

A Parallel Algorithm of Admission Control for Leaky Bucketed Multi-segment Arrival Envelope

Lain-Chyr Hwang¹, Yaotsu Chang², Chao-Wei Chou³

Department of {¹Electrical Engineering, ²Applied Mathematics, ³Information Management}

I-Shou University, Taiwan

E-mail: {¹lain, ²ytchang, ³cwchou}@isu.edu.tw

Abstract

Owing to the importance of quality of service (QoS) in the future communication networks, in spite of wired networks or wireless ones, the control mechanisms are important issues. The connection admission control (CAC) is the basic and necessary control mechanism for provision of QoS. This paper studies the CAC for a very general arrival envelope, called leaky bucketed multi-segment arrival envelope. By theoretical approach, we derived the minimum delay that can be provided by the network and developed an efficient algorithm to calculate the minimum delay. With the result of the paper, many arrival processes can be exactly and fast calculated the minimum delay to assist the decision of CAC.

Keywords: Quality of service, connection admission control, leaky bucketed multi-segment arrival envelope.

1: Introductions

The future communication network should guarantee Quality of service (QoS) to the users. To promise the guarantee, the control mechanisms are necessary. One of the important control mechanisms is the connection (or call) admission control (CAC), which is the first gate of the network to provide QoS.

To satisfy the QoS of various applications in the Internet, the IETF has developed two service architectures, i.e., (integrated services, IntServ) [2] and (differentiated services, DiffServ) [1]. IntServ utilizes the RSVP (Resource Reservation Protocol) [3] to make the reservation of network resources for the connections. IntServ possesses good granularity but poor scalability. DiffServ solves the problem of scalability and suits to the Internet in the near future, although its granularity is not perfect.

The CAC is associated to the scheduling algorithms of networks. For providing QoS, Sariowan [6] proposed a scheduling algorithm called SCED (Service Curve based Earliest Deadline first policy), which is also a kind of EDF. Pyun [5] also studied the SCED, where the service curve is with single rate and is leaky bucketed. In [5, 6], both papers suggested the algorithm in [4] to lower the complexity in CAC. The problem studied in [4] is: The arrival envelope of a new connection is a piecewise linear curve that claims a maximum delay

requirement. Under this situation, should the network accept this new connection or not? [4] employed an algorithm to find two parameters in series, and then the minimum delay for the new connection provided by the network is the maximum value of the two parameters.

Owing to the arrival envelope discussed in [4] is not general enough, this paper generalize the arrival envelope to a kind of leaky bucketed multi-segment arrival envelope. As we know, the leaky bucketed arrival envelope is the most general arrival envelope. Our proposed leaky bucketed multi-segment arrival envelope is more general than the general leaky bucketed arrival envelope that is with single rate in general. Except a more arrival envelope is discussed, this paper also derived the minimum delay for the leaky bucket multi-segment arrival envelope and proposed a new algorithm that finds the two parameters in parallel, so our proposed algorithm will be more efficient than that in [4]. In this paper, the theoretical approach is employed to prove that our derivation of the minimum delay is correct and our proposed algorithm is efficient.

The remaining parts of the paper are organized as follows. Section 2 introduces the leaky bucketed multi-segment arrival envelope. The theoretical derivation of the minimum delay is approached in Section 4. Section 5 illustrates the parallel algorithm and its complexity. Finally, the conclusions are given in Section 6.

2: Leaky Bucketed Multi-segment Arrival Envelope

The multi-segment arrival envelope discussed in [4] is a continuous function starting from zero [4, Definition 1]. However, a very popular arrival envelope is leaky bucketed and discontinuous, so this paper discusses the leaky bucketed multi-segment arrival envelope, which is expressed as

$$A(t) = \begin{cases} 0, & t = 0, \\ \sum_{i=1}^N \rho_i : (\tau_i, \sigma_i), & t > 0. \end{cases} \quad (1)$$

where N is the number of segment of the arrival envelope $A(t)$, segment i of $A(t)$ is denoted by $\rho_i : (\tau_i, \sigma_i)$ that indicates the segment starts from (τ_i, σ_i) and is with

