Robust stabilization of constraint time delay systems with dynamic compensators

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Abstract

This paper is concerned with the problem of robust stabilization for uncertain time delay systems subject to saturating actuator. Under certain conditions, a dynamic compensator, which uses only the accessible output variables, is synthesized. In contrast to the previous works, the global stabilization can also be achieved even with an unstable system matrix. Last, an example is included to illustrate the results developed in this paper.

1. Introduction

Since time delay is frequently a source of instability and commonly exists in various engineering, biological and economical systems due to the finite speed of information processing, the problem of stabilization of time delay systems has received considerable attention over the past years and has been addressed in numerous studies. Numerous approaches, such as the pole placement approach via spectral decomposition [3] [10], the Riccati equation approach [7] [11] [14] and the finite spectrum assignment [6] [13], have been devoted to deal with this stabilization problem. In all the studies referenced above, all the state variables are assumed to be available for exact measurement and the dynamic range of actuators to be unlimited. Unfortunately, in practical control systems, the state variables are not always available for direct measurement and the practical actuators may saturate and this may lead to serious degradation of system performance and possibly to instability [9].

Recently, the stabilization of time delay systems with saturating actuators was investigated in Chou et. al. [2] and Oucheriah [8]. In Chou et. al. [2], a linear dynamic output feedback compensator is designed. However, no conditions to guarantee the existence of such compensator are given. In Oucheriah [8], the global stabilization of observer-based linear constrained uncertain time delay systems is considered. But the author uses a full order observer to estimate the state of the system and assumes the systems matrix A is stable. For the case of unstable matrix A , only local stabilization can be achieved by linear feedback control.

In this paper, global stabilization of time delay systems subject to saturating actuator is considered. In contrast to Oucheriah [8], the global stabilization can also be achieved even with an unstable matrix A. Under certain conditions, a dynamic compensator, which uses only the output variables, is synthesized. The dynamic compensator is presented by employing the Razumikhin-type theorem [5] and the concept of quadratic stability. Finally, an example is given to illustrate the results developed in this paper.

For simplicity, in the following section, W^T denotes the transpose of W, and $\|W\|$ represents the Euclidean norm when W is a vector or the induced norm when W is a matrix. I_n is the identity matrix of $n \times n$. $\lambda(W)$ denotes an eigenvalue of W and $\lambda_{\max}(W)$ represents the eigenvalue of W with the maximum real part.

2. System description

Consider the constraint uncertain time-delay systems described by the following linear differential-difference equations:

$$\dot{x}(t) = Ax(t) + A_1 x(t-\tau) + \Delta_0(x(t),t) + \Delta_1(x(t-\tau),t) + Bsat(u(t)) \quad (1.a)$$

$$y(t) = Cx(t) \quad (1.b)$$

$$x(t) = \psi(t), \quad t \in [-\tau, 0] \quad (1.c)$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input and $y(t) \in R^r$ is the output of the system. $\tau \in R^+$ is the delay, and $\psi(t)$ is a continuous vector-valued initial function. A, A_1, B and C are constant matrices of appropriate dimensions. The uncertainties Δ_0 and Δ_1 are unknown and represent the nonlinear parameter perturbations with respect to the current state $x(t)$ and delayed state $x(t-\tau)$, respectively. In general it is assumed that $\Delta_0(x(t),t)$ and $\Delta_1(x(t-\tau),t)$ are bounded, i.e.

$$\|\Delta_0(x(t),t)\| \le \beta_0 \|x(t)\| \quad (2.a)$$

$$\|\Delta_1(x(t-\tau),t)\| \le \beta_1 \|x(t-\tau)\| \quad (2.b)$$

where β_0 and $\beta_1 > 0$ are given. The saturation function $sat(u_i(t))$ is defined as follows

$$sat(u_i(t)) = \begin{cases} u_H^i & \text{if } u_i(t) > u_H^i \\ u_i(t) & \text{if } -u_L^i \le u_i(t) \le u_H^i \\ -u_L^i & \text{if } u_i(t) < -u_L^i \end{cases}, i=1,2,\ldots,m \quad (3)$$

$sat(u(t)) = [sat(u_1(t)),\ldots,sat(u_m(t))]^T$ and $u_H^i,\ u_L^i \in R^+ (i=1,\ldots,m)$ are actuator limitations. Since

$$\left|sat(u_i(t) - \tfrac{1}{2}u_i(t))\right| \le \frac{1}{2}|u_i(t)| \quad (4)$$

, one has

$$\left\|sat(u(t) - \tfrac{1}{2}u(t))\right\| \le \frac{1}{2}\|u(t)\|. \quad (5)$$

This property will be useful later.

