

(d).Emission in the participating media.

Fig.1 Phenomena of light interacting with participating media

## 2.BACKGROUNDS

### 2.1 Light Interact with Participating Media

Light transport through participating media, some phenomena occur[15]. Such as absorption, in-scattering, out-scattering and emission of radiant energy as shown in Fig. 1. Summation of these four contributions of illumination yields[8]:

$$\frac{dL(x, \omega, \lambda)}{dx} = \sigma_a(x, \lambda)L_e(x, \omega, \lambda) + \sigma_s(x, \lambda) \int_{\Omega} L(x, \omega', \lambda) f(x, \omega', \omega) d\omega' - \sigma_a(x, \lambda)L(x, \omega, \lambda) - \sigma_s(x, \lambda)L(x, \omega, \lambda) \quad (1)$$

Where:

$\sigma_a(x, \lambda)$  is the absorption coefficient of participating media of wavelength  $\lambda$  at point  $x$ .

$\sigma_s(x, \lambda)$  is the scattering coefficient of participating media of wavelength  $\lambda$  at point  $x$ .

$L_e(x, \omega, \lambda)$  is the radiance of wavelength  $\lambda$  emitting at  $x$

toward direction  $\omega$ .

$L(x, \omega', \lambda)$  is the radiance of wavelength  $\lambda$  incident from  $\omega'$  direction arriving at point  $x$ .

$f(x, \omega', \omega)$  is the phase function, representing the fraction of radiance radiated toward point  $x$  from direction  $\omega'$  into  $\omega$  direction.

$\Omega$  represents the integration of direction over whole sphere of point  $x$ .

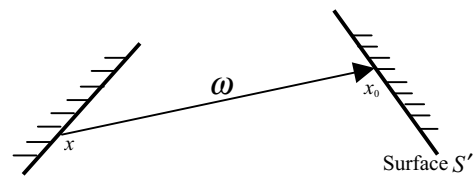


Fig. 2 A ray is cast from  $x$  along  $\omega$  direction, intersecting with nearest surface  $S'$  at  $x_0$ .

As shown in Fig. 2, a ray, which travels along the  $\omega$  direction, is shot from point  $x$  and intersects the nearest surface  $S'$  at point  $x_0$ . Integrating both sides of equation (1) along a path  $\omega$  from  $x_0$  to  $x$  yielding:

$$L(x, \omega, \lambda) = \int_{x_0}^x \tau(x', x, \lambda) \sigma_a(x', \lambda) L_e(x', \omega, \lambda) dx' + \int_{x_0}^x \tau(x', x, \lambda) \sigma_s(x', \lambda) \int_{\Omega} f(x', \omega', \omega) L(x', \omega', \lambda) d\omega' dx' + \tau(x_0, x, \lambda) L(x_0, \omega, \lambda) \quad (2)$$

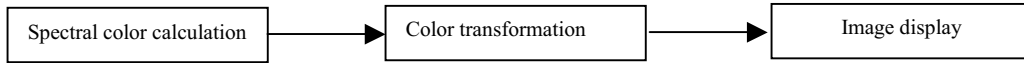
Equation (2) describes the radiance of point  $x$  incident

from  $\omega$  direction is composed of three contributions.  $\int_{x_0}^x \tau(x', x, \lambda) \sigma_a(x, \lambda) L_e(x', \omega, \lambda) dx'$  represents the radiance of self-emitting along the path from  $x_0$  to  $x$ .  $\int_{\Omega} \tau(x', x, \lambda) \sigma_s(x', \lambda) \int_{\Omega} f(x', \omega', \omega) L(x', \omega', \lambda) d\omega' dx'$  represents the radiance scattered into  $\omega$  direction along the path from  $x_0$  to  $x$ .  $\tau(x_0, x, \lambda) L(x_0, \omega, \lambda)$  is the contribution of radiance incident from  $\omega$  direction at point  $x_0$ .  $\tau(x_0, x, \lambda)$  is the transmittance of participating media, which is the ratio of radiance of wavelength  $\lambda$  at  $x_0$  and radiance of wavelength  $\lambda$  at  $x$  after light travels from  $x_0$  to  $x$ . In another word,  $\tau(p, q, \lambda)$  can be expressed as:

