Nonlinear PI Controllers for Distributed Bioreactor Systems

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Abstract

In this paper, the feedback linearization technique on the basis of equilibrium manifold to figure out a nonlinear PI configuration is proposed. Based on the optimization-based tuning procedure, the flexible tuning procedure can ensure the closed-loop performance. If the off-line identification and the sinusoid function validation are added, the observer-based PI control as the extended output feedback design is successfully applied for a distributed bioreactor system.

1. Introduction

Continuous cultures of some microorganism such as *Saccharomyces cerevisiae* and *Zymomonas mobilis* have been known to exhibit highly oscillatory behavior during routine operation (Jöbses et al., 1985; Strässle et al., 1988). Hjortsø and Nielsen (1995) developed the population balance equation (PBE) to describe the oscillating behavior of the microbial process. However, these oscillations caused by the spontaneous synchronization are probably attributable to system perturbations (Hjortsø and Nielsen, 1994). Moreover, Kurtz et al. (1998) and Zhu et al. (2000) recently proposed the nonlinear state feedback control and linear model-based predictive control for these self-oscillating systems, respectively. Notably, these bioprocesses are class of PBE-based distributed parameter systems, and both manipulated variables by exploiting feed substrate concentration and dilution rate are usually taken into account. In our opinions, i) the conventional control designs are difficult to stabilize self-oscillating, nonlinear distributed systems; ii) the linear model-based control designs are inadequate to reduce the large plant/model mismatch; iii) the multivariable control strategies hardly ensure the favorable cost/benefit ratio.

To our knowledge, the control of self-oscillating and/or chaotic behavior of non-distributed systems has been addressed in some literatures (Pérez and Albertos, 2004; Pellegrini and Biardi, 1990; Wu, 2000a). It is noted that the traditional PI control cannot effectively reduce the sustained oscillation problem. Referring the recent issues, Alvarez-Ramirez (1999) indicated that the specific PI configuration could be used to stabilize a class of nonlinear systems, and Wu (2004) also demonstrated that the nonlinear PI/PID control would be effectively implemented for the output regulation of polymerization processes. Besides, it has been verified that the feedback linearization algorithm can guarantee the asymptotical output regulation of time-varying uncertain nonlinear systems (Marino and Tomei, 1993). Intuitively, we think that the self-oscillating bioprocesses can be reduced as a time-varying uncertain nonlinear system, and the hybrid control configuration associated with PI control and nonlinear linearizing control is probably practical and feasible design for the stabilization of self-oscillating, distributed systems.

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2. Process Model

Consider a binary fission organism in a continuous bioreactor, in which the growth rate of cell age is assumed unity and the age distribution is governed by the population balance equation. Referring to the modeling result in Kurtz et al. (1998), the process model is described by

$$\frac{\partial W}{\partial t}+\frac{\partial W}{\partial a}=-\left[D+\Gamma(a,S^t)\right]W \tag{1a}$$

$$\dot{m}_0=-Dm_0+\int_0^\infty \Gamma(a,S^t)W(a,t)da \tag{1b}$$

$$\dot{S}=D(S_f-S)-\kappa(S)\int_0^\infty W(a,t)da \tag{1c}$$

$$\dot{S}^t=\alpha(S-S^t) \tag{1d}$$

where process variables $W(a,t)$ represents the frequency of concentration of cells with the age a at time t, m_0 is denoted as the cell number concentration, S is the substrate concentration, and S' represents the 'effective' substrate concentration. The input variables are the dilution rate D and the feed substrate concentration S_f. Since Eq. 1a is a typically partial differential equation, the following initial and boundary conditions are required

$$\begin{aligned} W(a,0)&=W_0(a)\\ W(0,t)&=2\int_0^\infty \Gamma(a,S')W(a,t)da \end{aligned} \tag{2}$$

where $\Gamma(a,S')$ represents the cell division intensity modeled by

$$\Gamma(a,S')=\begin{cases} 0 & a<a_c \\ \dfrac{1}{\varepsilon}(a-a_c)^j & a\ge a_c,\ j\ge 2 \end{cases} \tag{3}$$

where ε is a constant and the critical age of division a_c is described by

$$a_c=\pi_0+\pi_1/S' \tag{4}$$

where π_0 and π_1 are specified constants. The yield kinetic $\kappa(S)$ in Eq. 1c is based on the Monod expression

$$\kappa(S)=\frac{\mu_m S}{K+S} \tag{5}$$

where μ_m and K are constants. The adaptivity parameter α in Eq. 1d is denoted as a coefficient for changes in S and the response in cell metabolism.