slope ρ_i , $\sum_{i=1}^N \rho_i : (\tau_i, \sigma_i)$ is employed to indicate a curve consisting of N segments $\rho_i : (\tau_i, \sigma_i)$, $i = 1, \dots, N$, and $\rho_i \geq 0$, $\rho_N = 0$, $\tau_{i+1} > \tau_i$, $\tau_1 = 0_+ = 0 + \delta$, $\delta \rightarrow 0$, $\sigma_{i+1} \geq \sigma_i$, and $\sigma_1 \geq 0$. The general mathematical expression of segment i $\rho_i : (\tau_i, \sigma_i)$ is

$$\rho_i : (\tau_i, \sigma_i) = \rho_i(t - \tau_i) + \sigma_i, \tau_i \leq t < \tau_{i+1}, i = 1, \dots, N, (2)$$

and we let $\tau_{N+1} = \infty$. In general case, the slope of segment N is 0, which means that the arrival of the connection will terminate at some time, so the arrival envelope will keep at a constant. The slopes of the other segments, except the segment N , may also be 0, such that the function of the leaky bucketed multi-segment arrival envelope is not necessarily one-to-one. Furthermore, because of the shaping of the leaky bucket, the leaky bucketed multi-segment arrival envelope may be discontinuous. In the most common case, the origin of the arrival envelope is discontinuous ($\sigma_1 > 0$), but after that the arrival envelope is continuous. However, it is not necessary to be so, i.e., there may be discontinuous points in the successive segments. If there are no discontinuous point in the successive segments and $\sigma_1 = 0$, the arrival envelope is not leaky bucketed.

The leaky bucketed multi-segment arrival envelope results in that the available curve [4] is discontinuous and is with multi-segment. Here, we used curve instead of function, because a discontinuous impulse in a curve will be though as a vertical line segment. The expression of the available curve is

$$F(t) = \sum_{i=1}^M \eta_i : (\alpha_i, \beta_i), t \geq 0 (3)$$

where M is the number of the available curve, $\alpha_{i+1} > \alpha_i$, $\alpha_1 = 0$, $\beta_{i+1} \geq \beta_i$, $\beta_1 = 0$, $\eta_i \geq 0$, and η_i , $i > 1$, is not constrained.

3: The Minimum Delay

The core in the connection admission control for EDF scheduler is to find the minimum delay that can be provided by the network system, e.g. routers. Hence, this section is devoted to finding the minimum delay offered by the network system when a new connection is with leaky bucketed multi-segment arrival envelope and asks the admission to the network. If the minimum delay offered by the network is less than or equal to the delay requirement claimed by the connection, the new connection is granted to connect and transmit. In this paper, although we approach the derivation of minimum delay of the leaky bucketed multi-segment arrival envelope by the observations of [4] after some enhancements, we illustrate the correction of the observations for the leaky bucketed multi-segment arrival envelope by conscientious theoretical derivation.

With our results, the study of [4] becomes a special case of our study.

Because the contact of the available curve F and the arrival envelope A may be a line segment, not necessary be a point only as specified in [4], this section will prove that the end points of the contact must be the convex point of the available curve or the concave point of the arrival envelope in the Theorem 1. Furthermore, we will derive the minimum delay and prove that it is really the minimum right shift to make the arrival envelope wholly under the available curve in Theorem 2.

Because both the available curve and the arrival envelope have discontinuous points, this paper extend the definitions of convex point and concave points to include the discontinuous points provided that we defined the discontinuous impulse as a vertical line segment. In this way, the slope of the line segment of the upward impulse is $+\infty$ and that of the downward impulse is $-\infty$. According to this definition, to decide a flex point is either a convex point or a concave point can be done by the ordinary definition. As a consequence, the starting point of an upward impulse is a convex point and the terminal point of an upward impulse is a concave point. Similarly, the starting point of a downward impulse is a concave point and the terminal point of a downward impulse is a convex point. In order to be consistent with the counting of flexion points and line segments of continuous functions, the two end points is considered as a flexion point. In this way, the number of the flexion points of a curve is the same as the number of the line segments, excluding the vertical line segment, of the curve. Note that the origins of the available curve and the arrival envelope are both convex points.

