# Translation, Rotation, and Scale-Invariant Template Matching 

Yi-Hsien Lin, Chin-Hsing Chen, Chih-Cheng Wei<br>Department of Electrical Engineering<br>National Cheng Kung University, Tainan, 701, Taiwan, ROC<br>yihsien@csu.edu.tw, chench@eembox.ncku.edu.tw,weisuper@ms31.hinet.net


#### Abstract

A new template matching algorithm that enables fast matching between a reference template and a scene image regardless of its translation, rotation and scaling is proposed in this paper. In the beginning, the ring projection transform process is used to convert the $2 D$ template into a 1 D signal. Then, the template matching is performed by constructing a parametric template vector of the 1D signal with differently scaled templates of the object. Our approach is conceptually simple, easy for implementation and very efficient for template matching (no iterative steps are involved). Experimental results have shown that the proposed algorithm is not only efficient and robust to detect the target object with the changes of translation, orientation and scale, but also can estimate the scaling value of the target object in the input scene.


## 1: INTRODUCTIONS

Template matching is the process of determining the optimal match between the same scenes taken at different times or under different conditions and the template known according to some similarity measure. It is an important issue in image processing with many applications including remote sensing, medical imaging, and automatic inspection in industry. Over the last three decades, template matching has been widely studied and many registration algorithms have been proposed in the literatures [1-3]. Ideally, template matching should be invariant to translation, rotation and scaling in the input scene. However, most of the proposed techniques are not simultaneously invariant under the changes of translation, orientation and scale in the input scene or are too computationally expensive, which is not practical in the real world applications.

Although, various rotation-invariance approaches for template matching have been reported [4-9], the matching results of these schemes are sensitive to scale changes. Tanaka et al. [10] proposed a template matching method in which a parametric template space is constructed from a given set of template images with differently rotated image or differently scaled image for an object, and then quickly matched to a scene image. However, the matching scheme becomes impractical when both of arbitrary rotation and scale changes are present for a search object. Therefore the template
matching algorithm faces great challenges of computational efficiency, accuracy, and the input scene with the changes of translation, rotation and scale.

In this paper, a new template matching algorithm that enables fast matching between a reference template and a scene image regardless of its translation, rotation and scaling is proposed. In the beginning, our approach uses the ring projection transform (RPT) process to convert the 2D template in a circular region into a 1D gray-level signal as a function of radius. The advantages of the RPT process are that it owns the characteristic of rotation invariance and reduces the computational complexity of normalized correlation (NC). Then, the template matching is performed by constructing a parametric template (PT) vector of the 1D gray-level signal with differently scaled templates of the object. The merits of our approach are that not only can rotation invariance be obtained by the RPT process, but also scale invariance is achieved by the PT vector method. Besides, the approach does not need feature-based corresponding-points matching and allows estimation of the model parameters by a direct linear calculation rather than an iterative calculation, and therefore is computationally efficient.

## 2: THEORY

In this section, we present the necessary theory to match images which are translated, rotated, and scaled with respect to each other.

## 2.1: THE RING PROJECTION TRANSFORMATION (RPT) PROCESS

In order to deal with the changes of rotation for the input scene in the template matching, the RPT process [8-13] which reduces a 2 D image to a 1 D vector is used as the pre-process in our approach. Let's define a template to be $T(x, y)$ whose size is $M \times N$. The RPT process of the template is given as follows: First, the center point of the template $T(x, y)$ denoted as $\left(x_{c}, y_{c}\right)$ is derived, and subsequently, the Cartesian frame template $T(x, y)$ is transformed into a Polar frame based on the following relations:

$$
\begin{align*}
& x=r \cos \theta,  \tag{1}\\
& y=r \sin \theta, \tag{2}
\end{align*}
$$

Where $\quad \mathrm{r}=(\operatorname{int})\left(\sqrt{\left(\mathrm{x}-\mathrm{x}_{\mathrm{c}}\right)^{2}+\left(\mathrm{y}-\mathrm{y}_{\mathrm{c}}\right)^{2}}\right), \quad r \in[0, R]$, $R=\min (M, N), \quad \theta \in(0,2 \pi]$.