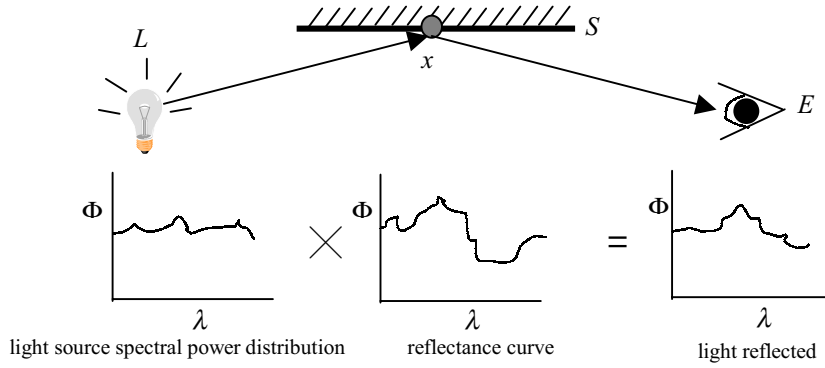
$$\begin{aligned} \tau(p, q, \lambda) &= \frac{\text{radiance of point } q}{\text{radiance of point } p} \\ &= e^{-\int_p^q (\sigma_a(x, \lambda) + \sigma_s(x, \lambda)) dx} \end{aligned} \quad (3)$$

spectral color calculation is completed, the colors of images are represented in the form of spectra and then a color transformation is performed to transform the spectra colors to RGB colors model. Lastly, the RGB color representation of images can be displayed on monitors. Fig. 3(b) shows the basic concept of spectral color calculation based on physical theory of light transport. The light source- $L$  emits energy toward all sphere direction around the light source.  $x$  is a point at the surface- $S$ , the color of light which reflect to eye- $E$  is obtained from the multiplication of the spectral power distribution of light source and the reflectance curve of point  $x$  at surface  $S$ . The following equation describes the basic concept of spectral color calculation.

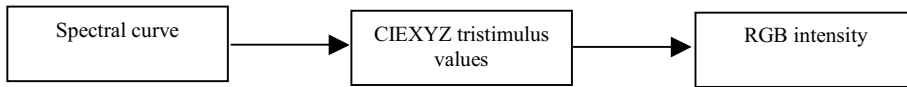
$$I_E(\lambda) = I_L(\lambda) \times \rho(\lambda)$$



(a).Processes of spectral color calculation for realistic image synthesis



(b). spectral color calculation for reflecting light



(c).Color transformation

Fig. 3 Concepts of spectral color calculation

## 2.2 Spectral Color Calculation

As described in previous section, using RGB model to calculate color for realistic image synthesizing will lead to errors of colors of synthesized images. A very limited portion of electromagnetic spectrum, which roughly ranged from 380nm to 780nm[16], dominates the color perception of human visual system. Thus the accurate color calculation technique should base on the calculation of spectral power over the visible portion of electromagnetic spectrum.

Fig. 3(a) shows processes needed for spectra color calculation and image display. Firstly, spectral color calculation is performed to synthesize images. After the

where  $I_E(\lambda)$  is the radiance of eye of wavelength  $\lambda$ ,  $I_L(\lambda)$  is the radiance of light source of wavelength  $\lambda$ ,  $\rho(\lambda)$  is the reflectance coefficient of surface  $S$  of wavelength  $\lambda$ .

Fig 3(c) shows steps of color transformation which transform the spectral curve to RGB intensity for display. Describes mathematically[17]:

$$\left. \begin{aligned} X &= k \int_{\lambda} \phi(\lambda) \bar{x}(\lambda) d\lambda \\ Y &= k \int_{\lambda} \phi(\lambda) \bar{y}(\lambda) d\lambda \\ Z &= k \int_{\lambda} \phi(\lambda) \bar{z}(\lambda) d\lambda \end{aligned} \right\} \quad (4)$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} XYZ \\ TO \\ RGB \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (5)$$

where  $k$  is a normalizing constant,  $\phi(\lambda)$  is spectral radiance,  $\bar{x}(\lambda)$ ,  $\bar{y}(\lambda)$  and  $\bar{z}(\lambda)$  are the so-called color matching functions for X, Y and Z primaries, respectively. Finally, the RGB intensity of the spectral curve is obtained from the multiplication of the XYZ to RGB matrix and the XYZ tristimulus.

### 3. THE METHOD OF SPECTRAL RENDERING FOR PARTICIPATING MEDIA

For simplicity, we assume that surfaces are either non-reflecting light sources or non-emitting reflector. Thus equation (2) can be written as either equation (6) or equation (7) depending on whether the surface is emitting or not.

$$L(x, \omega, \lambda) = \int_{x_0}^x \tau(x', x, \lambda) \sigma_a(x, \lambda) L_e(x', \omega, \lambda) dx' \quad (6)$$

$$L(x, \omega, \lambda) = \int_{x_0}^x \tau(x', x, \lambda) \sigma_s(x', \lambda) \int_{\Omega} f(x', \omega', \omega) L(x', \omega', \lambda) d\omega' dx' + \tau(x_0, x, \lambda) L(x_0, \omega, \lambda) \quad (7)$$

