Control of Uncertain Non-Minimum Phase CSTRs Using Sliding Mode Control Techniques

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Abstract

In this paper, a novel sliding mode control (SMC) scheme is developed for continuous stirred tank reactors (CSTRs) in the presence of simultaneously the non-minimum phase behavior and process uncertainties. For circumventing the negative effect of the inverse response, in the control scheme a statically equivalent output map (SEOM) is incorporated. Based on utilizing a zero-placement method, an effective algorithm for synthesizing the SEOM is presented. Through the help of the SEOM, a gain adaptive sliding mode controller is constructed to provide stable and robust closed-loop performance. In addition, the potential use of a sliding observer along with the proposed scheme is investigated in this work. Extensive simulation results reveal that the proposed design methodology is applicable and promising for the regulation control of CSTRs in the presence of non-minimum phase and process uncertainties.

1. Introduction

In chemical industry, continuous stirred tank reactor (CSTR), which is commonly used to convert reactants into products, is the central part of the whole process. For the purpose of achieving high conversion and economic benefits, CSTRs are usually operated around some certain equilibrium points which correspond to an optimal yield or an optimal productivity of the process [1]. A typical model for an exothermic reaction in a CSTR would incorporate Arrhenius reaction rates in a multi-dimensional set of coupled nonlinear ordinary differential equations. However, owing to the complexities of the consecutive and side reactions inside, the CSTR may exhibit a variety of exotic behavior as a function of the parameters of the system. These strongly pronounced nonlinearity and variety of the process dynamics such as inverse response and parameter uncertainties can make the control of CSTRs a challenging problem for process control engineers.

In the recent years, there has been a growing interest in the development of diversified control systems for the operation of CSTRs and significant developments have been made in the field of nonlinear process control [2-7]. Generally, one of the most difficult control problems arose from the operation of the CSTR would be the treatment of the inverse response behavior [8]. An inverse response means that the initial response of the process is in a direction opposite to its final response during a dynamic testing. For CSTRs, it can be caused by competing effects of two reaction dynamics. A process having inverse response behavior is often called the non-minimum phase process in the literature. As for nonlinear, non-minimum phase processes like many CSTRs, there has been attracted considerable attention during the past decade [8-14].

Despite the significant progress and remarkable interest in nonlinear, non-minimum phase processes, the control of CSTRs with the existence of simultaneously the nonlinearities, uncertainties and inverse response behavior has been rarely addressed [15-17]. This is because that the mathematical model for CSTRs could be inaccurate owing to that in the reaction systems there will unavoidably face with unmodeled side reaction dynamics, unknown internal or external noises and environmental influences, etc. The presence of these uncertainties could lead to the discrepancy between the true process and the formulated mathematical model used for controller design. Especially, the presence of model uncertainties can induce additional difficulty in obtaining an inverse of the process and can thus make many existing schemes fail to control the nonlinear uncertain CSTRs. Some exceptional successes in the design of a robust control system for a certain class of nonlinear, uncertain, non-minimum phase processes are reviewed as follows. Based on the approximate input/output linearization and a special state transformation, Jo et al. [15] investigated the problem of robust stabilization for a class of nonlinear systems with non-minimum phase behavior and mismatched uncertainties. Utilizing the framework of Hauser et al. [10], Wu [16] introduced an approximate state output feedback control scheme for asymptotic output regulation of an uncertain non-minimum phase system. Using the method of system center, Shkolnikov and Shtessel [17] employed some linear algebraic methods and sliding mode control approach to develop a method for asymptotic output tracking control of a class of causal non-minimum phase uncertain nonlinear processes.

In this paper, we consider the design of a sliding mode control (SMC) scheme for the nonlinear regulation of

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CSTRs whose dynamics present simultaneously the non-minimum phase behavior and process uncertainties. Based on the concept of statically equivalent minimum phase output, an auxiliary output along with a new design algorithm is proposed such that the zero offset performance and minimum phase behavior can be ensured under process variations and uncertainties. By means of the designed auxiliary output and the input-output feedback linearization, a sliding mode controller which is able to curb the effect of uncertainties and can thus provide robust control performance is proposed. Additionally, the potential use of a sliding observer along with the proposed scheme for practical implementation is also explored in this paper.

2. A sliding mode control scheme for nonlinear, uncertain, non-minimum phase CSTRs

2.1 Control system configuration and system description

Consider an open-loop stable single-input/single-output nonlinear uncertain CSTR described by

$$\dot{\mathbf{x}} = (\mathbf{f}(\mathbf{x}) + \Delta\mathbf{f}(\mathbf{x})) + (\mathbf{g}(\mathbf{x}) + \Delta\mathbf{g}(\mathbf{x}))\, u \tag{1a}$$

$$y = h(\mathbf{x}) \tag{1b}$$

where $\mathbf{x}$ is the n-dimensional state vector, and y and u are, respectively, the output and manipulated input of the process. $\mathbf{f}(\cdot)$, $\mathbf{g}(\cdot)$, $\Delta\mathbf{f}(\cdot)$ and $\Delta\mathbf{g}(\cdot)$ are smooth vector fields on an open set $U \in R^n$ and $h(\cdot)$ a smooth function on U. Without loss of generality, we assume that the origin $\mathbf{x} = 0$ is a uniformly asymptotically stable equilibrium point of the unforced nominal system and $h(\mathbf{x})$ vanishes at that equilibrium point. In other words, $\mathbf{f}(0) = 0$ and $h(0) = 0$. This means that y represents the tracking error. It should be noted that a given model could be easily rewritten in this form by defining appropriate deviation variables.

The system under consideration is assumed to be a non-minimum phase one, i.e., it has unstable zero dynamics in the sense defined in Byrnes and Isidori [18]. To control this kind of processes, in this paper we propose a sliding mode control system configuration as shown in Fig. 1. For clear presentation, we describe the design of the whole control scheme through individual parts.

2.2 Design of a statically equivalent output map (SEOM)

To compensate the undesirable inverse response, in the proposed scheme the auxiliary output $h_s(\mathbf{x})$ should be statically equivalent to the original output $h(\mathbf{x})$, i.e., $h_s(\mathbf{x})$ should have the same static gain as the actual process output $h(\mathbf{x})$ and make a $u - y_s$ system in the minimum phase despite of the influence of process uncertainties. Before embarking on the design of a statically equivalent output map (SEOM), $h_s(\mathbf{x})$, for nonlinear uncertain processes, we first review a zero-assignment method.

