# An Image Resolution Enhancement Algorithm Using Undecimated Wavelet Transform

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## ABSTRACT

*The image resolution reconstruction is a partial technology in many application including medical imaging, satellite imaging, and video applications. And Wavelet transform is the most exciting development in the last decade. Due to the wavelet representation has characteristics of the efficient time-frequency localization and the multi-resolution analysis, the wavelet transforms are suitable for processing the image resolution enhancement. Therefore, this method focuses on wavelet-based image resolution enhancement and proposes a framework of image resolution enhancement.*

*This paper proposes a wavelet-domain image resolution enhancement algorithm which is based on the estimation of detail wavelet coefficients at high resolution scales. The method exploits shape function according to wavelet coefficient correlation in a local neighborhood and employs undecimated discrete wavelet transform to estimate the unknown detail coefficients. The simulation results objectivity show that the proposed method is considerably superior to conventional image interpolation techniques.*

## 1: INTRODUCTION

The image resolution reconstruction is proved to be useful in many practical cases where multiple frames of the same scene can be obtained, including medical imaging, satellite imaging, and video applications. For surveillance or forensic purposes, a digital video recorder (DVR) is currently replacing the CCTV system, and it is often needed to magnify objects in the scene such as the face of a criminal or the license plate of a car. Another application is conversion from an NTSC video signal to an HDTV signal since there is a clear and present need to display a SDTV signal on the HDTV without visual artifacts.

Some existing image magnification methods such as bilinear and spline interpolations generate blurred images since they ignore the image content and interpolation by the neighborhood. Image interpolation generates a larger image from a smaller size image. Two different types of interpolation can be classified as enlargement and zooming. In the enlargement case, the method decomposes an image into a collection of basis functions and then stretches those basis functions to enlarge the image. In this category, some well known methods are pixel replication, zero padding in the frequency domain and zero padding in the wavelet domain. The enlargement method works very well when the image resembles the basis functions. The second type of interpolation is zooming. In the zooming case, one would like to add in extra detail as the image is enlarged. This type of interpolation requires an image model, in order to predict lost detail. In this category, there is a range of different interpolation techniques. Some well known techniques are linear interpolation and cubic interpolation. More recently interesting results have been obtained using wavelets to model real images and in particular, to model smoothness found in real images. By studying regularity measures of the image, details are added to the image by constraining the image to the known regularity. The troubling problem of image interpolation is that the image magnifies many times and loses the sharpness of the picture. Many researches discuss debluring and approximately divide into two types: one is using the unsharp masking that is a common method. And the other is modeling the edges and filtering the image with nonlinear filters to enhance the high frequency coefficients to sharpe the edge. In this paper, we propose a wavelet based method which estimates the high-frequency wavelet coefficients to sharpe the edge and reconstruct the high resolution image.

Wavelet transform is useful to image resolution enhancement. A common feature is the assumption that the image pixel values are seemed to be the low-pass filtered subband coefficients of a wavelet-transformed high-resolution image and used to estimate the detail wavelet coefficients in high-pass subbands. The trivial approaches approximate the HR image by filling the unknown subbands with zeros and applying the inverse wavelet transform. More sophisticated methods have attempted to estimate these unknown detail wavelet coefficients.

The detail coefficients are estimated using the coarser subband coefficients. In [1,2], Only the significant coefficients are estimated by the evolution of wavelet coefficients. Significant magnitude coefficients correspond to image discontinuities and consequently only the portrayal of edges in the enhanced resolution image can be targeted while smoother regions are not accommodated. In [3, 4], the sign estimation of descendant coefficients in wavelet subbands is generally assumed to be random and is relied upon. HMT-based methods model the unknown wavelet coefficients as belonging to mixed Gaussian distributions which are symmetrical around the zero mean [5, 6]. HMT-based method models the most probable state for the wavelet coefficients to be estimated. The posterior state is using state-transition information from lower-resolution scales and the coefficient estimates are randomly generated using this distribution. We utilize existing correlations that between intra and inter subbands to estimate and enhance the detail wavelet coefficients.

The proposed method uses a wavelet-domain image resolution enhancement algorithm which is based on the estimation of detail wavelet coefficients. According to wavelet coefficient correlation in a local neighborhood, the proposed method exploits shape function to enhance the intensity of discontinuity and then employs undecimated discrete wavelet transform to estimate the unknown detail coefficients. Finally, the proposed method uses quadtree weight function to modify the estimated detail wavelet coefficients.