For the sake of simplicity, we suppose $\{A,B\}$ is controllable and $\{C,A\}$ is observable. Assume that the state of system (1) cannot be measured directly. Now we focus our attention on the following linear dynamic output feedback compensator.

$$\dot{x}_d(t) = A_d x_d(t) + B_d y(t) \quad (6.a)$$

$$u(t) = Sx_d(t) + Ty(t) \quad (6.b)$$

$$x_d(0) = x_{d0} \quad (6.c)$$

where $x_d(t) \in R^{\hat{n}}$, $\hat{n}$ is $\min(v_c - 1, v_o - 1)$, $v_c(v_o)$ is the controllability (observability) index of the system (1), and A_d, B_d, S, T have appropriate dimensions. Then the problem becomes how to choose the control parameter matrices of (6), A_d, B_d, S, T, such that the closed loop dynamic with saturation control can be globally stabilized.

3 Main results

By substituting (6) into the system (1), we obtain the closed-loop equations as

$$\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{A}_1\tilde{x}(t-\tau) + \tilde{\Delta}_0(\tilde{x}(t),t) + \tilde{\Delta}_1(\tilde{x}(t-\tau),t) - \tilde{B}[\frac{1}{2}\tilde{u}(t) - sat(\tilde{u}(t))] \quad (7.a)$$

$$y(t) = Cx(t) \quad (7.b)$$

$$\tilde{x}(0) = \tilde{x}_0 = [x^T(0)\ \ x_d^T(0)]^T \quad (7.c)$$

where

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ x_d(t) \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} A + \frac{1}{2}BTC & \frac{1}{2}BS \\ B_d C & A_d \end{bmatrix},$$

$$\tilde{A}_I = \begin{bmatrix} A_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \tilde{u}(t) = \begin{bmatrix} u(t) \\ 0 \end{bmatrix},$$

$$\widetilde{\Delta}_0(\widetilde{x}(t),t)=\begin{bmatrix}\Delta_0(x(t),t)\\0\end{bmatrix}$$

$$\widetilde{\Delta}_1(\widetilde{x}(t-\tau),t)=\begin{bmatrix}\Delta_1(x(t-\tau),t)\\0\end{bmatrix}$$

Since $\|x(t)\|\le\|\widetilde{x}(t)\|$, it is obvious that

$$\left\|\widetilde{\Delta}_0(\widetilde{x}(t),t)\right\|\le\beta_0\|\widetilde{x}(t)\| \qquad (8.a)$$

$$\left\|\widetilde{\Delta}_1(\widetilde{x}(t-\tau),t)\right\|\le\beta_1\|\widetilde{x}(t-\tau)\| \qquad (8.b)$$

Remark 1: From the results of Brasch and Pearson [1], it is seen that if (A,B,C) is controllable and observable, a compensator is sufficient to achieve arbitrary pole placement for $\widetilde{A}$ in the system consisting of the plant and the dynamic compensator in cascade. It also shows that a compensator of order $\hat{n}=\min(v_c-1,v_o-1)$ is sufficient to achieve this result.

For the constrained uncertain time delay system (7), sufficient conditions for robust stability are described via the Razumikhin-type theorem in the following theorem.

Theorem 1. Suppose the control parameters , A_d,B_d,S,T, are selected such that

(i) $\widetilde{A}+\widetilde{A}_1$ is a stable matrix.

(ii) The Hamiltonian matrix

$$H=\begin{bmatrix}(\widetilde{A}+\widetilde{A}_1) & DD^T\\ -\gamma I_{\overline{n}} & -(\widetilde{A}+\widetilde{A}_1)^T\end{bmatrix} \qquad (9)$$

has no eigenvalues on the imaginary axis for some $\gamma>0$, where $\overline{n}=n+\hat{n}$ and $D=\widetilde{A}_1[\widetilde{A}\quad \widetilde{A}_1\quad \beta_0 I_{\overline{n}}\quad \beta_1 I_{\overline{n}}\quad \frac{1}{2}\|[TC\quad S]\|\widetilde{B}]\in R^{\overline{n}\times(4\overline{n}+m)}$

(iii) $\gamma>2\|P\|(\beta_0+\beta_1\delta)+\left\|P\widetilde{B}\right\|\|[TC\quad S]\|+5\tau^2\delta^2$ · (10)

Where P is the solution of

$$(\widetilde{A}+\widetilde{A}_1)^T P+P(\widetilde{A}+\widetilde{A}_1)+PDD^T P+\gamma I_{\overline{n}}=0 \quad (11)$$

and

$$\delta=\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \qquad (12)$$

then the closed -loop system (7) is globally asymptotic stable.