2.2.1 An auxiliary output design method using zero assignment technique

Consider the following nonlinear non-minimum phase system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u \tag{2a}$$

$$y = h(\mathbf{x}) \tag{2b}$$

An auxiliary process which is statically equivalent to the nonlinear system of Eq. (2) can be given by

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u \tag{3a}$$

$$y_{eq} = h_{eq}(\mathbf{x}) \tag{3b}$$

where $h_{eq}(\cdot)$ is an auxiliary output to make the system locally minimum phase. A formulation for $h_{eq}(\cdot)$ using the original system dynamics can be given by [13]:

$$h_{eq}(\mathbf{x}) = h(\mathbf{x}) + \sum_{j=1}^{n-1} \lambda_j(\mathbf{x})\Phi_j(\mathbf{x}) \tag{4}$$

where

$$\Phi_j(\mathbf{x}) = \det\begin{bmatrix} f_j(\mathbf{x}) & g_j(\mathbf{x}) \\ f_n(\mathbf{x}) & g_n(\mathbf{x}) \end{bmatrix}, \quad j = 1,2,\ldots,n-1 \tag{5}$$

are functions vanishing on the equilibrium curve and $\lambda_j(\mathbf{x}), j = 1,2,\ldots,n-1$, are functions being chosen such that $h_{eq}(\mathbf{x})$ is statically equivalent to $h(\mathbf{x})$. Because the selection of $\lambda_j(\mathbf{x})$ are arbitrary, for simplicity we consider in this paper the subclass of Eq. (4) with constant weights λ_j, i.e.

$$h_{eq}(\mathbf{x}) = h(\mathbf{x}) + \sum_{j=1}^{n-1} \lambda_j \Phi_j(\mathbf{x}) \tag{6}$$

The adjustable weights λ_j can be obtained by the assignment of zeros. Let $(\mathbf{x}_s, u_s)$ be a reference equilibrium point, then we can define the zeros polynomials corresponding to $h(\mathbf{x})$ and $\Phi_j(\mathbf{x})$, $j = 1,2,\ldots,n-1$, respectively, as

$$P(s) = \frac{\partial h(\mathbf{x}_s)}{\partial \mathbf{x}} \mathrm{Adj}\left[sI - \left(\frac{\partial \mathbf{f}(\mathbf{x}_s)}{\partial \mathbf{x}} + u_s \frac{\partial \mathbf{g}(\mathbf{x}_s)}{\partial \mathbf{x}} \right) \right] \mathbf{g}(\mathbf{x}_s) = p_0 + p_1 s + \cdots + p_{n-1} s^{n-1} \tag{7}$$

and

$$Q_j(s) = \frac{\partial \Phi_j(\mathbf{x}_s)}{\partial \mathbf{x}} \mathrm{Adj}\left[sI - \left(\frac{\partial \mathbf{f}(\mathbf{x}_s)}{\partial \mathbf{x}} + u_s \frac{\partial \mathbf{g}(\mathbf{x}_s)}{\partial \mathbf{x}} \right) \right] \mathbf{g}(\mathbf{x}_s) = \tilde{q}_{j,1} s + \cdots + \tilde{q}_{j,n-1} s^{n-1}, \quad j = 1,2,\ldots,n-1 \tag{8}$$

Furthermore, let $z_j, j = 1,2,\ldots,n-1$, be the desirable zeros for $h_{eq}(\mathbf{x})$ at the reference equilibrium point. The given values of z_j and the requirement of static equivalence with $h(\mathbf{x})$ completely specifies the desirable zero polynomial for $h_{eq}(\mathbf{x})$ as [13]:

$$P^d(s) = p_0 \prod_{j=1}^{n-1} \left(1 - \frac{s}{z_j} \right) = p_0 + p_1^d s + \cdots + p_{n-1}^d s^{n-1} \tag{9}$$

The necessary values of the adjustable weights λ_j, which will make the zeros polynomial of $h_{eq}(\mathbf{x})$ equal to $P^d(s)$, can be obtained by solving the following equations

$$P(s)+\sum_{j=1}^{n-1}\lambda_j Q_j(s)=P^d(s) \tag{10}$$

or explicitly from the following system of linear equations

$$\begin{bmatrix} \tilde{q}_{11} & \tilde{q}_{21} & \cdots & \tilde{q}_{n-1,1} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{q}_{1,n-1} & \tilde{q}_{2,n-1} & \cdots & \tilde{q}_{n-1,n-1} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_{n-1} \end{bmatrix} = \begin{bmatrix} p_1^d - p_1 \\ \vdots \\ p_{n-1}^d - p_{n-1} \end{bmatrix} \tag{11}$$

It should be noted that the solution of the above linear equations will admit a unique solution as long as $\left[\frac{\partial \mathbf{f}(\mathbf{x}_s)}{\partial \mathbf{x}}+u_s\frac{\partial \mathbf{g}(\mathbf{x}_s)}{\partial \mathbf{x}}\right]$ and $\mathbf{g}(\mathbf{x}_s)$ from a controllable pair, i.e., the columns $[\tilde{q}_{j1},\tilde{q}_{j2},\ldots,\tilde{q}_{j,n-1}]^T$ is linear independent. Also, the presented technique for the design of a statically equivalent minimum phase output is based on a perfect nominal model of the process. If the process model is perfect, the designed auxiliary output, $h_{eq}(\mathbf{x})$, is statically equivalent to the actual process output, $h(\mathbf{x})$, and thus makes $u-y_{eq}$ system minimum phase. However, as the process model is not so perfect or the process is subject to model uncertainties, the minimum phase behavior would not be ensured perfectly. In other words, if the controller is designed on the base of the auxiliary output, the closed-loop system may hardly guarantee zero offset performance when facing with process uncertainties. To overcome the drawbacks, in the next subsection we will propose a new algorithm and a modified synthetic output for uncertain non-minimum phase process having uncertainties.

2.2.2 Synthesis of a statically equivalent output map (SEOM) for use under process uncertainties

The purpose of this subsection is two folds. The first one is to ensure the minimum phase behavior under the influence of process uncertainties and the other one is to guarantee the statically equivalent output property of y_s. To meet the first goal, a new algorithm for redesign of λ_j is proposed. The idea is based on shifting the desired zeros to make the constructed minimum output invariant despite the influence of the process uncertainties. The design procedure is summarized as follows:

Initialization: Choose the desired zeros, $z_j^0 \in \mathrm{LHP}$, at the reflections of the RHP zeros with respect to the imaginary axis. Also, set $\Delta z_j > 0$ for pole shifting. Let $i=1$ and $z_j^i = z_j^0$.

Step 1: Set $P^d(s)$ based on the zeros z_j^i. Calculate $\lambda_j^i,\ j=1,2,\ldots,n-1$, from Eq. (11) and then construct $h_{eq}^i(\mathbf{x})$ according to Eq. (6).

Step 2: Check whether $h_{eq}^i(\mathbf{x})$ is minimum phase or not under process uncertainties by Monte Carlo simulations. If yes, stop. Otherwise, go next step.

Step 3: Shifting the desired zeros by $z_j^{i+1}=z_j^i-\Delta z_j$, then set $i=i+1$ and go back to step 1.

Once the present design procedure has converged, one can construct a minimum phase output map for the original uncertain process. However, the obtained output map, though is minimum phase, may not assure that the output is statically equivalent to the actual output under process uncertainties. To gain the statically equivalent property for the second goal, we suggest the following auxiliary output for control:

$$y_s \equiv h_s(\mathbf{x}) = y + (y_{eq}-y)e^{-\lambda_s t} \tag{12}$$

where $y_{eq}=h_{eq}(\mathbf{x})$, $y=h(\mathbf{x})$ and $\lambda_s>0$ is the tuning constant. The role of λ_s in this auxiliary output map is to make a smooth transition from the minimum phase one to actual process output. Actually, the selection of λ_s depends on the process dynamic characteristics. From Eq. (12), it is clear that, for small t, y_s is approximately equal to the minimum phase output map, y_{eq}. While for a larger t, y_s is approaching to the actual process output. That is, y_s appears to be a statically equivalent output map (SEOM) to the actual process output, which ensures no steady state offset and minimum phase behavior despite of the influence of process uncertainties.