The RPT of the template $T(x, y)$ at radius r , denoted as $P_{T}(r)$, is defined as

$$
\begin{equation*}
P_{T}(r)=\frac{1}{S_{r}} \sum_{k} T\left(r \cos \theta_{k}, r \sin \theta_{k}\right), \tag{3}
\end{equation*}
$$

where $S_{r}$ is the total number of pixels falling on the circle of radius, $r=0,1,2, \ldots, R$. Since $P_{T}(r)$ is defined as the mean of pixel intensities along the circle whose radius to the center of the template is $r$ as shown in Fig. 1, $P_{T}(r)$ values of all rings in the template have equal importance in the computation of correlation [8, 10]. Furthermore, since a RPT is constructed along circular rings of increasing radii, the derived 1D ring-projection template is invariant to rotation of its corresponding 2D image template. In order to calculate RPT along concentrate circles effectively, we use a lookup table whose diameter is set to the size of the template [9]. Then a RPT is obtained simply by summing up the pixel values along a concentric circle within the template results.

For the matching process, the NC is adopted in the measurement of similarity. Let

$$
\begin{equation*}
\overrightarrow{P_{T}} \triangleq\left[P_{T}(0), P_{T}(1), \ldots, P_{T}(R)\right] \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
\overrightarrow{P_{S}} \triangleq\left[P_{S}(0), P_{S}(1), \ldots, P_{S}(R)\right] \tag{5}
\end{equation*}
$$

represent the ring-projection vectors of the reference template and the scene subimage, respectively. The NC between the ring-projection vectors $\overrightarrow{P_{T}}$ and $\overrightarrow{P_{S}}$, denoted by $\left\langle\overrightarrow{P_{T}}, \overrightarrow{P_{S}}\right\rangle$, is defined as

$$
\begin{align*}
& \left\langle\vec{P}_{T}, \vec{P}_{S}\right\rangle= \\
& \frac{\left((R+1) \sum_{r=0}^{R} P_{T}(r) P_{S}(r)-\sum_{r=0}^{R} P_{T}(r) \sum_{r=0}^{R} P_{s}(r)\right)^{2} \times 100}{\left((R+1) \sum_{r=0}^{R} P_{T}(r)^{2}-\left(\sum_{r=0}^{R} P_{T}(r)\right)^{2}\right)\left((R+1) \sum_{r=0}^{R} P_{S}(r)^{2}-\left(\sum_{r=0}^{R} P_{S}(r)\right)^{2}\right)} . \tag{6}
\end{align*}
$$

With this definition, the value of $\left\langle\overrightarrow{P_{T}}, \overrightarrow{P_{S}}\right\rangle$ is unaffected by rotations and linear changes (constant gain and offset) in the reference template and the scene subimage. In addition, the dimensional length of the ring-projection vector is only $(\mathrm{R}+1)$. This significantly increases the computational efficiency for $\left\langle\overrightarrow{P_{T}}, \overrightarrow{P_{S}}\right\rangle$.

## 2.2: TEMPLATE MATCHING BY THE PARAMETRIC TEMPLATE (PT) VECTOR METHOD

The proposed method presented here is inspired by the PT method [11]. However, the PT method incurs the increase of computational complexity when the scene images involve the changes of
scale and rotation. In order to obtain rotation and scale invariance in the matching process, a simply approach using a PT vector is proposed. In our approach, a PT vector $\overrightarrow{P_{T_{p}}}$ is constructed from a set of base ring-projection vectors ( $\overrightarrow{P_{t_{0}}}, \overrightarrow{P_{t_{1}}}, \ldots, \overrightarrow{P_{t_{N}}}$ ) that are the RPTs including the template image and its differently scaled images as given by