To prove the theorem, we need the following lemma and observation.

Lemma [4]: For $\gamma\in R^+$, define the $2\overline{n}\times 2\overline{n}$ Hamiltonian matrix as

$$H=\begin{bmatrix}(\widetilde{A}+\widetilde{A}_1) & DD^T\\ -\gamma I_{\overline{n}} & -(\widetilde{A}+\widetilde{A}_1)^T\end{bmatrix}, \qquad (13)$$

Assume that (i) $\widetilde{A}+\widetilde{A}_1$ is a stable matrix, and (ii) H has no eigenvalues on the imaginary axis. Then the algebraic Riccati equation (ARE) of (11) has a positive definite solution P.

Observation 1: Consider the positive definite function

$$V(\widetilde{x}(t))=\widetilde{x}^T(t)P\widetilde{x}(t),\qquad \widetilde{x}(t)\in R^{\overline{n}},t\in R^+$$

where P is the solution of ARE(11). Assume

$$V(\widetilde{x}(t+\theta))<q^2V(\widetilde{x}(t)),\ q>1,\ \theta\in[-\tau,0], \quad (14.a)$$

then

$$V(\widetilde{x}(t+2\theta))<q^2V(\widetilde{x}(t+\theta))<q^4V(\widetilde{x}(t)),\ q>1,\theta\in[-\tau,0] \quad (14.b)$$

Equation (14) implies

$$\|\widetilde{x}(t+\theta)\|<q\delta\|\widetilde{x}(t)\|,\ q>1,\ \theta\in[-\tau,\ 0], \quad (15.a)$$

and

$$\|\widetilde{x}(t+2\theta)\|<q^2\delta\|\widetilde{x}(t)\|,\ q>1,\ \theta\in[-\tau,\ 0], \quad (15.b)$$

where δ is as shown in (12).

Proof: of Theorem 1: First, the closed-loop system (7) with saturation actuators can be expressed as

$$\begin{aligned}\dot{\widetilde{x}}(t)&=(\widetilde{A}+\widetilde{A}_1)\widetilde{x}(t)-\widetilde{A}_1\int_{t-\tau}^{t}\dot{\widetilde{x}}(\lambda)d\lambda+\widetilde{\Delta}_0(\widetilde{x}(t),t)\\&+\widetilde{\Delta}_1(\widetilde{x}(t-\tau),t)-\widetilde{B}[\frac{1}{2}\widetilde{u}(t)-sat(\widetilde{u}(t))]\\&=(\widetilde{A}+\widetilde{A}_1)\widetilde{x}(t)-\widetilde{A}_1\int_{t-\tau}^{t}[\widetilde{A}x(\lambda)+\widetilde{A}_1\widetilde{x}(\lambda-\tau)\\&+\widetilde{\Delta}_0(\widetilde{x}(\lambda),\lambda)+\widetilde{\Delta}_1(\widetilde{x}(\lambda-\tau),\lambda)-\widetilde{B}[\frac{1}{2}\widetilde{u}(\lambda)\\&-sat(\widetilde{u}(\lambda))]d\lambda+\widetilde{\Delta}_0(\widetilde{x}(t),t)+\widetilde{\Delta}_1(\widetilde{x}(t-\tau),t)\\&-\widetilde{B}[\frac{1}{2}\widetilde{u}(t)-sat(\widetilde{u}(t))]\end{aligned} \qquad (16)$$

Suppose A_d, B_d, S, T are selected such that $(\widetilde{A}+\widetilde{A}_1)$ is a stable matrix and H has no eigenvalues on the imaginary axis, then ARE(11) has a positive definite solution P by the above lemma. Let the quadratic Lyapunov function be constructed as

$$V(\widetilde{x}(t)) = \widetilde{x}^T(t)P\widetilde{x}(t), \quad \widetilde{x}(t)\in R^{\bar{n}}, t\in R^+ \qquad (17)$$