2.3 Design of a sliding mode controller

Based on the synthesized auxiliary output y_s, the nonlinear uncertain model used for controller design is given by

$$\dot{\mathbf{x}} = (\mathbf{f}(\mathbf{x})+\Delta\mathbf{f}(\mathbf{x}))+(\mathbf{g}(\mathbf{x})+\Delta\mathbf{g}(\mathbf{x}))\,u \tag{13a}$$

$$y_s = h_s(\mathbf{x}) \tag{13b}$$

It should be noted here that the present system is minimum phase under uncertainties and the auxiliary output y_s is statically equivalent to the actual process output y. Let the Lie derivative of a smooth function $h_s(\mathbf{x})$ along a vector field $\mathbf{g}(\mathbf{x})$ be defined as:

$$L_{\mathbf{g}}h_s(\mathbf{x}) = \frac{\partial h_s(\mathbf{x})}{\partial \mathbf{x}}\mathbf{g}(\mathbf{x}) = \sum_{i=1}^{n}\frac{\partial h_s(\mathbf{x})}{\partial x_i}g_i(\mathbf{x}) \tag{14}$$

In terms of Lie derivative, the relative degree of the system (13) is defined as

$$\rho = \min\left\{m : L_{\mathbf{g}}L_{\mathbf{f}}^{m-1}h_s(\mathbf{x})\neq 0\right\} \tag{15}$$

Similarly, let $\kappa=\min\left\{m : L_{\Delta\mathbf{f}}L_{\mathbf{f}}^{m-1}h_s(\mathbf{x})\neq 0\right\}$ and $w=\min\left\{m : L_{\Delta\mathbf{g}}L_{\mathbf{f}}^{m-1}h_s(\mathbf{x})\neq 0\right\}$ be the relative degrees of the uncertainties $\Delta\mathbf{f}$ and $\Delta\mathbf{g}$, respectively. Also, in the paper, we assume the uncertainties satisfy the generalized matching condition, i.e., $w\geq\rho=\kappa$. Under the above-mentioned assumptions, there exists a coordinate transformation of

$$\left[\boldsymbol{\xi}^T,\boldsymbol{\eta}^T\right]^T = \mathbf{T}(\mathbf{x}) = \left[h_s(\mathbf{x}),L_{\mathbf{f}}h_s(\mathbf{x}),\cdots,L_{\mathbf{f}}^{\rho-1}h_s(\mathbf{x}),\eta_1(\mathbf{x}),\cdots,\eta_{n-\rho}(\mathbf{x})\right]^T \tag{16}$$

By applying on this coordinate transformation, we can transfer the nonlinear uncertain system of Eq. (13) to its normal form as

$$\dot{\xi}_i = \xi_{i+1}, \quad i = 1, 2, \cdots, \rho - 1 \tag{17a}$$

$$\dot{\xi}_\rho = [b(\xi, \boldsymbol{\eta}) + \Delta b(\xi, \boldsymbol{\eta})] + [a(\xi, \boldsymbol{\eta}) + \Delta a(\xi, \boldsymbol{\eta})]u \tag{17b}$$

$$\dot{\boldsymbol{\eta}} = \mathbf{q}(\xi, \boldsymbol{\eta}) + \boldsymbol{\varphi}(\xi, \boldsymbol{\eta}) \tag{17c}$$

$$y_s = \xi_1 \tag{17d}$$

where $a(\xi, \boldsymbol{\eta})$, $\Delta a(\xi, \boldsymbol{\eta})$, $b(\xi, \boldsymbol{\eta})$, $\Delta b(\xi, \boldsymbol{\eta})$, $\mathbf{q}(\xi, \boldsymbol{\eta})$ and $\phi_i(\xi, \boldsymbol{\eta})$ are given, respectively, by

$$a(\xi, \boldsymbol{\eta}) = L_{\mathbf{g}} L_{\mathbf{f}}^{\rho-1} h_s \circ \mathbf{T}^{-1}(\xi, \boldsymbol{\eta}) \tag{18}$$

$$\Delta a(\xi, \boldsymbol{\eta}) = L_{\Delta\mathbf{g}} L_{\mathbf{f}}^{\rho-1} h_s \circ \mathbf{T}^{-1}(\xi, \boldsymbol{\eta}) \tag{19}$$

$$b(\xi, \boldsymbol{\eta}) = L_{\mathbf{f}}^{\rho} h_s \circ \mathbf{T}^{-1}(\xi, \boldsymbol{\eta}) \tag{20}$$

$$\Delta b(\xi, \boldsymbol{\eta}) = L_{\Delta\mathbf{f}} L_{\mathbf{f}}^{\rho-1} h_s \circ \mathbf{T}^{-1}(\xi, \boldsymbol{\eta}) \tag{21}$$

$$q_i(\xi, \boldsymbol{\eta}) = L_{\mathbf{f}} T_{\rho+i}(\mathbf{x}), \quad i = 1, 2, \cdots, n - \rho \tag{22}$$

$$\phi_i(\xi, \boldsymbol{\eta}) = L_{\Delta\mathbf{f}} T_{\rho+i}(\mathbf{x}) + L_{\Delta\mathbf{g}} T_{\rho+i}(\mathbf{x})\, u \quad, i = 1, 2, \ldots, n - \rho \tag{23}$$

and

$$\mathbf{x} = \mathbf{T}^{-1}(\xi, \boldsymbol{\eta}) \tag{24}$$

The nonlinear state feedback control law that provides input-output linearization of the nominal system can be expressed as

$$u = \frac{v - b(\xi, \boldsymbol{\eta})}{a(\xi, \boldsymbol{\eta})} \tag{25}$$

where v is the new controller to be designed for various purpose of control. In this paper, we propose the use of the sliding mode controller of Chen and Dai [19]. The sliding mode controller design procedure is stated as follows. First, define a sliding surface $\delta(t)$ as

$$\delta = \mathbf{c}^T \xi = \sum_{i=1}^{\rho} c_i \xi_i \,, \quad c_\rho = 1. \tag{26}$$

where the coefficients c_i are chosen such that the polynomial $\Gamma(\lambda) = \lambda^{\rho-1} + c_{\rho-1}\lambda^{\rho-2} + \cdots + c_2\lambda + c_1$ has all roots in the open left-half complex plane. Based on the sliding surface being selected, the next step is to synthesize the control law for achieving some certain robust stability and system performance. In this paper, we adopt a gain adaptive sliding mode control law v as [19]

$$v = -k\delta - \mathrm{sat}(\delta/\beta)[b_{\min}^{-1}(f_{\max} + |\delta|)] \tag{27}$$

where

$$f_{\max} = \sup_{(\xi,\boldsymbol{\eta})\in\mathbf{T}(U)} \left| \Delta b(\xi, \boldsymbol{\eta}) - \Delta a(\xi, \boldsymbol{\eta}) \frac{b(\xi, \boldsymbol{\eta})}{a(\xi, \boldsymbol{\eta})} \right| \tag{28}$$

$$b_{\min} = 1 - \sup_{(\xi,\boldsymbol{\eta})\in\mathbf{T}(U)} \left| \frac{\Delta a(\xi, \boldsymbol{\eta})}{a(\xi, \boldsymbol{\eta})} \right| \tag{29}$$

$$\mathrm{sat}(\delta/\beta) = \begin{cases} \delta/\beta, & \text{if } |\delta/\beta| < 1 \\ \mathrm{sign}(\delta/\beta), & \text{if } |\delta/\beta| \geq 1 \end{cases} \tag{30}$$

and the gain k is adaptively tuned by $\dot{k} = \tilde{\gamma}\delta^2$ ($\tilde{\gamma} > 0$). Also, in the control law, β is the user-specified boundary layer thickness used to eliminate the input chattering.