This paper is organized as follows. Section 2 is a review of previous work and introduces some of the definitions and concepts. Section 3 presents our proposed method to enhance image resolution in wavelet-domain. Section 4 shows simulation results and section 5 concludes.

## 2. BACKGROUND

Interpolation is often utilized in enlarging image, because it is easy and has a low computational complexity. However it produces obvious artifacts. Some researches in enlarging image use Markov model to predict unknown pixel, these methods drawbacks are the requirement of training data and computation complexity. In order to avoid the above lack, we use wavelet transform to obtain high-resolution image. Following, we introduce the detail of resolution enhancement, wavelet transform and the concepts of wavelet-based image resolution enhancement.

### 2.1 Resolution Enhancement

The approach to image interpolation, which we call prediction of image detail, can be explained in Fig. 1. The high resolution image is represented as the signal X at the input to the filter bank. The low resolution image, more coarsely sampled image, is the result of a low-pass filtering operation followed by decimation to give the signal A. The low-pass filter, L, represents the effects of the image acquisition system. The original high-resolution image X filters with the high pass filter H to obtain the detail signals D. If we have perfect reconstruction filter bank, we can reconstruct the original image, it would then be possible to. In resolution enhancement, it’s assumption that the signal A is known and the detail signal D is unknown, and the estimation of the signal D is a critical point to decide the visual quality image. Many recent researches discuss the estimation technologies.

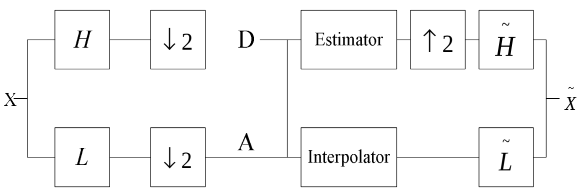



Fig. 1: Problem Model

In the problem of resolution enhancement, most of the difficulties arise in areas around edges and sharp changes. Around edges, many resolution enhancement methods tend to smooth and blur image detail. Fortunately, most of the signal information is often carried around edges and areas of sharp changes and can be used to predict these missing details from a sampled image.

### 2.2 Wavelet Transform

Due to the wavelet representation has characteristics of the efficient time-frequency localization and the multi-resolution analysis; the wavelet transforms are suitable for processing image resolution enhancement. The wavelet transform is useful to image resolution enhancement because it hierarchically decomposes the data into different subbands. This structure efficiently represents edges in the high frequency subbands, as well as smooth regions in the low frequency subbands.

### 2.2.1 Discrete Wavelet Transform

Discrete wavelet transform (DWT) decompose the input image in order to decorrelate the image data. The DWT can be thought of as a linear transform from input data to output data. The output consists of two parts each of which is half the size of the input signal. One part represents a low frequency, low-resolution result and the input; the other part represents high frequency differences between the low-resolution results of the input.

The first level of the wavelet transform represents applying DWT once vertically on each column of the image, and once horizontally on each row. The resulting image would then consist of four subbands representing the four different orientations, as shown in Fig. 2. The LL subband represents a low-resolution result of the image, while the other subbands represent high frequency detail information necessary to reconstruct the image. Each successive level of the wavelet transform operates only on the LL subband data produced by the previous level. This is because only the LL subband data still contains highly correlated pixels. Figure 3 shows three levels of wavelet transform applied to an image.

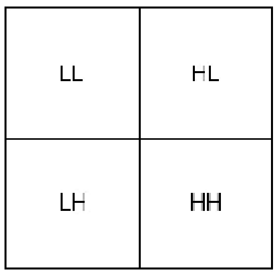



Fig. 2: Subbands of different orientation after applying one level of wavelet transform

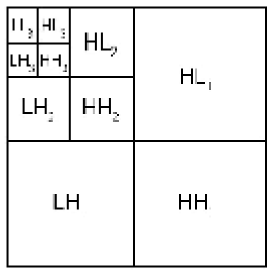



Fig. 3: Subbands formed three level wavelet transform

### 2.2.2 Undecimated Wavelet Transform

Unlike the DWT, which downsamples the approximation coefficients and detail coefficients at each decomposition level, the UWT does not incorporate the downsampling operations. Thus, the approximation coefficients and detail coefficients at each level are the same length as the original signal.