Now, we can specify and prove the Theorem 1. The main action in the connection admission control is to put the arrival envelope A wholly under the available curve F and under this constraint to find the minimum shift of A . If the minimum shift meets the minimum delay requirement of the connection, then the system accepts the connection. Otherwise, the new connection is rejected. To process the finding of the minimum shift, it can be processed by shifting the arrival envelope A from the most right-hand side toward the left until the arrival envelope A contacts with the available curve F . At this moment, the distance between the origin of the arrival envelope A and the origin of the coordinates is the minimum delay.

It is pointed out that the contact of the arrival envelope A and the available curve F must be a convex point of F or a concave point of A . Here, we modify this concept, because the contact of the arrival envelope A and the available curve F may be a line segment. The modified specification and the proof are given in Theorem 1.

Theorem 1: The end points of the contact of the arrival envelope A and the available curve F must be the convex point of the available curve or the concave

point of the arrival envelope. If the contact is not a line segment but a point, it can be thought as a line segment with zero length and the two end points becomes a point.

Proof: Prove it by contradiction. Assume the end points of contact are neither the convex point of the available curve nor the concave point of the arrival envelope. Let an end point be denoted by P , and denote the segments of A (F) at the left-hand side and right-hand side of P by P_{A-} (P_{F-}) and P_{A+} (P_{F+}), respectively, which may be a same segment, and denotes the slope of P_{A-} (P_{F-}) and P_{A+} (P_{F+}) by $A'(P_{A-})$ ($F'(P_{F-})$) and $A'(P_{A+})$ ($F'(P_{F+})$), respectively. Because P is not the concave point of the arrival envelope A , we have

$$A'(P_{A-}) \leq A'(P_{A+}) \quad (4)$$

Similarly, P is not the convex point of the available curve F , so

$$F'(P_{F-}) \geq F'(P_{F+}) \quad (5)$$

Three conditions are considered to prove that (4) and (5) can not be satisfied simultaneously under the constraint that the A must be under the F .

1. P is the right end point of the contact segment: Because the A must be under the F , it implies $A'(P_{A-}) = F'(P_{F-})$ and $A'(P_{A+}) < F'(P_{F+})$. If (4) $A'(P_{A-}) \leq A'(P_{A+})$ is satisfied, then $F'(P_{F-}) = A'(P_{A-}) \leq A'(P_{A+}) < F'(P_{F+})$, i.e., $F'(P_{F-}) < F'(P_{F+})$ to conflict with (5). On the other hand, if (5) $F'(P_{F-}) \geq F'(P_{F+})$ is satisfied, then $A'(P_{A+}) < F'(P_{F+}) \leq F'(P_{F-}) = A'(P_{A-})$, i.e., $A'(P_{A+}) < A'(P_{A-})$ to conflict with (4).
2. P is the left end point of the contact segment: Because the A must be under the F , it implies $A'(P_{A-}) > F'(P_{F-})$ and $A'(P_{A+}) = F'(P_{F+})$. If (4) is satisfied, then $F'(P_{F-}) < A'(P_{A-}) \leq A'(P_{A+}) = F'(P_{F+})$, i.e., $F'(P_{F-}) < F'(P_{F+})$ to conflict with (5). On the other hand, if (5) is satisfied, then $A'(P_{A+}) = F'(P_{F+}) \leq F'(P_{F-}) < A'(P_{A-})$, i.e., $A'(P_{A+}) < A'(P_{A-})$ to conflict with (4).
3. The contact is a single point P : Because the A must be under the F , it implies $A'(P_{A-}) > F'(P_{F-})$ and $A'(P_{A+}) < F'(P_{F+})$. If (4) is satisfied, then $F'(P_{F-}) < A'(P_{A-}) \leq A'(P_{A+}) < F'(P_{F+})$, i.e., $F'(P_{F-}) < F'(P_{F+})$ to conflict with (5). On the other hand, if (5) is satisfied, then $A'(P_{A+}) < F'(P_{F+}) \leq F'(P_{F-}) < A'(P_{A-})$, i.e., $A'(P_{A+}) < A'(P_{A-})$ to conflict with (4).