If surface  $S'$  is an emitting light source, equation (6) holds. Otherwise, if  $S'$  is a non-emitting reflector, equation (7) holds. Equation (7) can be further rewritten as:

$$\begin{aligned} L(x, \omega, \lambda) &= (1 - \tau(x_0, x, \lambda)) \cdot \\ &\frac{\int_{x_0}^x \tau(x', x, \lambda) \sigma_s(x', \lambda) \int_{\Omega} f(x', \omega', \omega) L(x', \omega', \lambda) d\omega' dx'}{(1 - \tau(x_0, x, \lambda))} \\ &+ \tau(x_0, x, \lambda) L(x_0, \omega, \lambda) \\ &= (1 - \tau(x_0, x, \lambda)) \cdot \frac{M}{(1 - \tau(x_0, x, \lambda))} + \tau(x_0, x, \lambda) \cdot S \end{aligned} \quad (8)$$

where

$$M = \int_{x_0}^x \tau(x', x, \lambda) \sigma_s(x', \lambda) \int_{\Omega} f(x', \omega', \omega) L(x', \omega', \lambda) d\omega' dx'$$

and  $S = L(x_0, \omega, \lambda)$ .

The first term of right hand side of equation (8) represents the radiance contribution of interaction with participating media. The last term of right hand side of equation (8) represents the radiance contribution of surfaces which light intersected with. The reason of the division by  $(1 - \tau(x_0, x, \lambda))$  after the multiplication by  $(1 - \tau(x_0, x, \lambda))$  of the first term of right hand side of equation (8) is to meet the requirement of stochastic process, which will be described below, of rendering of participating media.

In this paper, we assume that the participating media of the rendering environment is isotropically scattering and homogeneous. For isotropically scattering media equation (3) can be rewritten as:

$$\tau(x_0, x, \lambda) = e^{-(\sigma_a(\lambda) + \sigma_s(\lambda)) \cdot d} \quad (9)$$

where  $d$  is the distance from  $x_0$  to  $x$ . For homogeneous media:

$$f(x', \omega', \omega) = \frac{1}{4\pi} \quad (10)$$

By equation (8) the rendering of participating media can be decomposed as below[3]:

1. Shot a ray from  $x$  along  $\omega$  direction intersecting the nearest surface  $S'$  at  $x_0$ .
2. Choose a random number  $\xi$ .
3. If  $\xi \leq \tau(x_0, x, \lambda)$  then computes the contribution of  $S$ ,

$$\text{otherwise computes } \frac{M}{1 - \tau(x_0, x, \lambda)}.$$

A problem arises when the spectral rendering is applied to render participating media. If all the waves of visible portion of the spectra go the same path from eyes to light sources in one trial of cast ray,  $\tau(x_0, x, \lambda)$  can't compute properly.  $\tau(x_0, x, \lambda)$  is different from wavelengths to wavelengths is the reason for the problem depicted above. Thus a revised method is to compute the contribution of radiance of every wavelength separately. Technically the CIEXYZ tristimulus values of arbitrary point  $v$  can be computed as equation (4). For casting  $N$  different paths and every path carry only one wavelength which is uniformly selected, the CIEXYZ tristimulus values are obtained from the following equation:



(a). Without participating media (b). Rendering participating media with 5 samples of wavelength. (c). Rendering participating media with 80 samples of wavelength. The color of participating media is more blueish than the color of participating media in Fig. 4(b).

Fig. 4 Scenes rendered

$$\left. \begin{aligned} X &\approx k \sum_{i=1}^N \phi(\lambda_i) \bar{x}(\lambda_i) \\ Y &\approx k \sum_{i=1}^N \phi(\lambda_i) \bar{y}(\lambda_i) \\ Z &\approx k \sum_{i=1}^N \phi(\lambda_i) \bar{z}(\lambda_i) \end{aligned} \right\} \quad (11)$$

Under such condition every cast ray carry only one wavelength's power, that is One Path One Wavelength and abbreviated as OPOW.

#### 4. RESULTS

We have implemented the rendering method proposed in this paper. Due to the spectral rendering and rendering with participating media are both time consuming, 64 computers are off-line to render the scenes of Fig. 4(b) and Fig 4(c) corporately. The 64 computers are all the same, containing an Intel Pentium-II 300 MHZ CPU, 64 MB RAM and running with Microsoft Windows 98 OS. The results of spectral rendering with participating media are shown in Fig. 4. Fig. 4(a) is a scene without participating media. Fig. 4(b) and Fig. 4(c) are the scenes with presence of participating media, and synthesized with 5 and 80 samples of wavelengths respectively. Three images of Fig. 4 are all of the same size, 128X128. Taking a sampling rate of 12100 samples per pixel, the average rendering

shows the RMS error for scenes synthesized with various samples of wavelength. The image of using 80 samples of wavelength is the standard image for the calculation of RMS error. The curve of Fig. 6 is plotted using the values of the images of RMS errors rendered using 5, 10, 20, 40 and 80 samples of wavelength.