By using the sliding mode control law of Eq. (27), the closed-loop system can be represented by

$$\dot{\xi} = \mathbf{A}_c \xi + \mathbf{B}_c [v + b_s^{-1}(f_s + \mathbf{c}^T \xi)] \tag{31a}$$

$$\dot{\boldsymbol{\eta}} = \mathbf{q}(\xi, \boldsymbol{\eta}) + \boldsymbol{\varphi}(\xi, \boldsymbol{\eta}) \tag{31b}$$

where

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \ldots & 1 & 0 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 1 \\ -c_1 & -c_2 & -c_3 & -c_4 & \ldots & -c_{\rho-1} & -1 \end{bmatrix} \tag{32}$$

$$\mathbf{B}_c = [0 \ \cdots \ 0 \ \ b_s]^T \tag{33}$$

$$f_s := f_s(\xi, \boldsymbol{\eta}) = \Delta b(\xi, \boldsymbol{\eta}) - \Delta a(\xi, \boldsymbol{\eta}) \frac{b(\xi, \boldsymbol{\eta})}{a(\xi, \boldsymbol{\eta})} \tag{34}$$

and

$$b_s := b_s(\xi, \boldsymbol{\eta}) = 1 + \frac{\Delta a(\xi, \boldsymbol{\eta})}{a(\xi, \boldsymbol{\eta})} \tag{35}$$

Following the similar analysis procedure as stated in Chen and Dai [19], the closed-loop control system is robustly stabilized and possesses the robust properties of uniform ultimate boundedness and uniform stability. Essentially, the sliding mode control law in Eq. (27) consists chiefly of the four parameters: $\mathbf{c}$, $\tilde{\gamma}$, $k(0)$ and β. The coefficient vector, $\mathbf{c}$, can be viewed as the weighting factor for the state vector ξ. The parameter $\tilde{\gamma}$ is a positive constant related to the tuning rate of the adaptive gain. In the light of the tuning rule $\dot{k} = \tilde{\gamma}\delta^2$, a non-negative initial setting of k, $k(0)$, is sufficient to guarantee the negative feedback. Notice again that the parameter β is introduced to eliminate the input chattering.

3. Regulation control of a nonisothermal Van de Vusse reactor in the presence of non-minimum phase behavior and process uncertainties

Consider a Van de Vusse reactor in which the following series/parallel reactions [20, 21] are taking place:

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C \tag{36a}$$

$$2A \xrightarrow{k_3} D \tag{36b}$$

where A is the reactant, B the desired product, and C and D are unwanted by-products. An industrial example is the production of cyclopentenol (B) from cyclopentadiene (A) by acid-catalyzed electrophilic addition of water in dilute solution, where cyclopentanediol (C) and dicyclopentadiene (D) are produced as side products [22]. The reaction rates of A and B are assumed to be

$$r_A = -k_1(T)C_A - k_3(T)C_A^2 \tag{37a}$$

$$r_B = k_1(T)C_A - k_2(T)C_B \tag{37b}$$

where the rate coefficients $k_i(T)$ are given by Arrhenius expressions:

$$k_i(T) = k_{i0}\exp(-E_i/RT) \tag{38}$$

In the rate equations, C_A and C_B represent, respectively, the concentrations of the species A and B inside the reactor. The volume of the CSTR, V, is assumed to be constant during operation. The feed stream consisting of pure A is fed to the CSTR at an inlet flow rate of F. Besides, the concentration of A in the feed stream is $C_{A0} = 5\ \text{gmol}\cdot\text{L}^{-1}$ and the feed temperature is $T_0 = 403.15$ K. By means of the material balances for species A and B and the energy balance for the reactor, the dynamics of this CSTR can be modeled as follows:

$$\frac{dC_A}{dt} = -k_1(T)C_A - k_3(T)C_A^2 + (C_{A0} - C_A)u_m \tag{39a}$$

$$\frac{dC_B}{dt} = k_1(T)C_A - k_2(T)C_B - C_B u_m \tag{39b}$$

$$\frac{dT}{dt} = \frac{1}{\rho_s C_P}[(-\Delta H_1)k_1(T)C_A + (-\Delta H_2)k_2(T)C_B + (-\Delta H_3)k_3(T)C_A^2 + Q_s] + (T_0 - T)u_m \tag{39c}$$

where T is the reactor temperature, $u_m = F/V$ denotes the dilution rate, ΔH_i is the reaction heats, ρ_s and C_P are the density and specific heat of the reaction mixture, respectively, and $-Q_s$ the cooling rate per unit volume. The values for the model parameter constants and operation conditions are listed in Table 1. The control objective is to maintain the process output C_B as close as possible to the set point (steady-state value) by adjusting the dilution rate, $u_m = F/V$.

From an open-loop test as shown in Fig. 2, it can be easily observed that the equilibrium curve of this nonisothermal Van de Vusse reactor consists of two branches: a high temperature-high production rate branch, and a low temperature-low production rate branch. Considering the fact that there are always limits on the flow rate and the operating temperature, the high temperature-high production rate branch seems to be more desirable and economical for the operation of the reactor. Thus, in this paper the CSTR operation along the high temperature-high production rate branch of the equilibrium curve is considered. Let the reference steady-state be chosen as $C_{Ad} = 1.25\ \text{mol}\cdot\text{L}^{-1}$, $C_{Bd} = 0.90\ \text{mol}\cdot\text{L}^{-1}$ and $T_d = 407.15$ K, we have the steady state dilution rate of $u_d = 19.5218\ \text{hr}^{-1}$. Around this steady-state, the linearization model of this process is given by the following transfer function as

$$G(s) = \frac{-0.9s^2 + 100.3636s + 1233.0296}{s^3 + 162.8s^2 + 7592s + 115300} \tag{40}$$

This shows that the process exhibits locally asymptotically stable and locally non-minimum phase owing to no RHP pole (-96.518 and $-33.141 \pm 9.8118i$) and the presence of a RHP zero (-11.1673 and +122.6824). Since this Van de Vusse reactor presents non-minimum phase behavior around the reference steady-state, the conventional feedback linearization cannot be directly applied to this nonlinear process.