Consequently, (4) and (5) can not be satisfied simultaneously, so the end points of the contact of the arrival envelope A and the available curve F must be the convex point of the available curve or the concave point of the arrival envelope.

From Theorem 1, it is proven that even for the discontinuous multi-segment arrival envelope and

available curve the phenomenon observed is the same as that of [4] provided that the impulses are taken as vertical line segments. The result of Theorem 1 facilitates the approach to find the minimum delay. It reveals that only the convex points of available curve and the concave points of the arrival envelope need to be checked to find the minimum delay. The minimum delay is found to be the maximum value of the shifts of arrival envelope when the convex points of available curve contact the arrival envelope or the concave points of arrival envelope contacts the available curve. The mathematic expression of the minimum delay is given below. Moreover, the next theorem will prove that the minimum delay obtained is really the minimum to make the arrival envelope wholly under the available curve. Before that, we define key contact point in Definition 1 and prove a corollary from Theorem 1.

Definition 1: The key contact point is the point of convex point of available curve or the point of concave point of arrival envelope that decide the minimum delay. When the key contact point contacts the other curve, the shift of the arrival envelope is the minimum delay.

It may be not unique for the key contact point, i.e., the number of key contact points is larger than or equal to 1. The minimum delay found from these key contact point are equal, so the minimum delay can be found from any key contact point.

Corollary 1: (1) If a key contact point P is a concave point of arrival envelope A , then $A'(P_{A+}) = 0$ and $F'(P_{F-}) = 0$ can not be satisfied simultaneously.

(2) If a key contact point P is a convex point of available curve F , then $F'(P_{F+}) > 0$ and $F(x) \geq F(y)$, where y is the horizontal coordinate of P and x is the horizontal coordinate of any convex point of F after P .

Proof: (1) If $A'(P_{A+}) = 0$ and $F'(P_{F-}) = 0$ can be satisfied simultaneously, the curve of A can be further moved left and does not cause the curve A over the curve F . It means the P is not a key contact point to conflict the premise that P is a key contact point, so $A'(P_{A+}) = 0$ and $F'(P_{F-}) = 0$ can not be satisfied simultaneously when a concave point of arrival envelope P is a key contact point.

(2) First, we prove that $F(y)$ is not less than the value of any convex point of F after P . Because A is a non-decreasing function, it implies $A(x) \geq A(y)$ for $x > y$. Because the curve F is above the curve A , it yields $F(x) \geq A(x)$ for all x such that $F(x) \geq A(y) = F(y)$ for $x > y$. The horizontal coordinate of a convex point of F after P is larger than y , so the value of the convex point is larger than or equal to $F(y)$. Next, we prove that $F'(P_{F+}) > 0$. Because A is a non-decreasing function, it implies $A'(P_{A+}) \geq 0$. And, it is obvious that $F'(P_{F+}) < 0$ is impossible, otherwise the curve A will be above the curve F after the point P , so it is sufficient to prove $F'(P_{F+})$

$\neq 0$ to illustrate $F'(P_{F+}) > 0$. Because the slope of the last segment of A is 0 and the network capacity is generally constant, the slope of the last segment of F is positive. If $F'(P_{F+}) = 0$, then there exists convex points of F , otherwise the slope of the last segment of F is impossible to be positive. Because the value of any convex point is not less than the value of F at P , the right end point Q of the segment P_{F+} is a convex point. Because the curve A is not above the curve F , the slope of the segment between P and Q of A is 0. It implies the convex point P of F is not a key contact point, because the most right point of the curve A with the equal value of the convex point P is a point after Q rather than P . It is a contradiction, so $F'(P_{F+}) \neq 0$. Consequently, the only possible condition is $F'(P_{F+}) > 0$.

