Combined Power Allocation and Diversity for Multi-Cell Networks

Jui Teng Wang
Graduate Institute of Communication Engineering
National Chi Nan University, Puli, Nantou, Taiwan
jtwang@ncnu.edu.tw

ABSTRACT

An efficient method for deriving the optimal feasible power and weight of combined power allocation and diversity is proposed in this paper. Instead of solving a constrained optimization problem where both the variables of power and the variables of weight are involved, this method simply solves a set of equations where only the variables of power are involved.

1: INTRODUCTIONS

Power control and diversity are two effective techniques to enhance the signal to interference and noise ratio (SINR) for wireless networks. Power control can be realized by allocating different power levels to the links that have different link gains. In general, the link with larger link gain is supposed to have smaller power level and the link with smaller link gain is supposed to have larger power level. Nevertheless, larger power level may cause more interference to other users. Therefore, an optimum transmitter power control was proposed in [3] to achieve the balancing of the carrier to interference ratio (CIR) and the minimal power consumption.

On the other hand, diversity exploits the random nature of radio propagation by finding independent (or at least highly uncorrelated) signal paths for communication. If one radio path undergoes a deep fade, another independent path may have a strong signal. By having more than one path to select from, the SINR at the receiver can be improved. The antenna array is an example of the space diversity, which uses a beamformer to increase the SINR for a particular direction. As reported in [1], the minimum variance distortionless response (MVDR) beamformer can maximize the SINR for a fixed power allocation.

In [1], a joint optimal power control and beamforming that uses the MVDR beamformer was proposed for the wireless networks. It was shown that the power and weight of the proposed algorithm in [1] converge to the optimal power and weight that minimize the total power consumption. However, this algorithm needs to solve a constrained optimization problem where both the variables of power and the variables of weight are involved. Apparently, this may consume much computational resource, so we propose in this paper an efficient method to derive the optimal power and weight for combined power allocation and diversity. This method is executed by solving a set of equations where only the variables of power are involved.

The rest of this paper is organized as follows. Section 2 describes the investigated system model. Sections 3 and 4 propose the combined power allocation and diversity and the iterative algorithm, respectively. Numerical results are presented in Section 5. Finally, we state our conclusions in Section 6.

2: SYSTEM MODEL

We consider the reverse link of a wireless network and assume there are N active base stations in the network with K_i users connected to base station i, $1 \le i \le N$. Notice that K_i is constant during the process of power control and all users use the same frequency band with bandwidth f_w. Assume the receiver of each base station exploits a diversity scheme with M diversity branches and neglect the thermal noise. The pair (i, k) is used to denote the kth user connected to the ith base station. Consider user (i, k). Let P_{ik}, r_{ik}, s_{ik}, and w_{ik}^j represent its transmitting power, transmitting rate, message signal and weight on the jth diversity branch, respectively. Also, let x_i^j represent the total received signal at the jth diversity branch of the ith base station. Then the output of the combiner for user (i, k) is given by

$$y_{ik} = \sum_{j=1}^{M} (w_{ik}^j)^* x_i^j \ .$$

Note that

$$x_i^j = \sum_{n=1}^{N} \sum_{l=1}^{K_n} \sqrt{P_{nl}}\, a_{(n,l)i}^j s_{nl} \ ,$$

where $a_{(n,l)i}^j$ denotes the array gain between user (n, l) and base station i on the jth diversity branch. Furthermore, the received signal of user (i, k) at the jth diversity branch is denoted by d_{ik}^j, which can be expressed as

$$d_{ik}^j = \sqrt{P_{ik}}\, a_{(i,k)i}^j s_{ik} \ .$$

The received SINR (per bit) for user (i, k) is given by

$$E_{ik} \equiv (\frac{E_b}{I_0}) = \frac{E(\mathbf{w}_{ik}^H \mathbf{d}_{ik} \mathbf{d}_{ik}^H \mathbf{w}_{ik}) / r_{ik}}{[E(\mathbf{w}_{ik}^H \mathbf{x}_i \mathbf{x}_i^H \mathbf{w}_{ik}) - E(\mathbf{w}_{ik}^H \mathbf{d}_{ik} \mathbf{d}_{ik}^H \mathbf{w}_{ik})] / f_w} \ ,$$