To apply the proposed scheme to this Van de Vusse reactor, we first define the deviation variables: $x_1 = C_A - C_{Ad}$, $x_2 = C_B - C_{Bd}$, $x_3 = T - T_d$, $u = u_m - u_d$ and let $x_{1d} = C_{Ad}$, $x_{2d} = C_{Bd}$, $x_{3d} = T_d$, $x_{10} = C_{A0}$ and $x_{30} = T_0$. It is further assumed that in this reaction system there exists an unmodeled first-order side reaction from A and an error in measuring the inlet flow rate, F. Under such a situation, the process is described by the following uncertain model:

$$\dot{\mathbf{x}} = (\mathbf{f}(\mathbf{x}) + \Delta\mathbf{f}(\mathbf{x})) + (\mathbf{g}(\mathbf{x}) + \Delta\mathbf{g}(\mathbf{x}))u \tag{41a}$$

$$y = h(\mathbf{x}) \tag{41b}$$

where

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ f_3(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} -k_1(x_3+x_{3d})\cdot(x_1+x_{1d}) - k_3(x_3+x_{3d})\cdot(x_1+x_{1d})^2 \\ +k_1(x_{3d})\cdot x_{1d} + k_3(x_{3d})\cdot x_{1d}^2 - x_1\cdot u_d \\ k_1(x_3+x_{3d})\cdot(x_1+x_{1d}) - k_2(x_3+x_{3d})\cdot(x_2+x_{2d}) \\ -k_1(x_{3d})\cdot x_{1d} + k_2(x_{3d})\cdot x_{2d} - x_2\cdot u_d \\ \frac{1}{\rho_s C_P}[-\Delta H_1 k_1(x_3+x_{3d})\cdot(x_1+x_{1d}) - \Delta H_2 k_2(x_3+x_{3d}) \\ \cdot(x_2+x_{2d}) - \Delta H_3 k_3(x_3+x_{3d})\cdot(x_1+x_{1d})^2 + \Delta H_1 k_1(x_{3d}) \\ \cdot x_{1d} + \Delta H_2 k_2(x_{3d})\cdot x_{2d} + \Delta H_3 k_3(x_{3d})\cdot x_{1d}^2] - x_3\cdot u_d \end{bmatrix} \tag{42}$$

$$\mathbf{g}(\mathbf{x}) = \begin{bmatrix} x_{10} - x_{1d} - x_1 \\ -x_{2d} - x_2 \\ x_{30} - x_{3d} - x_3 \end{bmatrix} \tag{43}$$

$$h(\mathbf{x}) = x_2 \tag{44}$$

$$\Delta\mathbf{f}(\mathbf{x}) = [-e_f x_1 \quad 0 \quad 0]^T \tag{45}$$

and

$$\Delta\mathbf{g}(\mathbf{x}) = [e_g \quad e_g \quad e_g]^T \tag{46}$$

of which $0.1 \le e_f \le 0.3$ and $0.1 \le e_g \le 0.3$. To design a statically equivalent output map (SEOM) for the proposed SMC scheme, we calculate Φ_1 and Φ_2 from the process model as

$$\begin{aligned}\Phi_1(\mathbf{x}) = {} & [-k_1(x_3+x_{3d})\cdot(x_1+x_{1d}) - k_3(x_3+x_{3d})\cdot(x_1+x_{1d})^2 \\ & + k_1(x_{3d})\cdot x_{1d} + k_3(x_{3d})\cdot x_{1d}^2 - x_1\cdot u_d](x_{30} - x_{3d} - x_3) \\ & - \{\frac{1}{\rho_s C_P}[-\Delta H_1 k_1(x_3+x_{3d})\cdot(x_1+x_{1d}) - \Delta H_2 k_2(x_3+x_{3d}) \\ & \cdot(x_2+x_{2d}) - \Delta H_3 k_3(x_3+x_{3d})\cdot(x_1+x_{1d})^2 + \Delta H_1 k_1(x_{3d}) \\ & \cdot x_{1d} + \Delta H_2 k_2(x_{3d})\cdot x_{2d} + \Delta H_3 k_3(x_{3d})\cdot x_{1d}^2] - x_3\cdot u_d\}(x_{10} - x_{1d} - x_1)\end{aligned} \tag{47a}$$

$$\begin{aligned}\Phi_2(\mathbf{x}) = {} & [k_1(x_3+x_{3d})\cdot(x_1+x_{1d}) - k_2(x_3+x_{3d})\cdot(x_2+x_{2d}) \\ & - k_1(x_{3d})\cdot x_{1d} + k_2(x_{3d})\cdot x_{2d} - x_2\cdot u_d](x_{30} - x_{3d} - x_3) \\ & - \{\frac{1}{\rho_s C_P}[-\Delta H_1 k_1(x_3+x_{3d})\cdot(x_1+x_{1d}) - \Delta H_2 k_2(x_3+x_{3d}) \\ & \cdot(x_2+x_{2d}) - \Delta H_3 k_3(x_3+x_{3d})\cdot(x_1+x_{1d})^2 + \Delta H_1 k_1(x_{3d})\cdot x_{1d} \\ & + \Delta H_2 k_2(x_{3d})\cdot x_{2d} + \Delta H_3 k_3(x_{3d})\cdot x_{1d}^2] - x_3\cdot u_d\}(-x_{2d} - x_2)\end{aligned} \tag{47b}$$

The zeros polynomial for the process output is obtained from the Eq. (40) to be $P(s) = -0.9s^2 + 100.3636s + 1233.0296$. Then, we also obtain the zeros polynomials of Φ_1 and Φ_2, respectively, as follows:

$$Q_1(s) = -654.7228s^2 - 173130.6147s \tag{48a}$$

$$Q_2(s) = -535.3284s^2 + 33842.4452s \tag{48b}$$

Based on the previous information and process model, one can synthesize an auxiliary output as

$$y_{eq} = h_{eq}(\mathbf{x}) = x_2 + \lambda_1 \Phi_1(\mathbf{x}) + \lambda_2 \Phi_2(\mathbf{x}) \quad (49)$$

It should be mentioned again that the zeros polynomial around the given steady-state, i.e. $P(s) + \lambda_1 Q_1(s) + \lambda_2 Q_2(s)$, can be made equal to a given polynomial $P^d(s)$ by appropriately adjusting the weight parameters λ_1 and λ_2. Let $z^0 = [-11.1673 \ \ -1226824]$ and $\Delta z = [0.2 \ \ 2]$, and following the proposed searching algorithm, we have $\lambda_1 = -6.2415 \cdot 10^{-4}$ and $\lambda_2 = -2.5991 \cdot 10^{-3}$. The open-loop simulations shown in Fig. 3 reveal that the obtained weights of λ_1 and λ_2 are able to make y_{eq} the minimum phase under the uncertainty variations. However, this figure also shows that the synthetic output y_{eq} is still unable to give statically equivalent to the actual process output. Therefore, it is desirable to design a SEOM, y_s, for the CSTR. Now, by using the auxiliary output of y_s designed based on Eq. (12), we simulate the open-loop system response with various values of λ_s. From Fig. 4, it is evident to observe that the constructed auxiliary output y_s constitutes a statically equivalent output for the actual process despite of the influence of process uncertainties. Also observed is that the parameter λ_s controls the transition behavior of y_s. The larger the values of λ_s, the faster transition response of y_s.

Having constructed y_s, we are ready for the design of a sliding mode controller. Based on the auxiliary output y_s, it is easily verified that the characteristic indexes for $\mathbf{f}$, $\Delta\mathbf{f}$ and $\Delta\mathbf{g}$ are to be $\rho = \kappa = w = 1$, which satisfy the condition of $w \geq \rho = \kappa$. To implement the sliding mode controller, the value of $f_{\max}$ and $b_{\min}$ should be predetermined. With the values of $e_f = e_f^0 \pm \tau$ and $e_g = e_g^0 \pm \tau$, where $\tau = 0.1$, $e_f^0 = 0.2$ and $e_g^0 = 0.2$, we use the estimated maximum bound values of $f_{\max} = 7$ and $b_{\min} = 0.3$ for the sliding mode controller. The other parameters are set as $\lambda_s = 0.3$, $c_1 = 1.0$, $k(0) = 1.0$, $\tilde{\gamma} = 3$ and the boundary layer thickness $\beta = 0.01$. In order to verify the regulation ability of the proposed strategy, we suppose that the system states are perturbed to move away from their steady states to be $x_1(0) = 1.0$, $x_2(0) = 0.2$ and $x_3(0) = 1.0$ initially. Up to this point, we are ready to investigate the following important issue regarding the application of the proposed scheme.