From Theorem 1, Corollary 1-(2), and [4, (10), (11)], we can express the minimum delay for the leaky bucketed multi-segment arrival envelope under a EDF scheduler as

$$d_{\min} = \max(m_x, m_y) \quad (6)$$

$$m_x = \max\{u - \max\{A^{-1}(F(u))\}: u \in V_{F+}, F(u) < A(\infty)\} \quad (7)$$

$$m_y = \max\{F^{-1}(A(a)) - a: a \in A_A\} \quad (8)$$

where f^{-1} is the pseudo inverse function of f . When f is not a one-to-one function, f^{-1} is a set and is expressed as [4, (6)]

$$f^{-1}(y) = \{x: f(x) = y\}, \quad (9)$$

$\max\{f^{-1}(y)\}$ is the maximum value among the set $f^{-1}(y)$, V_{F+} is the set of all the convex points of F with positive slope of its right-hand side segment, A_f is the set of all the concave points of a function f . The f in (9) may be A or F . Because the slope of a segment of A may be 0 in this paper, the pseudo inverse function of A is necessary when the inverse function of A does not exist. With regard to the available curve F , it may be a many-to-one function by nature.

In the following, we will prove the minimum delay derived in (6) is the minimum shift to cause the arrival envelope wholly under the available curve. First, a lemma is proven.

Lemma 1: The shift amount found by (6)-(8) results in all the concave points of arrival envelope A is under the available curve F and all the convex points of available curve F is above the arrival envelope A .

Proof: From (7), for a $u \in V_{F+}$, $F(u) < A(\infty)$, there exists $F(u) = A(u - s)$, where s is the shift amount at this moment of the contact of A and F and $s \leq m_x \leq d_{\min}$. Because A is a non-decreasing function, $F(u) = A(u - s) \geq A(u - d_{\min})$, which implies this convex point

of F must not be under the curve A when the shift is d_{\min} . That is, all the convex points in the set V_{F+} all above the curve A . Furthermore, all the values of the other convex points of F are not less than the minimum of the values of the convex points in V_{F+} , so the points in V_{F+} are also above the curve A .

Theorem 2: The minimum delay obtained by (6) is really the minimum right shift amount to cause the arrival envelope wholly under the available curve.

Proof: We will first illustrate the arrival envelope A is wholly under the available curve F by Lemma 1, and then illustrate the minimum delay obtained by (6) is really the minimum shift. Assume the curve A is above the curve F after a contact point P when the shift is d_{\min} . From Lemma 1, it shows that after the point P , F must surpass the curve A before the next concave point of A or the next convex point of F . Assume Q is the next concave point of A or the next convex point of F , which is nearer to the point P . Because the curve A is above the curve F just after P , it has $F'(P_{F+}) < A'(P_{A+})$. From Lemma 1, the curve A has no concave points between P and Q , such that curve A is a convex curve between P and Q , i.e., the slope is non-decreasing, and the curve F has no convex points between P and Q , such that curve F is a concave curve between P and Q , i.e., the slope is non-increasing. These kinds of slopes of A and F cause that the curve F can not surpass the curve A before the point Q , so there is no such point P , i.e., all the points of curve A is below the curve F . Next, assume that d_{\min} is not the minimum shift and there exists a shift $d' < d_{\min}$. For $m_x \geq m_y$, the key contact point is a convex point of F . Assume a key contact point is at $(u_x, F(u_x))$ of curve F and $(u_x - m_x, A(u_x - m_x))$ of curve A , then $u_x - m_x = \max\{A^{-1}(F(u_x))\}$. When $d' < d_{\min} = m_x$, one has $u_x - d' > u_x - m_x = \max\{A^{-1}(F(u_x))\}$ such that $A(u_x - d') \neq F(u_x)$. Because A is a non-decreasing function, it has $F(u_x) = A(u_x - m_x) < A(u_x - d')$, which indicates the curve F is under the curve A when the shift is d' . Hence, it is impossible to have a shift $d' < d_{\min}$ for $m_x \geq m_y$. On the other hand, for $m_x < m_y$, assume P is a contact point that is a concave point of A , when the shift is d_{\min} . If P of A can be move more left, then it must be $F'(P_{F-}) \leq 0$, otherwise the P of A will be over the curve F after the shift left. Because A is a non-decreasing function, it must be $F'(P_{F+}) \geq 0$ to avoid the curve F below the curve A after the point P . For $F'(P_{F+}) = 0$, it must be $A'(P_{A+}) = 0$, and from Corollary 1-(1), one has $F'(P_{F-}) \neq 0$. Consequently, it must be $F'(P_{F-}) < 0$, which indicates the P of F is a convex point. Then it results in $m_x = m_y$, which conflicts with the premise $m_x < m_y$, so $F'(P_{F+}) \neq 0$, i.e., $F'(P_{F+}) > 0$. However, because $F'(P_{F-}) \leq 0$, the point P of F is still a convex point and it still conflicts with the premise $m_x < m_y$. Therefore, the curve A can not be moved left and it can be $F'(P_{F-})$