$$= \frac{\mathbf{w}_{ik}^H \Omega_{ik} \mathbf{w}_{ik}}{\mathbf{w}_{ik}^H \Phi_i \mathbf{w}_{ik} - \mathbf{w}_{ik}^H \Omega_{ik} \mathbf{w}_{ik}} \frac{f_w}{r_{ik}},$$

where $\mathbf{w}_{ik} = \{w_{ik}^j\}$, $\mathbf{x}_{ik} = \{x_{ik}^j\}$, $\mathbf{d}_{ik} = \{d_{ik}^j\}$, $\Phi_i = E(\mathbf{x}_i \mathbf{x}_i^H)$ and $\Omega_{ik} = E(\mathbf{d}_{ik} \mathbf{d}_{ik}^H)$. Note that Φ_i and Ω_{ik} are the correlation matrixes for the total received signal and the received signal of interest, respectively. Assume the message signals are uncorrelated with zero mean and $E(|s_{ik}|^2) = 1$, then we have

$$\Omega_{ik} = P_{ik} \mathbf{a}_{(i,k)i} \mathbf{a}_{(i,k)i}^H$$

and

$$\Phi_i = \sum_{n,l} P_{nl} \mathbf{a}_{(n,l)i} \mathbf{a}_{(n,l)i}^H,$$

where $\mathbf{a}_{(i,k)i} = \{a_{(i,k)i}^j\}$. Therefore, we have

$$E_{ik} = \frac{P_{ik} \mathbf{w}_{ik}^H \mathbf{a}_{(i,k)i} \mathbf{a}_{(i,k)i}^H \mathbf{w}_{ik}}{\sum_{n,l} P_{nl} \mathbf{w}_{ik}^H \mathbf{a}_{(n,l)i} \mathbf{a}_{(n,l)i}^H \mathbf{w}_{ik} - P_{ik} \mathbf{w}_{ik}^H \mathbf{a}_{(i,k)i} \mathbf{a}_{(i,k)i}^H \mathbf{w}_{ik}} \frac{f_w}{r_{ik}}.$$

The MVDR combining is accomplished by minimizing the interference and noise subject to $\mathbf{w}_{ik}^H \mathbf{a}_{(i,k)i} = 1$. It was shown that the weight for the MVDR combining is given by [8]

$$\tilde{\mathbf{w}}_{ik} = \frac{(\Phi_i - \Omega_{ik})^{-1} \mathbf{a}_{(i,k)i}}{\mathbf{a}_{(i,k)i}^H (\Phi_i - \Omega_{ik})^{-1} \mathbf{a}_{(i,k)i}}. \qquad (1)$$

As a result, the received SINR with the MVDR combining for user (i, k) can be expressed as

$$E_{ik} = P_{ik} (\mathbf{a}_{(i,k)i}^H (\Phi_i - \Omega_{ik})^{-1} \mathbf{a}_{(i,k)i})(f_w / r_{ik}). \qquad (2)$$

Let Q_{ik} denote the SINR requirement of user (i, k), then it needs to find the power and weight that satisfy $E_{ik} \ge Q_{ik}$, and such power and weight are called the feasible power and weight.

3: COMBINED POWER ALLOCATION AND DIVERSITY

The optimal feasible power and weight for combined power allocation and diversity can be obtained by solving the following optimization problem:

$$\min_{\mathbf{W},\mathbf{P}} \sum_{i,k} P_{ik}$$
$$\text{subject to } E_{ik} \ge Q_{ik} \text{ for all } i \text{ and } k,$$

where $\mathbf{P} = \{P_{ik}\}$ and $\mathbf{W} = \{\mathbf{w}_{ik}\}$. In this section, we will show that with the MVDR combining, the optimal feasible power and weight satisfy a set of equations that makes the received SINR equal to the SINR requirement for each user. To facilitate the discussions, we use the following notations. For two sets $\mathbf{A} = \{A_{ik}\}$ and $\mathbf{B} = \{B_{ik}\}$, $\mathbf{A} \le \mathbf{B}$ and $\mathbf{A} = \mathbf{B}$ mean $A_{ik} \le B_{ik}$ and $A_{ik} = B_{ik}$ for all i and k, respectively. Moreover, let $\mathbf{E} = \{E_{ik}\}$ and $\mathbf{Q} = \{Q_{ik}\}$, we have the theorem below.