$$
\begin{equation*}
\overrightarrow{P_{T_{p}}} \triangleq \frac{\overrightarrow{P_{t_{0}}} \omega_{0}+\overrightarrow{P_{t_{1}}} \omega_{1}+\cdots+\overrightarrow{P_{t_{N}}} \omega_{N}}{\left|\overrightarrow{P_{t_{0}}} \omega_{0}+\overrightarrow{P_{t_{1}}} \omega_{1}+\cdots+\overrightarrow{P_{t_{N}}} \omega_{N}\right|}, 0.0 \leq \omega_{i} \leq 1.0, \sum_{i=0}^{N} \omega_{i}=1 . \tag{7}
\end{equation*}
$$

The NC between the scene subimage vector $\overrightarrow{P_{S}}$ and a PT vector $\overrightarrow{P_{T_{p}}}$ becomes $\left\langle\overrightarrow{P_{T_{p}}}, \overrightarrow{P_{S}}\right\rangle$. Subsequently, the problem we want to solve becomes a constrained optimization, that is

$$
\begin{equation*}
\max _{\{\overline{\{\omega}\}}\left\langle\overrightarrow{P_{T_{p}}}, \overrightarrow{P_{S}}\right\rangle \text { subject to } \sum_{i=0}^{N} \omega_{i}=1 \tag{8}
\end{equation*}
$$

This problem can be solved by the Lagrange Multiplier method. The solution $\vec{\omega}$ is given by

$$
\begin{equation*}
\vec{\omega}=\frac{H^{-1} \vec{G}}{\left(\vec{n} \bullet H^{-1} \vec{G}\right)}, \tag{9}
\end{equation*}
$$

where $\vec{\omega}, H, \vec{G}, \vec{n}$ are

$$
\vec{\omega} \triangleq\left[\begin{array}{c}
\omega_{0}  \tag{10}\\
\vdots \\
\omega_{N}
\end{array}\right],
$$

$H \triangleq\left[\begin{array}{ccc}\left\langle\overrightarrow{P_{t_{0}}}, \overrightarrow{P_{t_{0}}}\right\rangle & \cdots & \left\langle\overrightarrow{P_{t_{0}}}, \overrightarrow{P_{t_{N}}}\right\rangle \\ \vdots & \ddots & \vdots \\ \left\langle\overrightarrow{P_{t_{N}}}, \overrightarrow{P_{t_{0}}}\right\rangle & \cdots & \left\langle\overrightarrow{P_{t_{N}}}, \overrightarrow{P_{t_{N}}}\right\rangle\end{array}\right]$,
$\vec{G} \triangleq\left[\begin{array}{c}\left\langle\overrightarrow{P_{s}}, \overrightarrow{P_{t_{0}}}\right\rangle \\ \vdots \\ \left\langle\overrightarrow{P_{s}}, \overrightarrow{P_{t_{N}}}\right\rangle\end{array}\right]$,
$\vec{n} \triangleq\left[\begin{array}{c}1 \\ \vdots \\ 1\end{array}\right]$.
Next, the estimated scaling value sv of the scene subimage is found in terms of the following equation:

$$
\begin{equation*}
s v=\sum_{i=0}^{N} \omega_{i} s_{i} \tag{14}
\end{equation*}
$$

where $s_{i}, 0 \leq i \leq N$, denotes the differently scaling values generated by scaling the template image, respectively.

The approach enables fast matching in the matching algorithm. Since the RPT process reduces a 2D image to a 1 D vector, the computational efficiency is significantly increased. Additionally, the correlation matrix H can be determined in the training phase; the optimal parameters $\vec{\omega}$ and the scaling value are directly obtained
from the correlation vector $\vec{G}$ and the correlation matrix H in the matching phase. No iterative steps are involved, so the execution time is deterministic.