> 0 . In a word, no matter the key contact point is the concave point of A or the convex point of F , the d_{\min} is the minimum shift amount and the curve A can not be move left further.

4: The Parallel Algorithm

```

 $m_x = 0;$ 
for  $i = 2$  to  $m$  //Move the pointer of  $\Theta$ 
{ //Move the pointer of  $W_A$ 
   $j = \max\{k: A(a_k) < \{F(V_{F+})\}_i\};$ 
   $m_x = \max(m_x, \{T_{F+}\}_i - A_{-j}(\{F(V_{F+})\}_i));$ 
}

 $m_y = 0;$ 
 $q = 1;$ 
 $a = \{A_A\}_q;$ 
 $i = 0;$ 
 $v' = 0;$ 
do while ( $v' \leq \max\{F(V_{F+})\}$ )
{  $k = \min\{j: \{I\}_j > i\};$  //Move the pointer of  $\Theta$ 
   $v'' = \{F(V_{F+})\}_k;$ 
   $E = A_{F+} \cap [v', v''];$ 
  do while ( $a \leq \max\{A_A\}$  and  $A(a) < v''$ )
  {  $q++;$  //Move the pointer of  $A_A$ 
     $a = \{A_A\}_q;$ 
    //Move the pointer of  $E$ 
     $l = \max\{p: \{E\}_p < A(a)\};$ 
     $m_y = \max(m_y, F_{-(i+l-1)}(A(a)) - a);$ 
  }
   $i = k;$ 
   $v' = v'';$ 
}

 $d_{\min} = \max(m_x, m_y);$ 

```

Figure 1 The parallel algorithm for the minimum delay.

Utilize an import new observation, the algorithm of [4, Fig. 5], which is with complex $O(K^2N)$, can be simplified to the algorithm in Figure 2 and to have the low complex the same as that of the algorithm of [4, Fig. 17]. The new algorithm developed in this paper is called the parallel algorithm for the minimum delay. Furthermore, the parallel process makes the process to calculate the minimum delay faster. The new algorithm calculate the m_x and the m_y in parallel instead of in series as [4, (10)-(12) and Fig. 17].

Now, describe the important new observation. To find m_y , the concave point(s) of arrival envelop A contacts the available curve F to cause A wholly under F . Assume a contact point is at $(\tau, F(\tau))$ of F and $(\tau + m_y, A(\tau + m_y))$ of A when curve A shifts m_y . Because A is a non-decreasing function, it must be $F(t) \geq F(\tau)$ for all $t > \tau$. If not so, assume $F(\gamma) < F(\tau)$ for $t = \gamma > \tau$. Because $F(\tau) = A(\tau + m_y) < A(\gamma + m_y)$, it results in an unreasonable phenomenon $F(\gamma) < A(\gamma + m_y)$, i.e., the arrival envelope A is above the available curve F . Owing to this

observation, we know that the key contact point of a concave point of A must be on the line segment with convex point P of F as starting point and the height of the key contact point is not higher than the heights of all the convex points of F after P . In another word, the key contact point is lower the lowest convex points of F on the right hand side of the key contact point and the height of the lowest convex point is higher than the point P . In a word, the height of the key contact point is between the height of a certain convex point P of F and the height of the lowest convex point among the convex right to P .