Theorem 1: If there exist $\mathbf{P}$ and $\mathbf{W}$ such that $\mathbf{E} \ge \mathbf{Q}$, then there exist $\hat{\mathbf{P}} \le \mathbf{P}$ and $\hat{\mathbf{W}} = \{\hat{\mathbf{w}}_{ik}\}$ such that $\hat{\mathbf{E}} = \mathbf{Q}$, where $\hat{\mathbf{w}}_{ik} = \tilde{\mathbf{w}}_{ik}(\hat{\mathbf{P}})$.

The above theorem can be proved by using the facts that the MVDR combining maximizes the received SINR for a fixed power allocation and the optimal feasible power is the power that makes the received SINR equal to the SINR requirement for each user.

According to Theorem 1, if there exist feasible power and weight for all users, the optimal feasible weight is determined by the MVDR combining so that the following set of equations holds.

$$E_{ik} = P_{ik} (\mathbf{a}_{(i,k)i}^H (\Phi_i - \Omega_{ik})^{-1} \mathbf{a}_{(i,k)i})(f_w / r_{ik}) = Q_{ik}$$
$$\text{for all } i \text{ and } k. \qquad (3)$$

In practice, we can first solve (3) for the optimal feasible power and then use the optimal feasible power to calculate the optimal feasible weight by (1).

The computational complexity for solving (3) can be reduced by solving only N equations rather than $\sum_{i=1}^{i=N} K_i$ equations. Consider users (i, k) and (i, j). According to (3), we have

$$P_{ik} (\mathbf{a}_{(i,k)i}^H (\Phi_i - \Omega_{ik})^{-1} \mathbf{a}_{(i,k)i})(f_w / r_{ik}) = Q_{ik}, \qquad (4)$$

$$P_{ij} (\mathbf{a}_{(i,j)i}^H (\Phi_i - \Omega_{ij})^{-1} \mathbf{a}_{(i,j)i})(f_w / r_{ij}) = Q_{ij}. \qquad (5)$$

Dividing (4) by (5) yields

$$\frac{P_{ik}}{P_{ij}} = \frac{(\mathbf{a}_{(i,j)i}^H (\Phi_i - \Omega_{ij})^{-1} \mathbf{a}_{(i,j)i})(f_w / r_{ij}) Q_{ik}}{(\mathbf{a}_{(i,k)i}^H (\Phi_i - \Omega_{ik})^{-1} \mathbf{a}_{(i,k)i})(f_w / r_{ik}) Q_{ij}}.$$

Assume user (i, r) is the representative user connected to the ith base station and let

$$P_{ik} = f_{ik} P_{ir} \text{ for all } k, \qquad (6)$$

where

$$f_{ik} = \frac{(\mathbf{a}_{(i,r)i}^H (\Phi_i - \Omega_{ir})^{-1} \mathbf{a}_{(i,r)i})(f_w / r_{ir}) Q_{ik}}{(\mathbf{a}_{(i,k)i}^H (\Phi_i - \Omega_{ik})^{-1} \mathbf{a}_{(i,k)i})(f_w / r_{ik}) Q_{ir}}. \qquad (7)$$

We can first solve the following N equations for P_{ir}:

$$E_{ir} = P_{ir} (\mathbf{a}_{(i,r)i}^H (\Phi_i - \Omega_{ir})^{-1} \mathbf{a}_{(i,r)i})(f_w / r_{ir}) = Q_{ir}$$
$$\text{for all } i, \qquad (8)$$

and then calculate P_{ik} for all k by (6) and (7).