According to the concept of quadratic stability, one knows that if the Lyapunov derivative $\dot{V}(\widetilde{x}(t))$ is small than zero, then the system (16) is asymptotically stable.
Then the derivative of the quadratic Lyapunov function (17) along the trajectory of system (16) is

$$\begin{aligned}\dot{V}(\widetilde{x}(t)) &= \widetilde{x}^T(t)[(\widetilde{A}+\widetilde{A}_1)^T P + P(\widetilde{A}+\widetilde{A}_1)]\widetilde{x}(t)\\ &-2\widetilde{x}^T(t)P\widetilde{A}_1\int_{t-\tau}^{t}[\widetilde{A}x(\lambda)+\widetilde{A}_1\widetilde{x}(\lambda-\tau)+\widetilde{\Delta}_0(\widetilde{x}(\lambda),\lambda)\\ &+\widetilde{\Delta}_1(\widetilde{x}(\lambda-\tau),\lambda)-\widetilde{B}[\tfrac{1}{2}\widetilde{u}(\lambda)-sat(\widetilde{u}(\lambda))]d\lambda\\ &+2\widetilde{x}^T(t)P[\widetilde{\Delta}_0(\widetilde{x}(t),t)+\widetilde{\Delta}_1(\widetilde{x}(t-\tau),t)-\widetilde{B}[\tfrac{1}{2}\widetilde{u}(t)-sat(\widetilde{u}(t))]]\end{aligned}$$

$$\begin{aligned}&\le \widetilde{x}^T(t)[(\widetilde{A}+\widetilde{A}_1)^T P + P(\widetilde{A}+\widetilde{A}_1)]\widetilde{x}(t)\\ &+2\int_{t-\tau}^{t}[\|\widetilde{x}^T(t)P\widetilde{A}_1\widetilde{A}\|\|x(\lambda)\|+\|\widetilde{x}^T(t)P\widetilde{A}_1\widetilde{A}_1\|\|\widetilde{x}(\lambda-\tau)\|\\ &+\beta_0\|\widetilde{x}^T(t)P\widetilde{A}_1\|\|\widetilde{x}(\lambda)\|+\beta_1\|\widetilde{x}^T(t)P\widetilde{A}_1\|\|\widetilde{x}(\lambda-\tau)\|\\ &+\frac{1}{2}\|\widetilde{x}^T(t)P\widetilde{A}_1\widetilde{B}\|\|[TC \quad S]\|\|\widetilde{x}(\lambda)\|]d\lambda+2\|\widetilde{x}^T(t)P\|[\beta_0\|\widetilde{x}(t)\|\\ &+\beta_1\|\widetilde{x}(t-\tau)\|]+\|\widetilde{x}^T(t)P\widetilde{B}\|\|[TC \quad S]\|\|\widetilde{x}(t)\|\end{aligned} \qquad (18)$$

where (2) and (5) have introduced.
Using the inequality, $2ab \le q^{-1}a^2 + qb^2$ for any a, $b\in R$ and $q>0$, one obtains

$$\begin{aligned}&2\int_{t-\tau}^{t}\|\widetilde{x}^T(t)P\widetilde{A}_1\widetilde{A}\|\|x(\lambda)\|d\lambda\\ &\le \int_{t-\tau}^{t}\{\frac{1}{\tau}[\widetilde{x}^T(t)P\widetilde{A}_1AA^T A_1^T\widetilde{x}(t)]+\tau\widetilde{x}^T(\lambda)\widetilde{x}(\lambda)\}d\lambda\\ &=\widetilde{x}^T(t)P\widetilde{A}_1\widetilde{A}\widetilde{A}^T\widetilde{A}_1^T\widetilde{x}(t)+\tau\int_{t-\tau}^{t}\widetilde{x}^T(\lambda)\widetilde{x}(\lambda)d\lambda\end{aligned} \qquad (19)$$

$$\begin{aligned}&2\int_{t-\tau}^{t}\|\widetilde{x}^T(t)P\widetilde{A}_1\widetilde{A}_1\|\|\widetilde{x}(\lambda-\tau)\|d\lambda\\ &\le \widetilde{x}^T(t)P\widetilde{A}_1\widetilde{A}_1\widetilde{A}_1^T\widetilde{A}_1^T\widetilde{x}(t)+\tau\int_{t-2\tau}^{t-\tau}\widetilde{x}^T(\lambda)\widetilde{x}(\lambda)d\lambda\end{aligned} \qquad (20)$$