The undecimated discrete wavelet transform gives a denser approximation to the continuous wavelet transform than the approximation provided by the orthonormal discrete wavelet transform (DWT). The structure of a two level undecimated wavelet transform is shown in Fig. 4(a). For the inverse, we invert both the even part and the odd part, and then average the result, as in Fig. 5. The two level undecimated inverse wavelet transform is shown in Fig. 4(b).

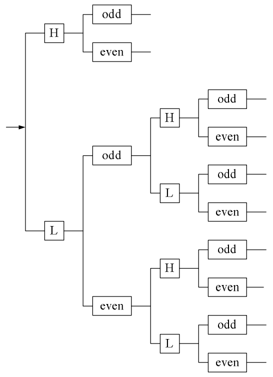



Fig. 4(a): The two-level undecimated discrete wavelet transform

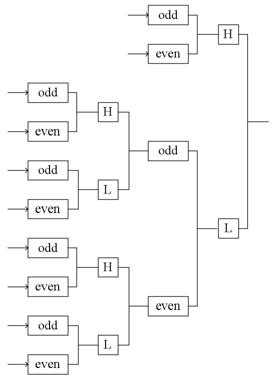



Fig. 4(b): The two-level undecimated inverse discrete wavelet transform

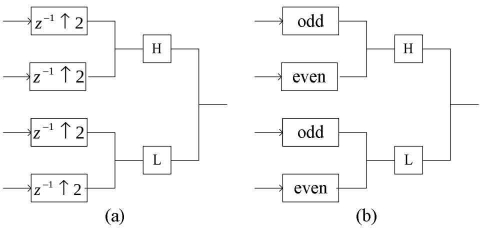



Fig. 5: Two equivalent representations for the building block for the undecimated inverse discrete wavelet transform

## PROPOSED ALGORITHM

When the image's resolution magnify, image may be blur and obtain poor visual quality. This proposed algorithm enhances the visual quality of enlarging image in wavelet domain. Because the image magnify, the blurred edge is an obvious problem. The proposed method suggests shape function to enhance edge information and undecimated discrete wavelet transform to estimate detail wavelet transform coefficients. Furthermore, quadtree weight function is adopted to adjust the detail wavelet transform coefficients. The visual quality of enlarging image is improved. The detailed descriptions of shape function, undecimated discrete wavelet transform and quadtree weight function are as follows.

### 3.1 Shape Function

The decimated discrete wavelet transform coefficients of real-world images tend to be separated to four subbands in Fig. 6. After wavelet transform, the full image energy will be congregated in low frequency. The coefficients imply noise and edge in high frequency subbands (LH, HL, and HH).

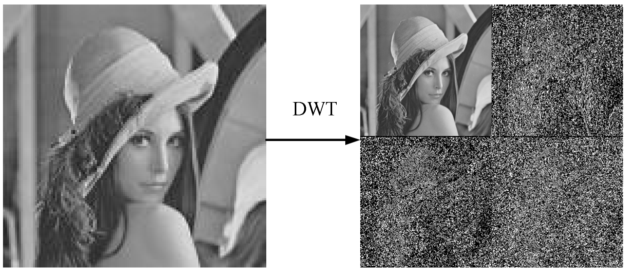



Fig. 6: Discrete Wavelet Transform

The algorithm treats the image (X) as the LL subband coefficients of an image that decomposed by decimated wavelet transform. And then the image (X) decomposes by undecimated wavelet transform to estimate the coefficients in LL, HL, and HH. If the method directly decomposes the image (X) by undecimated wavelet transform to estimate the coefficients in LL, HL, and HH. The discontinuations of the reconstructive image is blur and unobvious therefore the proposed algorithm uses shape function to enhance the edge of LL in order to preserve more edge information while estimating the coefficients in LL, HL, and HH. The more edge information preserve, the more clarity advance.