4: ITERATIVE ALGORITHM

An iterative algorithm for solving (3) is proposed in this section. For convenience, we let $\mathbf{W}^m = \{\mathbf{w}_{ik}^m\}$, $\mathbf{P}^m = \{P_{ik}^m\}$ and $\mathbf{E}^m = \{E_{ik}^m\}$ denote the weight set, transmitter power set and the set of SINR in the mth discrete time, respectively. Also, we let $P_{\max}$ represent the maximum power level for all users and assume the array gains are real numbers.

Step 1: Let $m = 0$ and measure the array gains $a_{(i,k)i}^j$ for all i and k.

Step 2: Let $P_{ik}^0 = P_{\max}$ for all i and k.

Step 3: Derive the correlation matrix Ω_{ik} by

$$\Omega_{ik} = P_{ik}^{m}\mathbf{a}_{(i,k)i}\mathbf{a}_{(i,k)i}^{H}.$$

Step 4: Determine the correlation matrix Φ_i by

$$\Phi_i = E(\mathbf{x}_i\mathbf{x}_i^{H}).$$

Step 5: Execute the MVDR combining by letting $\mathbf{w}_{ik}^{m} = \dfrac{(\Phi_i - \Omega_{ik})^{-1}\mathbf{a}_{(i,k)i}}{\mathbf{a}_{(i,k)i}^{H}(\Phi_i - \Omega_{ik})^{-1}\mathbf{a}_{(i,k)i}}$ and evaluate the received SINR after combining.

Step 6: Adjust the power level as

$$P_{ik}^{m+1} = \min(P_{\max}, \frac{Q_{ik}}{E_{ik}^{m}} * P_{ik}^{m}). \tag{9}$$

Step 7: Let $m = m+1$ and go to Step 3.

Note that (9) is the same as the power adjustment formula of the distributed constrained power control (DCPC) algorithm in [6]. Using the results in [1] and [6], we can prove that the power level obtained from the proposed algorithm converges to the solution of (3) if the solution of (3) meets the power constraint.

5: NUMERICAL RESULTS

In this section, we study a wireless network that is composed of 19 hexagonal cells. We assume that each base station is equipped with 4 diversity branches and the locations of the users are uniformly distributed over the cell area. The frequency bandwidth, the transmitting rate and the SINR requirement are set to 1 MHz, 10 Kbps and 10 dB, respectively. The power level for each user is constrained to $P_{\max}$ and a user is connected to the base station with the largest link gain to minimize its transmitting power level. The array gain $a_{(n,l)i}^{j}$ is modeled as $a_{(n,l)i}^{j} = S_{(n,l)i}^{j} / D_{(n,l)i}^{\alpha}$, where $S_{(n,l)i}^{j}$ is the shadowing factor between user (n, l) and base station i on the jth branch, $D_{(n,l)i}$ is the distance between user (n, l) and base station i, and α is a constant that models the large scale propagation loss. The shadowing factor models power variation due to shadowing. $S_{(n,l)i}^{j}$, $1 \le j \le M$, $1 \le n,i \le N$ and $1 \le l \le K_n$, are assumed to be independent, log-normal random variables with 0 dB expectation and σ standard deviation. The parameter value of σ in the range of 4-10 dB and the propagation constant α in the range of 3-5 usually provide good models for urban propagation [7]. In our simulations, we choose $\alpha = 4$ and $\sigma = 8$ dB.

In Figures 1 and 2, we plot the power level (normalized to $P_{\max}$) and the received SINR, respectively, against the number of iterations executed in the proposed iterative algorithm for several users. In these figures, we assume there are 8 users per cell. It can be seen from Figure 1 that the power levels are decreasing for all users. Furthermore, we found from Figure 2 that the received SINR converges to the SINR requirement for all users.