#### 5. CONCLUSION

Due to the difficulty of transformation of absorption coefficients and scattering coefficients from spectral domain to RGB color domain, rendering of participating media by spectra is needed to properly synthesize the scenes of participating media. A spectral rendering method for participating media is proposed in this paper. OPOW is used to carry the phenomenon of wavelength dependent of participating media out. Results show that colored participating media can be properly rendered using proposed method.

A major advantage of the proposed method is high parallel degree when parallel processing is proceeded. In general, a pixel is the basic unit of one task in parallel rendering, but one path is the basic unit of the proposed method. Since the path of one wavelength is not dependent on paths of the other wavelengths of the same pixel, and one path compute only one wavelength's data, the basic unit of one task can be reduced to one path in parallel rendering.

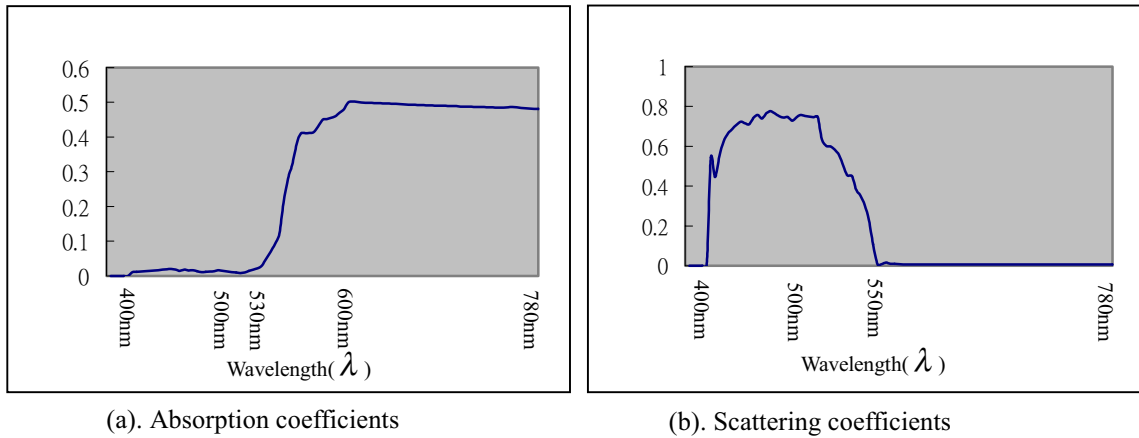


Fig. 5 Plots of absorption coefficients and scattering coefficients.

time of Fig. 4(b) and Fig. 4(c) are 462 minutes and 7892 minutes respectively. Because the samples of wavelength of Fig. 4(b) are less than the samples of wavelength of Fig. 4(c), the color of participating media of Fig. 4(c) are more blueish than the color of participating media of Fig. 4(b). By Fig. 4, we could speculate that rendering of participating media using RGB color model, which has only 3 color primaries, will lead to errors of color. Fig. 5 plots the absorption coefficients and scattering coefficients of various wavelengths of the participating media appeared in the scenes of Fig. 4(b) and Fig. 4(c). Fig. 5 shows that the scattering coefficients are greater from 400nm to 550nm and the absorption coefficients are greater from 600nm to 780nm. This indicates that the participating media rendered would be bluish. Fig. 6

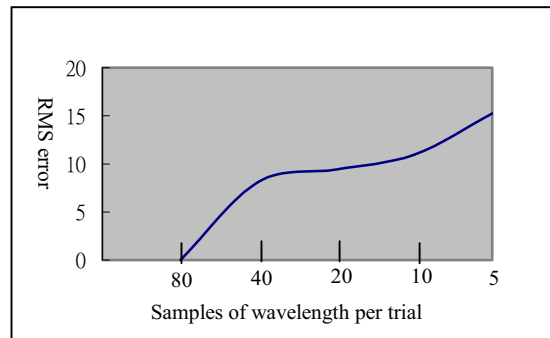


Fig. 6 RMS errors for various samples of wavelength.

Since the spectral rendering method is more time consuming than rendering using others color model, the

urgent work in future is to parallelize the method of spectral rendering. Other considerations about future works are to develop more efficient methods, such as applying important sampling for both light sources and wavelength selection.

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