Because 2-D wavelet transform use the direction of filtering (horizontal and vertical). We also use separate function to enhance the significant information such as discontinuations in the original image. Shape function formula is shown below：

$$Var_v = [2x(m,n) - (x(m,n+1) + (m,n-1))]/l \quad (3)$$

$$Var_h = [2x(m,n) - (x(m+1,n) + x(m-1,n))]/l \quad (4)$$

$$z(m,n) = x(m,n) + Var_v + Var_h \quad (5)$$

In the equations (3), (4), and (5), $x(m,n)$ is the current pixel to be modified using neighborhood correlation and $l$ is a scale factor. $z(m,n)$ is the enhancement pixel. $Var_v$ and $Var_h$ are the vertical and horizontal variances and both of them affect the enhancement intensity. The large $Var_h$ variance means that there is a horizontal boundary and then shape function heavily enhances the edge intensity. On the contrary, the small $Var_h$ variance means that there is no horizontal boundary and then shape function lightly enhances the edge intensity.

After enhance the shape information in original image, the enhancement image decomposes by undecimated wavelet transform to estimate the high frequency coefficients. And then the high-resolution image is obtained by applying the inverse wavelet transform and use the quadtree weight function to adjust the estimated detail wavelet transforms.

### 3.2 Undecimated Discrete Wavelet Transform

The traditional DWT has exerted a remarkable influence on several signal processing applications such as denoising, estimation, and compression. However, in signal denoising, the DWT is known to create artifacts around the discontinuities of the input signal. These artifacts degrade the performance of the threshold-based denoising algorithm. It has been shown that many of the artifacts could be suppressed by a redundant representation of the signal.

The undecimated discrete wavelet transform (UDWT) differs from the traditional DWT because it does not employ a decimator after filtering. This is also known as the redundant or translation invariant DWT. The absence of a decimator leads to a redundant input signal representation. This makes a denser approximation to the continuous wavelet transform than that of the DWT. The translation invariant property of the UDWT makes it preferable for use in various signal processing applications, as it relies heavily on spatial information.

Let L(z) and H(z) respectively the low-pass (LP) and high-pass (HP) filters, let X(z) be the low-resolution image.

$$LL_0(z) = L_{col}(z)L_{row}(z)X(z) \quad (6)$$

$$HL_0(z) = L_{col}(z)H_{row}(z)X(z) \quad (7)$$

$$LH_0(z) = H_{col}(z)L_{row}(z)X(z) \quad (8)$$

$$HH_0(z) = H_{col}(z)H_{row}(z)X(z) \quad (9)$$

This algorithm applies the wavelet transform but omits down-sampling. Figure 7 shows that the 'lena' test image is decomposed by undecimated discrete wavelet transform into four subbands. The most edge informations are the coefficients in the high frequency subbands therefore the proposed method employs the four subbands that decomposed by undecimated discrete wavelet transform to estimate the high frequency subbands coefficients and composes these subbands to high-resolution image by common discrete wavelet transform. The invest discrete wavelet transform illustrats in Fig. 8. The interpolative reconstruction is achieved by the synthesis wavelet filter pairs and, as a consequence, the selection of a mother wavelet which better models the regularity of natural images yields better results. For example, the well-established wavelet, Daubechies 9/7 wavelets, are expected to generate better results than Haar wavelets.

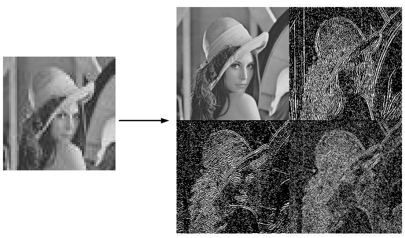


Fig. 7: Undecimated Discrete Wavelet Transform

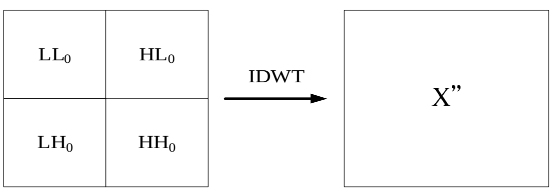



Fig. 8: Inverse Discrete Wavelet Transform

### 3.3 Quadtree Weight Function

The wavelet coefficients of natural images have two primary properties: non-Gaussianity and persistency. Persistency refers to the observation that the magnitudes of wavelet coefficients corresponding to the quad-tree. This property is exploited in state-of-the-art wavelet-based image compression algorithms such as Shapiro's embedded zerotree wavelet (EZW) and Said and Pearlman's set partitioning in hierarchical trees (SPIHT).