3.1 The role of the SEOM

Fig. 5 shows that, without the aid of y_s, the sliding mode controller of Chen and Dai [19] is unable to control this non-minimum phase CSTR, resulting an unstable closed-loop system. Once y_s is incorporated in the control scheme as that shown in Fig. 1, the SMC system is stabilized and the control performance is satisfactory. A reason for this is that the incorporated SEOM is able to eliminate the undesirable inverse response and therefore the SMC is able to reach robust stability and system performance.

3.2 The presence of extra disturbances

The extra disturbances introduce significantly additional modeling errors, which leads the uncertainty vector $\Delta\mathbf{f}$ to be

$$\Delta\mathbf{f}(\mathbf{x}) = [-e_f x_1 \quad d_1 \quad d_2]^T \quad (50)$$

For simulation, we let $d_1 = 0.6$ and $d_2 = 2$. In designing SMC to deal with this extra disturbance, the value of $b_{\min} = 0.3$ is still suitable for this case, but $f_{\max}$ should be increased. With increasing $f_{\max}$ to 10, the simulation result is depicted in Fig. 6. From this figure, it is clear to observe that the SMC control system simply using y_{eq} results in a small offset on the steady state exists because the design of y_{eq} does not consider the influence of uncertainties. In contrast, owing to that the proposed SMC is designed on the base of the modified synthetic output, y_s, the SMC control system is capable of driving the process output gradually to achieve zero steady state offset performance even though diversified and extra uncertainties are imposed on the CSTR.

3.3 Parameter variations

When the kinetic parameters change and/or the model is not so accurate to present the actual process, there is considerable discrepancy between the actual process and the process model. To explore the plant uncertainties on the essential behavior of the control system, we assume that the process's kinetic parameters k_{10}, k_{20} and k_{30} have +25% variation from their nominal values after time of 0.5 hr while these parameter values in the model remain unchanged. In designing SMC to deal with these parameter variations, the value of $f_{\max}$ is set as 10 for accommodating these exceptional uncertainties. The closed-loop system performance is shown in Fig. 7, demonstrating that the proposed scheme is robust despite of the presence of the parameter variations.

3.4 Implementation with a sliding observer

In practice, it is often not possible to obtain all states of chemical processes via on-line measurements. In this paper, we propose the use of a sliding observer [23] for estimating the states of this nonlinear CSTR. To proceed, we assume that only the process output x_2 is measured, i.e. the states, x_1 and x_3, are unmeasured. Following the design methodology of Wang et al. [23], a nonlinear sliding observer is constructed based on the nominal system and the output measurement x_2 as follows:

$$\begin{aligned} \dot{\bar{x}}_1 = & -k_1(\bar{x}_3 + x_{3d}) \cdot (\bar{x}_1 + x_{1d}) - k_3(\bar{x}_3 + x_{3d}) \cdot (\bar{x}_1 + x_{1d})^2 \\ & + k_1(x_{3d}) \cdot x_{1d} + k_3(x_{3d}) \cdot x_{1d}^2 - \bar{x}_1 \cdot u_d + (x_{10} - x_{1d} - \bar{x}_1) \\ & \cdot u + k_{01}\mathrm{sat}(\bar{x}_2 / \beta_{ob}) \end{aligned} \quad (51a)$$

$$
\begin{aligned}
\dot{\bar{x}}_2 = {} & k_1(\bar{x}_3 + x_{3d}) \cdot (\bar{x}_1 + x_{1d}) - k_2(\bar{x}_3 + x_{3d}) \\
& \cdot (\bar{x}_2 + x_{2d}) - k_1(x_{3d}) \cdot x_{1d} + k_2(x_{3d}) \\
& \cdot x_{2d} - \bar{x}_2 \cdot u_d - (x_{2d} + \bar{x}_2) \cdot u + k_{02}\mathrm{sat}(\breve{x}_2 / \beta_{\mathrm{ob}})
\end{aligned} \tag{51b}
$$

$$
\begin{aligned}
\dot{\bar{x}}_3 = {} & \frac{1}{\rho_s C_P}[-\Delta H_1 k_1(\bar{x}_3 + x_{3d}) \cdot (\bar{x}_1 + x_{1d}) \\
& - \Delta H_2 k_2(\bar{x}_3 + x_{3d}) \cdot (\bar{x}_2 + x_{2d}) - \Delta H_3 k_3(\bar{x}_3 + x_{3d}) \\
& \cdot (\bar{x}_1 + x_{1d})^2 + \Delta H_1 k_1(x_{3d}) \cdot x_{1d} + \Delta H_2 k_2(x_{3d}) \cdot x_{2d} \\
& + \Delta H_3 k_3(x_{3d}) \cdot x_{1d}^2] - \bar{x}_3 \cdot u_d + (x_{30} - x_{3d} - \bar{x}_3) \cdot u \\
& + k_{03}\mathrm{sat}(\breve{x}_2 / \beta_{\mathrm{ob}})
\end{aligned} \tag{51c}
$$

where $\breve{x}_2 = x_2 - \bar{x}_2$ denotes the error between the measured output x_2 and the estimated output value $\bar{x}_2$. The observer boundary layer thickness, β_{ob}, and the switching gain, k_{02}, are set as $\beta_{\mathrm{ob}} = 0.05$ and $k_{02} = 2$, respectively. With the desired observer poles of $\tilde{p}_1 = \tilde{p}_2 = -4$ and following the design procedure of Wang et al. (1997), we have the remaining gains as $k_{01} = -2.9132$ and $k_{03} = 6.1208$.

Fig. 8 depicts the schematic diagram of the proposed scheme for this uncertain CSTR where the sliding observer, the modified synthetic output and the sliding mode controller are integrated as a whole. For simulation, we set the initial conditions of the estimated states as $\bar{x}_1(0) = 0.8$, $\bar{x}_2(0) = 0.2$ and $\bar{x}_3(0) = 1.3$ for the sliding observer. The closed-loop system performance is shown in Fig. 9. From this figure, it is clear to show that the nonlinear observer is capable of estimating system states despite of the influence of model uncertainties. Based on the above simulation results, it is evident that the proposed scheme with a sliding observer appears to be an effective and promising approach to control of nonlinear, uncertain, non-minimum phase chemical processes, providing excellent control performance without the need of full state measurements.

4. Conclusions

In this paper, a robust and systematic sliding mode strategy has been proposed for the regulation control of an uncertain CSTR in the presence of inverse response. To overcome the negative effect of inverse response behavior and eliminate the steady state offset, a new algorithm has been proposed such that the designed auxiliary output is statically equivalent to the actual output and makes the resultant system minimum phase despite the influence of the process uncertainties. With the incorporation of the constructed statically equivalent output map, a robust SMC scheme can be easily established to tackle with the difficult control problem of uncertain non-minimum phase CSTR. In addition, the potential use of a sliding observer along with the proposed scheme has also been investigated in this work. Extensive simulation results reveal that the proposed sliding mode control scheme for nonlinear, non-minimum phase, uncertain CSTRs is promising, which is able to overcome the negative effects of inverse response, unmodeled side reaction, measuring error, unmeasured disturbance and plant/model mismatch.