$$\begin{aligned}\dot{V}(\widetilde{x}(t)) &\le \widetilde{x}^T(t)[(\widetilde{A}+\widetilde{A}_1)^T P + P(\widetilde{A}+\widetilde{A}_1)+PDD^T P]\widetilde{x}(t)\\ &+[2\beta_0\|P\|+\|[TC \quad S]\|\|P\widetilde{B}\|]\|\widetilde{x}(t)\|^2+2\beta_1\|P\|\|\widetilde{x}(t)\|\|\widetilde{x}(t-\tau)\|\\ &+2\tau\int_{t-2\tau}^{t}\|\widetilde{x}(\lambda)\|^2 d\lambda+\tau\int_{t-\tau}^{t}\|\widetilde{x}(\lambda)\|^2 d\lambda\end{aligned}$$

$$\begin{aligned}&2\|\widetilde{x}^T(t)P\|[\beta_0\|\widetilde{x}(t)\|+\beta_1\|\widetilde{x}(t-\tau)\|]\\ &+\|\widetilde{x}^T(t)P\widetilde{B}\|\|[TC \quad S]\|\|\widetilde{x}(t)\|\end{aligned} \qquad (21)$$

$$\begin{aligned}&\le 2\beta_0\|P\|\|\widetilde{x}(t)\|^2+2\beta_1\|P\|\|\widetilde{x}(t)\|\|\widetilde{x}(t-\tau)\|\\ &+\|[TC \quad S]\|\|P\widetilde{B}\|\|\widetilde{x}(t)\|^2\end{aligned} \qquad (22)$$

Substituting (19)- (22) into (18) yields

$$\begin{aligned}\dot{V}(\widetilde{x}(t)) &\le \widetilde{x}^T(t)[(\widetilde{A}+\widetilde{A}_1)^T P + P(\widetilde{A}+\widetilde{A}_1)+PDD^T P]\widetilde{x}(t)\\ &+[2\beta_0\|P\|+\|[TC \quad S]\|\|P\widetilde{B}\|]\|\widetilde{x}(t)\|^2+2\beta_1\|P\|\|\widetilde{x}(t)\|\|\widetilde{x}(t-\tau)\|\\ &+2\tau\int_{t-2\tau}^{t}\|\widetilde{x}(\lambda)\|^2 d\lambda+\tau\int_{t-\tau}^{t}\|\widetilde{x}(\lambda)\|^2 d\lambda\end{aligned} \qquad (23)$$

where D is defined in theorem.
Applying the Razumikhin-type theorem with assumptions of (14)-(15), and substituting them into (23), we can obtain the following bound on $\dot{V}(x(t))$

$$\dot{V}(\widetilde{x}(t)) \le -w\|\widetilde{x}(t)\|^2 \qquad (24)$$

where

$$w=\gamma-2\|P\|(\beta_0+\beta_1\delta)+\|P\widetilde{B}\|\|[TC \quad S]\|+5\tau^2\delta^2 \qquad (25)$$

If condition (iii) of the present theorem is satisfied, then a sufficiently small q>1 exists such that w>0. Thus according to the Razumikhin type theorem, system (16) is asymptotically stable. This implies that system (1) and (7) are all also asymptotically stable. ■

Remark 2: In the above analysis, the constraint saturation which is considered inside the sector [0, 1] will bring about conservative results (Su et al., [12]). If the saturation nonlinearity, in actual system, is inside a finite part, inside the sector [a, 1], i.e. $\dfrac{|u_i(t)_{\max}|}{u_i^H \text{ (or } u_i^L)} \le \dfrac{1}{a}$, where $0\le a\le 1$, and $|u_i(t)_{\max}|$ is the maximum absolute value of each $u_i(t)$ for all t., the results can be much improved. From Fig. 2, it is obvious

$$\left|sat(u_i(t))-\tfrac{1}{2}(1+a)u_i(t)\right| \le \left|\tfrac{1}{2}(1-a)u_i(t)\right| \qquad (26)$$

Thus we can recast (5) as follows

$$\left\|sat(u(t))-\tfrac{1}{2}(1+a)u(t)\right\| \le \left\|\tfrac{1}{2}(1-a)u(t)\right\| \qquad (27)$$