The property of wavelet transform is the fact that the transform of a typical signal consists of a small number of large coefficients and a large number of small coefficients. Most wavelet coefficients have small values and contain very little signal information. However, a few wavelet coefficients have large values that represent significant signal information. In quadtree framework, each of these large coefficients also has a large child, since the children wavelet basis functions simply divide up the spatial support of the parent. In this paper, to determine the state of parent coefficients, the coefficients in the subband are divided into two types: one is significant type and the other is insignificant type. According to this property, each wavelet coefficient is modeled in the two states: 'significant' corresponding to wavelet component containing significant information or 'insignificant' representing coefficients with little signal energy.

After undecimated wavelet transform, the high-frequency coefficients is roughly estimated. In improvement of the perceptual quality, the adjustment of those high-frequency wavelet coefficients is important. The enhanced low-resolution image $\hat{X}$ is decomposed by discrete wavelet transform into four subbands that includes $LL_1$, $LH_1$, $HL_1$, and $HH_1$. We exploit the inter correlation between $LL_1$, $LH_1$, $HL_1$, and $HH_1$ to estimate the weight factor $w_c$ in high-frequency wavelet subbands $LH_0$, $HL_0$, and $HH_0$. In Fig. 9, the wavelet coefficient P is the parent of wavelet coefficients $C_1$,$C_2$, $C_3$ and $C_4$ and the wavelet coefficients $C_1$,$C_2$, $C_3$ and $C_4$ is the children of wavelet coefficient P. According to this quad-tree property, while the coefficient is classified to significant coefficient and its children is also significant. $s$ is the standard deviation of subband and $a$ is a weighting factor. If the parent coefficient is classified to significant, the children weight factor $w_c$ is $a \times w_p$, the other coefficients set to zero. The quadtree weight function formula is as follows:

$$w_c = \begin{cases} w_p \times a & \text{if } |w_p| \geq s \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

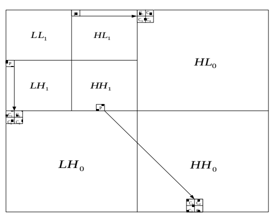



Fig. 9: Quad-tree Property in Wavelet Domain

### 3.4 Flowchart

The complete block diagram of the proposed algorithm presents in Fig. 10. First, the input is a high-resolution image. This image (HR) gets low-resolution image (LR) through down-sample (low-pass filter and down-sample) and then the low-resolution image uses shape function to enhance the low-resolution image. The enhanced low-resolution image is symbol as $\hat{X}$ . And then the proposed algorithm decomposes the enhanced low-resolution image into the LH, HL, and HH subband coefficients ( $\hat{X}_{HL}$ , $\hat{X}_{LH}$ , and $\hat{X}_{HH}$ ) by undecimated discrete wavelet transform and uses the quadtree weight function to adjust high-frequency subband coefficients. Finally, the proposed method synthesis $\hat{X}$ , $\hat{X}_{HL}$ , $\hat{X}_{LH}$ , and $\hat{X}_{HH}$ to get the output image by inverse discrete wavelet transform.

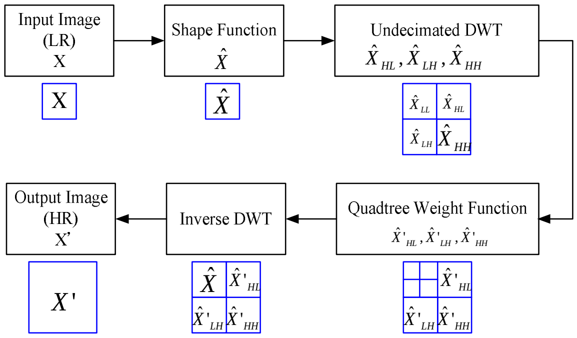


Fig. 10: Proposed Algorithm Flowchart

## 4.SIMULATION RESULTS

In our experiments, the test images are ‘Lena’ and ‘Baboon’. These images are regarded as the unknown HR originals. There are gray-level image with a size of 512 x 512 with 8 bits per pixels. We have filtered and downsampled the original images by factors of 2 for 2× resolution enhancement and used those as the available LR images. We have used the original images as ground truth and we have expressed the quality of the resolution enhancement process in terms of the well-established peak signal-to-noise ratio (PSNR) metric. PSNR value has been accepted as a widely used quality measurement in the field of image processing. PSNR value is a mathematics evaluation expression that can be calculated as