The parallel algorithm for the minimum delay is described in Figure 1, where f_{-j} is the inverse function of the j -th segment of function f . There are two pointers respectively pointing to the sets A_A and Θ , where Θ is a triplet as

$$\{\Theta\}_i = (\{I\}_i, \{T_{F+}\}_i, \{F(V_{F+})\}_i), \quad (10)$$

V_{F+} is sorted in ascending order, T_{F+} is the set of the times corresponding to the elements in V_{F+} , I is the set of the original index in the curve F corresponding to the elements in V_{F+} , and $\{E\}_i$ is the i -th element of the set E . That is, Θ saves not only the values of F , but also the corresponding time and index that indicates the order number of the convex point with positive slope of right-hand side in F . The two pointers advance one at a time without ever returning. It results in the complexity $O(|V_{F+}| + |W_A|) = O(M + N)$ at the worst case to find m_x , where W_f is the set of all flexion points of f and $|E|$ is the number of the elements of the set E .

To find m_y , the complex to locate the convex point of F , where the key contact point (a concave point of A) is on the segment starting form this convex point, is also $O(M + N)$. After locate the convex point, it needs to find the segment that contacts with the key contact point. For the worst case, all the concave points of F are checked and the complexity is $O(|A_F|)$. As a consequence, the overall complexity in find m_y is $O(|V_{F+}| + |W_A| + |A_F|) = O(M' + N)$ at the worst case, where M' is the number of the segments of F . The complexity of the parallel algorithm at the worst case is the same as the series algorithm in [4].

However, two point emphasized to illustrate the parallel algorithm can find the minimum delay faster than the series algorithm does. First, the two parameters should be calculated in series by the series algorithm, but they can be found in parallel by the parallel algorithm. It can save almost half of the processing time. Secondly, the complexity of the parallel algorithm found above is for the worst case. In general case, not all the convex points and all the line segments should be checked. In fact, after checking a convex point P , the next convex point needs to be checked is the lowest convex point Q after P . Furthermore, only the segments between P and Q need to be checked. In general, most convex points and segments of F can be skipped. It can also save almost half of the processing time.

5: Conclusions

Because the importance of the CAC to the provision of QoS, this paper developed a parallel algorithm to find the minimum delay in the CAC for the leaky bucketed multi-segment arrival envelope, which is a very general arrival envelope and can take the arrival envelope in [4] and the leaky bucketed arrival envelope as its special cases. Furthermore, the parallel algorithm developed in this paper is more efficient than the series algorithm in [4].

Owing the popularity of the scheduling mechanism EDF, the CAC collocates with EDF is a topic worth further study. What factors affect the decision of CAC except the arrival envelope? And how they affect the CAC? These topics are our future works.

References

- [1] S. Blake, D. Black, M. Carlson, E. Davies, Z. Wang, and W. Weiss, "An architecture for differentiated services," IETF RFC 2475, Dec. 1998.
- [2] R. Braden, D. Clark, and S. Shenker, "Integrated services in the Internet architecture: an overview," IETF RFC 1633, June 1994.
- [3] R. Braden, L. Zhang, S. Berson, S. Herzog, and S. Jamin, "Resource ReSerVation Protocol (RSVP)--Version 1 Functional Specification," IETF RFC 2205, Sep. 1997.
- [4] V. Firoiu, J. Kurose, and D. Towsley, "Efficient admission control of piecewise linear traffic envelopes at EDF schedulers," IEEE/ACM Trans. Networking, vol. 6, no. 5, pp. 558-570, Oct. 1998.
- [5] K. Pyun, and H.K. Lee, "The SCED service discipline with $O(1)$ complexity for deadline calculation," IEICE Trans, Comm., vol. E85-B, no. 5, pp. 1012-1019, May 2002.
- [6] H. Sariowan, R.L. Cruz, and G.C. Polyzos, "Scheduling for Quality of Service Guarantees via Service Curves," in Proc. IEEE ICCCN, Las Vegas, Nevada, pp. 512-520, Sep. 20-23, 1995.