We also recast the closed-loop equations (7) as

$$\dot{\tilde{x}}(t) = \tilde{A}_a\tilde{x}(t) + \tilde{A}_1\tilde{x}(t-\tau) + \tilde{\Delta}_0(\tilde{x}(t),t) + \tilde{\Delta}_1(\tilde{x}(t-\tau),t) - \tilde{B}[\frac{(1+a)}{2}\tilde{u}(t) - sat(\tilde{u}(t))] \quad (28)$$

where

$$\tilde{A}_a = \begin{bmatrix} A + \frac{(1+a)}{2}BTC & \frac{(1+a)}{2}BS \\ B_dC & A_d \end{bmatrix} \quad (29)$$

and we have the following improved corollary

Corollary 1: Suppose the saturation nonlinearity is inside a finite part, as shown in Fig. 2, and the control parameters , A_d, B_d, S, T, are selected such that

(i) $\tilde{A}_a + \tilde{A}_1$ is a stable matrix.

(ii) The Hamiltonian matrix

$$H = \begin{bmatrix} (\tilde{A}_a + \tilde{A}_1) & DD^T \\ -\gamma I_{\bar{n}} & -(\tilde{A}_a + \tilde{A}_1)^T \end{bmatrix} \quad (30)$$

has no eigenvalues on the imaginary axis for some $\gamma > 0$, where $\bar{n} = n + \hat{n}$ and $D = \tilde{A}_1[\tilde{A}_a \quad \tilde{A}_1 \quad \beta_0 I_{\bar{n}} \quad \beta_1 I_{\bar{n}} \quad \frac{(1-a)}{2}\|[TC \quad S\|\tilde{B}] \in R^{\bar{n}\mathrm{x}(4\bar{n}+m)}$.

(iii) $\gamma > 2\|P\|(\beta_0 + \beta_1\delta) + (1-a)\|P\tilde{B}\|\|[TC \quad S]\| + 5\tau^2\delta^2$ (31)

.

Where P is the solution of

$$(\tilde{A}_a + \tilde{A}_1)^T P + P(\tilde{A}_a + \tilde{A}_1) + PDD^T P + \gamma I_{\bar{n}} = 0 \quad (32)$$

then the closed -loop system (1) is globally asymptotic stable.

The proof of Corollary 1 is similar to the one in Theorem 1. Thus is omitted here

4. An Example

Consider the following linear constrained time delay system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & 0.2 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & -2 \end{bmatrix}\begin{bmatrix} x_1(t-\tau) \\ x_2(t-\tau) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} sat(u(t)) + \Delta_0(x(t),t) + \Delta_1(x(t-\tau),t)$$

$$y(t) = \begin{bmatrix} 0 & 0.4 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Where $\tau = 0.1$ and perturbations Δ_0 and Δ_1 are assumed to be bounded by $\beta_0 = 0.1$ and $\beta_1 = 0.2$, respectively. It is found that the system matrix A is unstable. Since $v_o = v_c = 2$, according to Brash and Pearson (1970), it is sufficient to achieve arbitrary pole placement for $\tilde{A}$ by only using a first-order compensators. Thus, let the first-order compensator as

$$\dot{x}_d(t) = -3x_d(t) - y(t)$$
$$u(t) = 0.5x_d(t) - 3y(t)$$

such that $\tilde{A}$ is stable and

$$\tilde{A} + \tilde{A}_1 = \begin{bmatrix} -2.8 & 0.1 & 0 \\ 0.1 & -2.4 & 0.25 \\ 0 & -0.4 & -3 \end{bmatrix}$$

is also stable. Then the ARE of (11) with $r = 0.25$ has a positive definite solution P as

$$P = \begin{bmatrix} 0.0449 & 0.0025 & 0.0001 \\ 0.0025 & 0.0717 & 0.0003 \\ 0.0001 & 0.0003 & 0.0417 \end{bmatrix}$$, and

$$\gamma - 2\|P\|(\beta_0 + \beta_1\delta) + \|P\tilde{B}\|\|[TC \quad S]\| + 5\tau^2\delta^2 = 0.0184 > 0$$

.

Therefore, the stability conditions in the Theorem 1 are satisfied and the stability of the closed loop system with saturating actuator can be guaranteed.

5. Conclusion

In this paper, a dynamic compensator to stabilize a class of uncertain constrained time delay systems is developed. In contrast to the previous work, the system matrix A can be unstable. For the case of unstable system matrix A, the globally stabilization can also be achieve by our results.

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