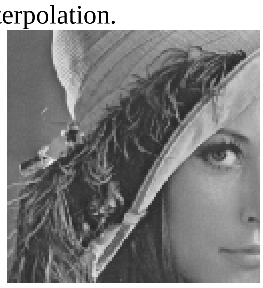
$$\text{PSNR}=10\log_{10}\frac{255^2}{\frac{1}{T}\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}\left(x_{i,j}-x'_{i,j}\right)^2} \qquad (11)$$

Simulation results obtained by simulating the above algorithm are shown in Tab. 1 where PSNR performance is tabulated for 2× enlargement factors. Table 1 show the comparison of the proposed algorithm and the standard bilinear interpolation approach with a number of alternative approaches in the wavelet domain. The wavelet-based resolution enhancement methods uses two well-known classes of analysis-synthesis filters namely Haar and Daubechies 9/7 and the unknown detail coefficients are estimated as zeros. The simulation results shows that PSNR of the proposed method is outperform than that of the other image resolution enhancement methods.

Tab. 1: PSNR results for 2× enlargement of ‘Lena’ and ‘Baboon’ test images (from 256×256 to 512×512)

| Method / Image | Lena(dB) | Baboon(dB) |
|---|---|---|
| Bilinear | 30.13 | 22.85 |
| NEDI [4] | 34.10 | 23.87 |
| WZP(Haar) | 31.46 | 23.61 |
| WZP(Db. 9/7) | 34.45 | 24.22 |
| Carey et al. [2] | 34.48 | 24.24 |
| HMM [5] | 34.52 | 24.24 |
| HMM SR [7] | 34.61 | 24.31 |
| WZP and CS [8] | 34.93 | 24.28 |
| Proposed Method | 35.46 | 24.33 |

Experimental results for ‘Lena’ test image are shown in Figure 11. The method ‘WZP’ is wavelet transform using zero padding. Figure 11 (a)~(d) are extracts from original and enlargements, Figure 11(a) is original image and this image down sample to be small size test image. Fig. 11 (b) shows that the enlargement slightly blur and the PSNR value is 34.45 dB. And the visually quality of discontinuous in enlargement by proposed method obviously outperform than that in enlargement by 9/7 wavelet zero padding. Fig 12 (a)~(c) are the residual image from original image by different resolution enhancement methods. Finally, the proposed method not only for 2x enlargement but also the other enlargement factors 4×, 8×, and 16×. Figure13 shows the comparison of ‘Lena’ enlargements that enlarge by bilinear and proposed algorithm with enlargement factors 4×, 8×, and 16×. The simulation result shows that the enlargement by the proposed method is exceptional in the enhancement of the discontinuous field and furthermore the visually quality of enlargement by the proposed method is more outstanding than that of enlargement by bilinear interpolation.


(a)


(b)


(c)


(d)

Fig. 11: Extracts from original and 2×enlargement images
(a)Original image
(b)enlargement by bilinear interpolation (PSNR=30.13 dB)
(c)enlargement by WZP (Db 9/7) (PSNR=34.45 dB)
(d)enlargement by proposed method (PSNR=35.46 dB)

(a)

(b)

(c)

Fig. 12: Different from original image
(a) Residual image using bilinear interpolation
(b) Residual image using WZP (Db. 9/7)
(c) Residual image using proposed method

(a)

(b)

(c)

(d)

(e)

(f)

Fig. 13: Enlargement image
(a), (c), (e) by bilinear interpolation for 4×, 8×, 16× enlargement
(b), (d), (f) by proposed algorithm for 4×, 8×, 16× enlargement

## 5: Conclusions

This wavelet-domain image resolution enhancement algorithm proposes shape function with wavelet coefficient correlation to enhance the discontinuances and employs undecimated wavelet transform to estimate the unknown detail coefficients. Finally, the proposed algorithm uses quadtree weight function to adjust the high frequency coefficients. Experimental results demonstrate that the performance of wavelet-based schemes outperforms that of conventional methods and shows that our method outperforms conventional image resolution enhancement methods such as bilinear interpolation, for a wide range of standard test images. More importantly the proposed method compares favorably with state-of-the-art competing methods operating in the wavelet domain both in objective and subjective terms.

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