Acknowledgement

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References

[1] Luyben, W. L.; Process modeling, simulation, and control for chemical engineers, McGraw-Hill, Singapore (1990).

[2] Iyer, N. M. and A. E. Farell; "Adaptive input-output linearizing control of a continuous stirred tank reactor," *Computers chem. Engng.*, **19**, 575-579 (1995).

[3] Lagerberg, A. and C. Breitholtz; "A study of gain scheduling control applied to an exothermic CSTR," *Chemical Engineering and Technology*, **20**, 435-444 (1997).

[4] Ge, S. S., C. C. Hang and T. Zhang; "Nonlinear adaptive control using neural networks and its application to CSTR systems," *Journal of Process Control*, **9**, 313-323 (1999).

[5] Alvarez-Ramirez, J. and A. Morales; "PI control of continuously stirred tank reactors: stability and performance," *Chemical Engineering Science*, **55**, 5497-5507 (2000).

[6] Wu, F.; "LMI-based robust model predictive control and its application to an industrial CSTR problem," *Journal of Process Control*, **11**, 649-659 (2001).

[7] Gopaluni, R. B., I. Mizumoto and S. L. Shah; "A robust nonlinear adaptive backstepping controller for a CSTR," *Industrial and Engineering Chemistry Research*, **42**, 4628-4644 (2003).

[8] Kravaris, C., P. Daoutidis and R. A. Wright; "Output feedback control of non-minimum phase nonlinear processes," *Chemical Engineering Science*, **49**, 2107-2122 (1994).

[9] Kravaris, C. and P. Daoutidis; "Nonlinear state feedback control of second-order non-minimum phase nonlinear systems," *Computers and Chemical Engineering*, **14**, 439-449 (1990).

[10] Hauser, J., S. Sastry and G. Meyer; "Nonlinear control design for slightly nonminimum phase systems: application to V/STOL aircraft," *Automatica*, **28**, 665-679 (1992).

[11] Wright, R. A. and C. Kravaris; "Non-minimum phase compensation for nonlinear processes," *AIChE J.*, **38**, 26-40 (1992).

[12] Aoyama, A., F. J. Doyle III and V. Venkatasubramanian; "Control-affine neural network approach for non-minimum-phase nonlinear process control," *J. Proc. Cont.*, **6**, 17-26 (1996).

[13] Kravaris, C., M. Niemiec, R. Berber and C. B. Brosilow; Nonlinear model-based control of nonminimum-phase processes. In R. Berber and C. Kravaris (Eds.), *Nonlinear Model Based Process Control*. Dordrecht: Kluwer, pp. 115-141 (1998).

[14] Niemiec, M. P. and C. Kravaris; "Nonlinear model-state feedback control for nonminimum-phase processes," *Automatica*, **39**, 1295-1302 (2003).

[15] Jo, N. H., J. Byun, H. Shim and J. H. Seo; "Robust

stabilization of nonminimum phase nonlinear systems," Proceedings of the American Control Conference, pp. 3359-3363, Philadelphia, Pennsylvania (1998).

[16] Wu, W.; "Approximate feedback control for uncertain nonlinear systems," *Ind. Eng. Chem. Res.*, **38**, 1420-1431 (1999).

[17] Shkolnikov, I. A. and Y. B. Shtessel; "Tracking in a class of nonminimum-phase systems with nonlinear internal dynamics via sliding mode control using method of system center," *Automatica*, **38**, 837-842 (2002).

[18] Byrnes, C. I. and A. Isidori; "Global feedback stabilization of nonlinear systems," Proc. IEEE CDC, pp. 1031-1035, Ft. Lauderdale, Florida, USA (1985).

[19] Chen, C. T. and C. S. Dai; "Robust controller design for a class of nonlinear uncertain chemical processes," *Journal of Process Control*, **11**, 469-482 (2001).

[20] Van de Vusse, J. G.; "Plug-flow-type reactor versus tank reactor," *Chem. Eng. Sci.*, **19**, 994-998 (1964).

[21] Kantor, J. C.; "Stability of state feedback transformations for nonlinear systems- Some practical considerations," Proceedings 1986 American Control Conference, pp. 1014-1016, Seattle, WA (1986).

[22] Engell, S. and K. U. Klatt; "Nonlinear control of a nonminimum-phase CSTR," Proceedings 1993 American Control Conference, pp. 2341-2945, San Fransico, CA (1993).

[23] Wang, G. B., S. S. Peng and H. P. Huang; "A sliding observer for nonlinear process control," *Chemical Engineering Science*, **52**, 787-805 (1997).

Table 1. Model parameters and operating conditions.

$k_{10} = 1.287 \cdot 10^{12}\ \text{h}^{-1}$	$\Delta H_1 = 4.2\ \text{kJ} \cdot \text{mol}^{-1}$
$k_{20} = 1.287 \cdot 10^{12}\ \text{h}^{-1}$	$\rho_s = 0.9342\ \text{kg} \cdot \text{L}^{-1}$
$k_{30} = 9.043 \cdot 10^{9}\ \text{L(mol} \cdot \text{h)}^{-1}$	$\Delta H_2 = -11\ \text{kJ} \cdot \text{mol}^{-1}$
$E_1/R = 9758.3\ \text{K}$	$\Delta H_3 = -41.85\ \text{kJ} \cdot \text{mol}^{-1}$
$E_2/R = 9758.3\ \text{K}$	$C_P = 3.01\ \text{kJ(kg} \cdot \text{K)}^{-1}$
$E_3/R = 8560\ \text{K}$	$Q_s = -451.509\ \text{kJ(L} \cdot \text{h)}^{-1}$

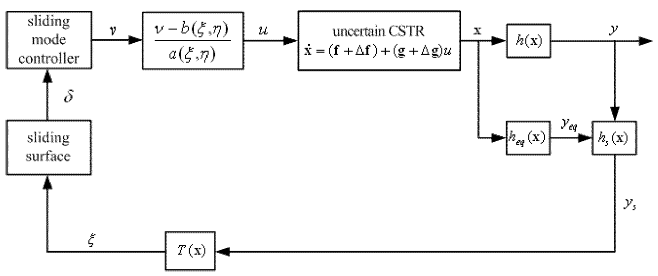

Fig. 1. Schematic diagram of the proposed sliding mode control scheme.

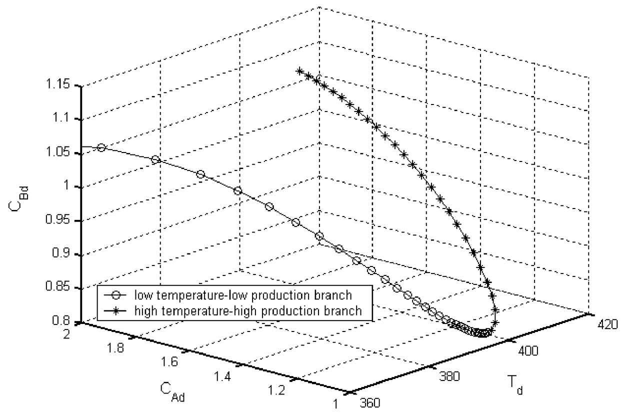

Fig. 2. An open-loop test of the CSTR.