Figure 3 demonstrates the average power per user, which is normalized to $P_{\max}$, against the number of users per cell for the following combinations: {MVDR combining, optimal power allocation} (our proposed scheme), {equal weight combining, optimal power allocation}, {MVDR combining, adaptive power allocation} and {equal weight combining, adaptive power allocation}, where the equal weight combining lets $w_{ik}^{j} = 1$ for $1 \le j \le M$, the optimal power allocation assigns the power level such that the received SINR is equal to the SINR requirement for each user and the adaptive power allocation lets the power level be inversely proportional to the received SINR. It can be seen from this figure that the combination with the MVDR combining and optimal power allocation gives the minimal power consumption. The reason is our proposed scheme can provide the optimal feasible power and weight that minimize the power consumption as implied by Theorem 1.

6: CONCLUSIONS

We have proposed in this paper a method to derive the optimal feasible power and weight for combined power allocation and diversity. Instead of solving a constrained optimization problem, this method simply solves a set of equations that makes the received SINR equal to the SINR requirement for each user. Simulation results show that the power and weight obtained from this set of equations can minimize the power consumption. To reduce the computational complexity, we further proposed another method where the number of equations can be reduced from the number of users to the number of base stations. It is clear that this method can save the computational resource.

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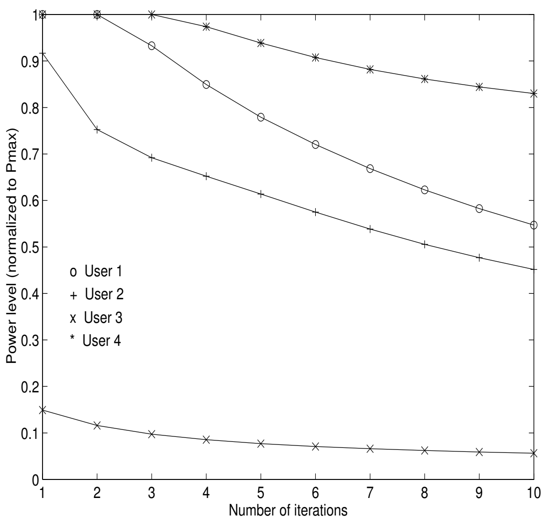

Figure 1. The power level (normalized to P_{max}) against the number of iterations executed in the proposed iterative algorithm.

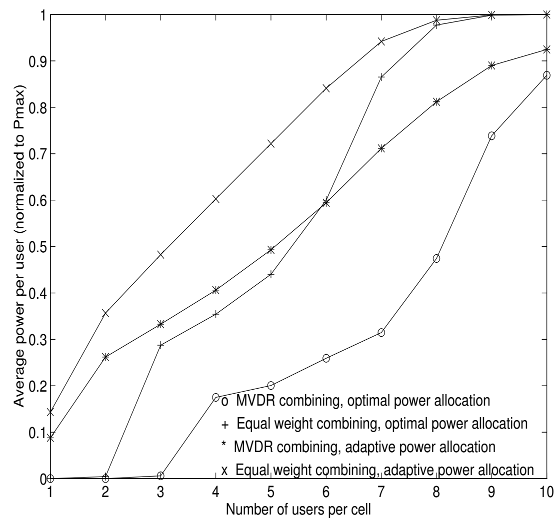

Figure 3. The average power per user (normalized to P_{max}) against the number of users per cell.

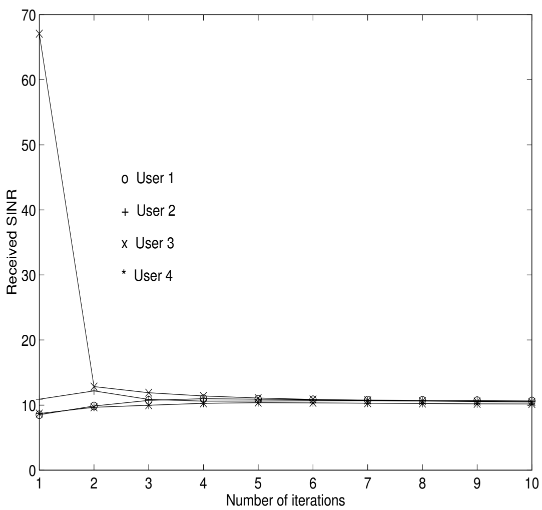

Figure 2. The received SINR against the number of iterations executed in the proposed iterative algorithm.