Remark 1: Basically, it is difficult to solve the above model equations due to the semi-infinite integral form. Referring to the issue in Kurtz et al. (1998), the coordinate transformation, $a'=1-e^{-a}$, is suggested to map [0, ∞) →[0, 1). Moreover, the transformed equation by Eq. 1a is rewritten as

$$\frac{\partial \tilde{W}}{\partial t}+(1-a')\frac{\partial \tilde{W}}{\partial a'}=-\left[D+\tilde{\Gamma}(a',S')\right]\tilde{W} \tag{6}$$

with respect to the renewal conditions by Eq. 2 is shown as

$$\begin{aligned} \tilde{W}(a',0)&=\tilde{W}_0(a')\\ \tilde{W}(0,t)&=2\int_0^1 \frac{\tilde{\Gamma}(a',S')\tilde{W}(a',t)}{1-a'}da' \end{aligned} \tag{7}$$

By Eqs 1b and 1c, the corresponding transformed equations are written as

$$\dot{m}_0=-Dm_0+\int_0^1 \tilde{\Gamma}(a',S')\frac{\tilde{W}(a',t)}{1-a'}da' \tag{8a}$$

$$\dot{S}=D(S_f-S)-\kappa(S)\int_0^1 \frac{\tilde{W}(a',t)}{1-a'}da' \tag{8b}$$

In this article, the process model is based on the transformed PBE model with initial and boundary conditions by Eqs 6-9. Using the numerical techniques introduced in the books of Marchuk (1982) and Schiesser (1991), the method of lines and finite differences can convert the PDE (Eq. 6) into the successive ODEs with time as the independent variable.

3. Nonlinear PI Control Designs

Because the process is a typically distributed model, Christofides and Daoutidis (1996), and Wu (2000b) used numerical techniques to establish a class of non-distributed nonlinear control systems. Since the PBE model has a uniform behavior after the large time, the steady-state cell distribution is considered. Suppose that the equilibrium $\tilde{W}_{ss}$ should be solved by

$$\frac{d\tilde{W}_{ss}}{da'} = -(D + \Gamma(a', S_{ss}))\tilde{W}_{ss}$$

$$0 = D(S_f - S_{ss}) - \kappa(S_{ss})\int_0^1 \frac{\tilde{W}_{ss}(a')}{1-a'}da' \quad (9)$$

and the approximate non-distributed model is shown as

$$\dot{x}_1 = -ux_1 + \int_0^1 \tilde{\Gamma}(a', x_3)\frac{\tilde{W}_{ss}(a')}{1-a'}da'$$

$$\dot{x}_2 = u(S_f - x_2) - \kappa(x_2)\int_0^1 \frac{\tilde{W}_{ss}(a')}{1-a'}da' \quad (10)$$

$$\dot{x}_3 = \alpha(x_2 - x_3)$$

where $x^T = [m_0, S, S']$. Assume that the control u=D, and the output $y = x_1$. Moreover, the output of the first-order reference model is shown as

$$y_m = y_{m,b} + \varepsilon_1 \int_0^t (y_{sp} - y)d\tau \quad (11)$$

where y_{sp} and $y_{m,b}$ represents the setpoint and initial of reference model, respectively.