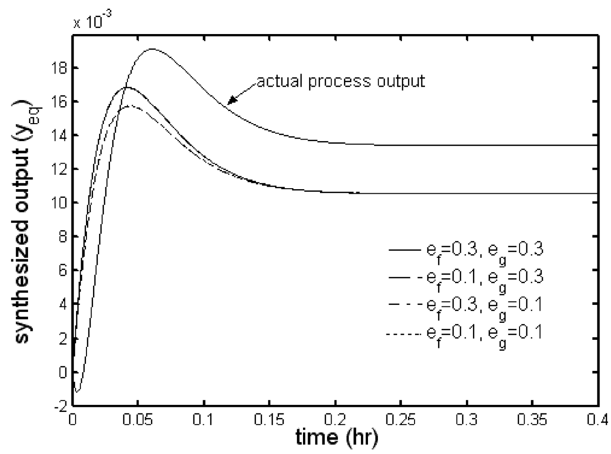

Fig. 3. Open-loop simulation under process parameter variations.

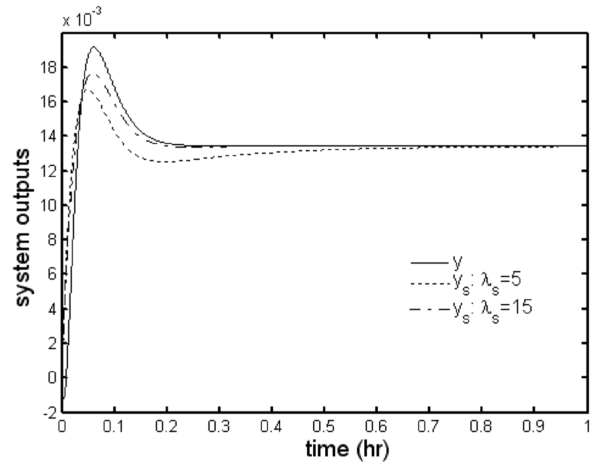

Fig. 4. Open-loop simulations using SEOM with designed values of λ_s.

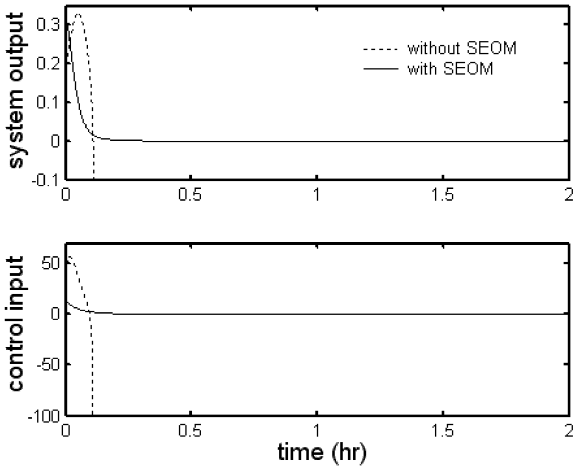

Fig. 5. Comparisons of the SMC design with and without the aid of the SEOM.

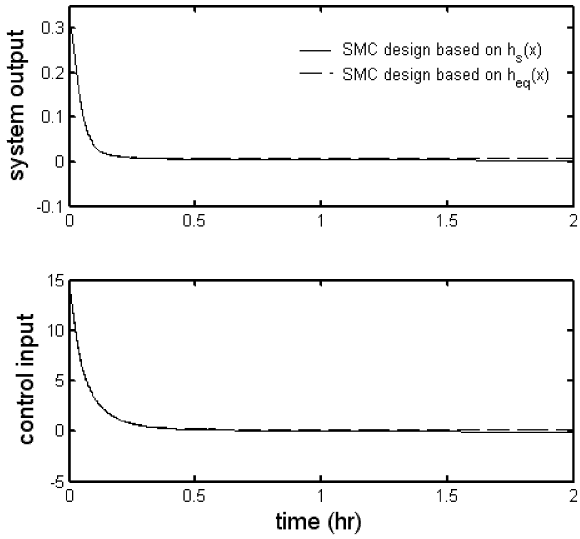

Fig. 6. Closed-loop system performance in face with the unmodeled side reaction, measuring error and extra disturbances.

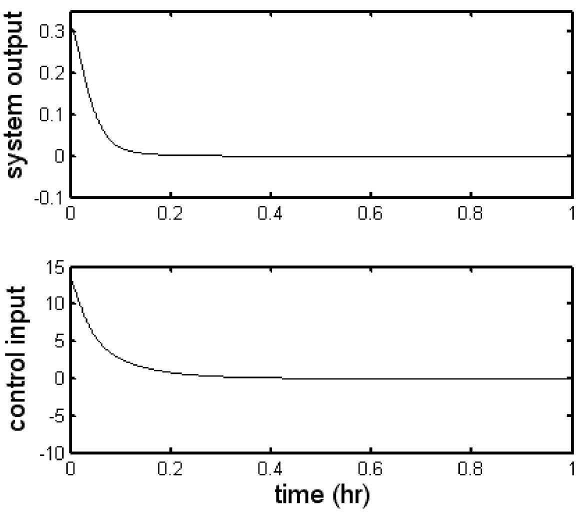

Fig. 7. Closed-loop system performance in face with parameter uncertainties.

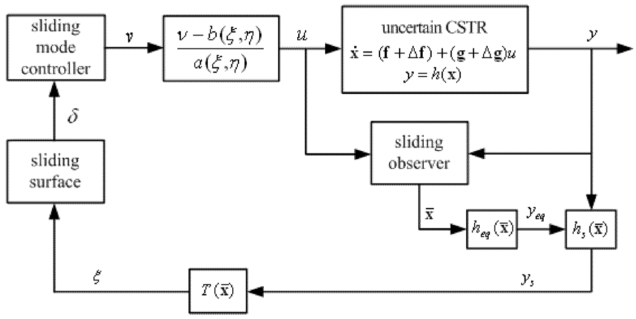

Fig. 8. Schematic diagram of the proposed sliding mode control scheme with a sliding observer.

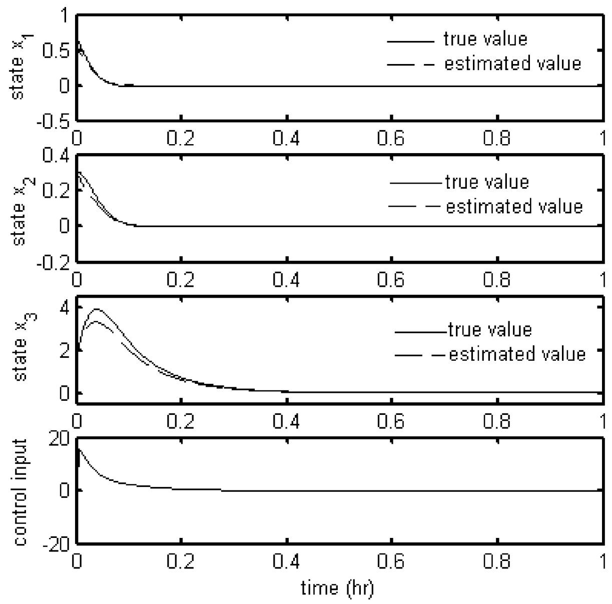

Fig. 9. System response under sliding observer with observer poles $\tilde{p}_1 = \tilde{p}_2 = -4$ for the case of existing unmodeled side reaction and measuring error.