## 3: RESULTS OF EXPERIMENT

In this section, based on two different test images and the corresponding template images shown in Figure 2, the performance of our proposed method was evaluated by applying some known transformation. The image transform is implemented using bicubic interpolation from the original image. The PT vector was generated by scaling the template image with a scaling value of 0.36 , $0.49,0.64,0.81,1,1.21,1.44,1.69$ and 1.96 respectively. We implemented this algorithm by C++ Builder 6.0 under a personal computer with an Intel's Pentium III 1 GHz processor. The searching region is from $\left(x_{c}-10, y_{c}-10\right)$ to $\left(x_{c}+10, y_{c}+10\right)$ exhaustively, where $\left(x_{c}, y_{c}\right)$ is the center position of the searching template. Table 1 and Table 2 summarize the results of the image A and B. Column "Original" and "Estimated" show the exact transformation parameters and the computed transformation parameters using our approach, respectively.

From the results, we can see that the estimated translation values are exact under the changes of translation, rotation and scale for the test images. By comparison of Table 1 and Table 2, the performance of the image B for the estimated scaling values is superior to the image A . The reason is that the template image size of the image A in Figure 2(a) is smaller than that of the image B in Figure 2(b). Obviously, the template image size affects the performance of the template matching.

## 4: CONCLUSION

In this paper, a new template matching approach using the RPT process and the PT vector method is proposed. The RPT process increases the computational efficiency and owns the characteristic of rotation invariance in template matching. The PT vector method provides a template matching with scale invariance. Furthermore, our approach is conceptually simple, easy for implementation and very efficient for template matching.

Experimental results have shown that the proposed algorithm is not only efficient and robust to detect the target object with the changes of translation, orientation and scale, but also can estimate the scaling value of the target object in the input scene. However, the proposed method in its current form can not estimate the rotation angle of the target object. It is worthwhile to extend the
proposed method for solving the problem in the future study.

## REFERENCES

[1]. L. G. Brown, "A Survey of Image Registration Techniques," ACM Computing Surveys, vol. 24, no. 4, pp. 326-376, 1992.
[2]. B. Zitová and J. Flusser, "Image Registration Methods: A Survey," Image and Vision Computing, vol. 21, no. 11, pp. 997-1000, 2003.
[3]. J. B. A. Maintz and M. A. Viergever, "A Survey of Medical Image Registration," Medical Image Analysis, vol. 2, pp. 1-36, 1998.
[4]. A. A. Cole-Rhodes, K. L. Johnson, J. LeMoigne and I. Zavorin, "Multiresolution Registration of Remote Sensing Imagery by Optimization of Mutual Information Using A Stochastic Gradient," IEEE Trans. on Image Processing, vol. 12, no. 12, pp. 1495-1511, 2003.
[5]. R. J. Prokop and A. P. Reeves, "A Survey of Moment-Based Techniques for Unoccluded Object Representation and Recognition," CVGIP Graph Models Image Process, vol. 54, no. 5, pp. 438-460, 1992.
[6]. D. M. Tsai and C. H. Chiang, "Rotation-Invariant Pattern Matching Using Wavelet Decomposition," Pattern Recognition Letters, vol. 23, pp. 191-201, 2002.
[7]. F. Ullah and S. Kanekoi, "Using Orientation Codes for Rotation-Invariant Template Matching," Pattern Recognition, vol. 37, no. 2, pp. 201-209, 2004.
[8]. Y. H. Lin, C. H. Chen and C. C. Wei, "New Method for Subpixel Image Matching with Rotation Invariance by Combining the Parametric Template Method and The Ring Projection Transform Process," Optical Engineering, vol. 45, no. 6, pp. 067202(1-9), 2006.
[9]. M. S. Choi and W. Y. Kim, "A Novel Two Stage Template Matching Method for Rotation and Illumination Invariance," Pattern Recognition, vol. 35, pp. 119-129, 2002.
[10]. D. M. Tsai and C. H. Chiang, "Rotation-Invariant Pattern Matching Using Wavelet Decomposition," Pattern Recognition Letters, vol. 23, pp. 191-201, 2002.
[11]. K. Tanaka, M. Sano, S. Ohara and M. Okudaira, "A Parametric Template Method and Its Application to Robust Matching," IEEE Conference on Computer Vision and Pattern Recognition, vol. 1, pp. 620-627, 2000.
[12]. Y. Y. Tang, H. D. Cheng and C. Y. Suen, "Transformation-Ring-Projection (TPR) Algorithm and Its VLSI Implementation," International Journal of Pattern Recognition and Artificial Intelligence, vol. 5, pp. 25-56, 1991.
[13]. Y. Y. Tang, B. F. Li, H. Ma and J. Liu, "Ring-Projection-Wavelet-Fractal Signatures: A Novel Approach to Feature Extraction," IEEE Trans. Circuits and System-II: Analog and Digital Signal Process, vol. 45, pp. 1130-1134, 1998.