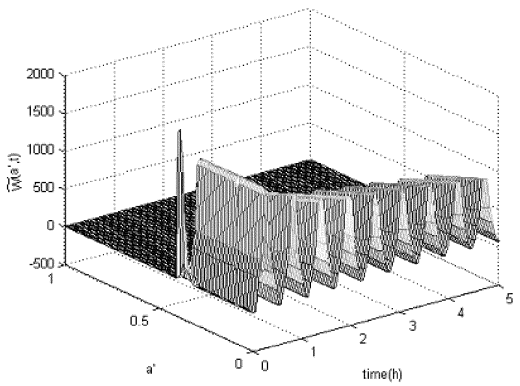

Fig. 1 Open-loop profiles

In fact, many states of bioprocesses are unavailable, and metabolic kinetics of processes are usually complex and uncertain. Inspired by the response of $\tilde{W}(a', t)$ in Figure 1, the steady-state profile under periodic oscillation is a characteristic to build up a training strategy. The off-line estimation mechanism is constructed by a sinusoid function

$$\Omega(t) = \alpha_3 \sin(\alpha_1 + \alpha_2 t) \quad (12)$$

where parameters α_i, i=1,2,3, should be identified during the off-line estimation/correction. Moreover, the estimated function of $\tilde{W}(a', t)$ is shown as

$$\hat{W}(a', t) = \tilde{W}_{ss}(a')(1 + \Omega(t)) \quad (13)$$

Then, the reduced nonlinear observer is expressed by

$$\dot{\hat{x}}_1 = -u\hat{x}_1 + \int_0^1 \tilde{\Gamma}(a', \hat{x}_3)\frac{\hat{W}(a', t)}{1-a'}da'$$

$$\dot{\hat{x}}_2 = u(S_f - \hat{x}_2) - \kappa(\hat{x}_2)\int_0^1 \frac{\hat{W}(a', t)}{1-a'}da' \quad (14)$$

$$\dot{\hat{x}}_3 = \alpha(\hat{x}_2 - \hat{x}_3)$$

where $\hat{x}_i$, i=1,2,3, represents the states of observer. Through the off-line validation, if the estimation errors given by

$$\min_{\alpha_i} \|x - \hat{x}\| \le \sigma, \sigma \ge 0 \quad (15)$$

can hold, then the state estimation is feasible. When only the cell number concentration (x_1) of process is available, the nonlinear PI control is written as

$$u_{OPI} = \tilde{u}_b(y) + K_c^{PI}(y)[(y_{sp} - y) + \frac{1}{\tau_I^{PI}}\int_0^t (y_{sp} - y)d\tau \quad (16)$$

with

$$\tilde{u}_b(y) = \frac{\varepsilon_2(y - y_{m,b}) + \theta(\hat{x}_3, t)}{y}$$

$$K_c^{PI}(y) = -\varepsilon_1 / y \quad (17)$$

$$\tau_I^{PI} = 1/\varepsilon_2$$

where $\theta(\hat{x}_3, t) = \int_0^1 \tilde{\Gamma}(a', \hat{x}_3)\frac{\hat{W}(a', t)}{1-a'}da'$.

Remark 2: The estimated function $\theta(\hat{x}_3, t)$ aims to take over the original function $\tilde{r}(x_3, t)$ as soon as possible. It is no doubt that this time-varying observer (Eq. 14) can sufficiently capture the steady-state behavior of the biosystem. However, the estimation error is inevitable due to model errors and simplified identification rule by

exploiting the sinusoid function $\bar{W}(a',t)$. Furthermore, a simple and on-line adjustable controller tuning algorithm is recommended. To our knowledge, one of tuning parameters ε_1 associated with the integral mode should be fixed to avoid the reset windup problem, and another parameter ε_2 is determined by solving the minimization of the following quadratic form

$$\min_{\varepsilon_2} J = \sum_{k=1}^{N} \left(y_{sp} - y(k\Delta t) \right)^2 \tag{18}$$

where Δt represents the sampling time.

4. Conclusions

In this paper, we focus on oscillating microbial cultures described by PBE models. According to the equilibrium manifold, a non-distributed and low-order model is developed. Based on a first-order reference model and the optimization-based tuning algorithm, the simple, nonlinear PI control is proposed. Since an off-line identification and validation of time-varying observer is employed, we think that this extended output feedback design is valuable in the possible real-time implementation.

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