Fig. 1 Concept of RPT for a template.


Fig. 2 Two different test images and the corresponding reference templates used in the experiment:
(a)The image $A$ and its reference template with size $137 \times 137$.
(b) The image $B$ and its reference template with size $257 \times 257$.

| Original |  |  | Estimated |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Translation | Rotation | Scale | Translation | Scale |  |
|  |  |  |  | Value | Error |
| $(28,25)$ | $40^{0}$ | 1.69 | $(28,25)$ | 1.77 | 0.08 |
| $(28,25)$ | $80^{0}$ | 1.69 | $(28,25)$ | 1.75 | 0.06 |
| $(28,25)$ | $-40^{0}$ | 1.69 | $(28,25)$ | 1.78 | 0.09 |
| $(28,25)$ | $-80^{0}$ | 1.69 | $(28,25)$ | 1.76 | 0.07 |
| $(20,25)$ | $40^{0}$ | 0.49 | $(20,25)$ | 0.53 | 0.04 |
| $(20,25)$ | $80^{0}$ | 0.49 | $(20,25)$ | 0.53 | 0.04 |
| $(20,25)$ | $-40^{0}$ | 0.49 | $(20,25)$ | 0.52 | 0.03 |
| $(20,25)$ | $-80^{0}$ | 0.49 | $(20,25)$ | 0.54 | 0.05 |

Table 1, Matching results of translated, rotated and scaled image for the Image A.

| Original |  |  | Estimated |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Translation | Rotation | Scale | Translation |  |  |
|  |  |  |  | Value | Error |
| $(10,10)$ | 400 | 1.69 | $(10,10)$ | 1.71 | 0.02 |
| $(10,10)$ | 800 | 1.69 | $(10,10)$ | 1.70 | 0.01 |
| $(10,10)$ | -40 0 | 1.69 | $(10,10)$ | 1.68 | 0.01 |
| $(10,10)$ | -80 0 | 1.69 | $(10,10)$ | 1.70 | 0.01 |
| $(20,20)$ | 400 | 0.49 | $(20,20)$ | 0.48 | 0.01 |
| $(20,20)$ | 800 | 0.49 | $(20,20)$ | 0.49 | 0.00 |
| $(20,20)$ | -40 0 | 0.49 | $(20,20)$ | 0.48 | 0.01 |
| $(20,20)$ | -80 0 | 0.49 | $(20,20)$ | 0.49 | 0.00 |
| Mean error of the estimated scaling value |  |  |  |  | 0.009 |
| Max error of the estimated scaling value |  |  |  |  | 0.02 |
| Min error of the estimated scaling value |  |  |  |  | 0 |
| Standard derivation of the estimated scaling value |  |  |  |  | 0.006 |
| Average run-time (sec) |  |  |  | 0.61 |  |

Table 2: Matching results of translated, rotated and scaled image for the Image B.

