Institutions for Debt Stabilization and Economic Development: Convergence Probability

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Abstract
The superconvergence probability is defined. The micro and macro financial policies and the optimum equilibrium are estimated. A new simulation method is provided.
Keywords:Superconvergence Probability; Equilibrium and optimal policies. Financial institutions.

1 Introduction

1.1 Motivation

The purpose of this study is to investigate the policy coordination and convergence of the micro and macro financial institutions. We detect the second-order impulse control through the optimal treatment of the policy variable. We prove a new simulation method where the equilibrium is the prediction while the traditional simulation is based calibrated coefficients of interpolation within the sample. Thus our convergence probability detects the various performance and significant level of alternative policy impacts; we do not use the judgment about simulation goodness of fitting the data. The recovery problem considered here is: How can we use the minimum required time to recover the economic growth?

The existence of conflicting theories implies that the first-order coefficient relationship between two variables is instable, indefinite, and nonstationary. For example, output has been negatively or positively affected by the government spending, the public education expenditure, and exports(Hsieh and Ho,2003; Yang,2003; Hsu and Li, 2003). In Taiwan, the government spending has a positive impact upon output; the education expenditure has a negative impact. In the autoregression analysis, non-stationarity is found and implies that the first-order coefficient may be greater than one, and that the second-order autoregressive coefficient can be positive or negative. In many countries, the behavior of real exchange rates shows a positive second-order autoregressive coefficient, while in some countries the negative second-order coefficient is found. The central banks' intervention and non-transparency distorted the analytical capability of the exchange rate behaviour. Theoretically, the optimal capital taxation (and the flat income tax rate) can be zero or positive when the government spending has a positive contribution and supplement to private investment (Chen, 2003). The conflicting theories sometimes may arise from sourceless transformation from symmetry to asymmetric relationship, including the instability of the coefficients, the positive equilibrium level, the instable foreign real interest rates, and the imperfect mobility of capital. For example, Mundell(1963 and1964) suggests the fixed exchange rate for the Asian countries and the gradual relaxation towards the flexible exchange rate through privatization process. However, a linear price model shows a structural break since 1989 in Taiwan when the price level has been stabilized and declined through the contractionary money supply. The theoretical price level, however, is determined by both fiscal and monetary policy(Ho, 2003). It is difficult to evaluate, simulate, and solve the linear simultaneous equations if simulations fail to provide the convergence probability and convergence time. The model's answer all depends upon the assumptions of, say, perfect capital mobility, the constant foreign interest rate as well as the constant first-order coefficient. Therefore, we find that the internal and the external balances may not be achieved through the zero inflation rate and the zero interest rate for both developing and developed industrial countries.

Consider the privatization process. In many countries, the large banks yield a large rate of return; whereas in some countries, the small banks yield a rate of return higher than the large banks. In statistic analysis, debt has an incentive leverage mechanism for managers to promote performance, but debt has insignificant effects in dynamic analysis (Dessi et al. 2003). The simulation often is a replication and based on the calibrated coefficient, and cannot detect the various convergence probability as the performance criterion of the scale economy. Thus we need a formula to estimate the optimal required capital for each bank. The managers and politicians are the firms' information insiders and tend to prolong no rule of law for privatization. Thus, an asymmetric rule of law has exacerbated the policy coordination. According to the report of the *International Monetary Fund*, in 2003 in Taiwan, the share of public debt in gross domestic product (GDP) has exceeded 33%; while the stable share of tax revenues fell to 13.5%. In 2001-2003, the growth rate of money supply declined. The zero inflation rate targeting leads to the declines of the annual output growth from 6% to 3%. The default rates of loans by six largest banks exceeded 20%, respectively, and need be reduced and stabilized (Carre, 2000). The privatization and the expansion of equity capital of the state-owned banks, including the Taipei bank and the Chang-Hua Bank, may involve insider trading. Either the state bank has been cheaply merged with the private Fu-Ban bank, or the expanded equity shares of the Chung Hua bank have been sold to political insiders at a price lower than the equity market price per share (Note 1). The Taipei bank paid as big as 1 billion dollar as the advice fee for the mergence and acquisition. In the United States, the merger is often supposed to pay the termination fee, say, as large as one billion dollars to the acquisited bank (Stouraitis, 2003). Few works have been done with the optimization problem at all horizons and without iterations. An related question arises: How can the micro and macro policies be coordinated to converge towards the unique maximum equilibrium of the attainable output growth within a finite time?.

1.2. Methodology

In this paper, following Hsieh(2003), our goal is to test the best estimator of parameter θ about the null hypothesis $H_0 : \theta = 0 \notin \theta(\mathrm{x}^*)$ against the alternative $H_1 : \theta \in \theta(\mathrm{x}^*) \neq 0$. We solve the simultaneous system as a non-constant entire function, where the equilibrium is not a constant. We estimate the optimal policy and the non-zero equilibrium which is robust to instable coefficients. We analyze the evolution of financial institutions and predictions, and focus on the minmax solution, maximizing the output or output growth and minimizing the cost of time and the control policy through, say, investment in knowledge-based pharmaceutical industries. We introduce dynamic quadratic regression. It provides dynamic statistic tests and indicates the convergence probability, the optimal policy, the optimum equilibrium, the required sample period, as well as prediction at all horizons. Without iterations, we provide a new simulation to test the hypotheses of conflicting theories.

In the econometric process, in Step 1, dynamic quadratic regression estimates the optimal policy and the nonzero equilibrium. In Step 2, we reduce the heteroschedasticity and the residual disturbances into a Brownian motion. In Step 3, we use the convergence probability to test changes in the non-zero equilibrium and policy. Without iterations, the nonsmooth function is regularized.

1.3. The preliminary results

First, the state-owned banks are reoriented towards the knowledge-based productive investment, say, in bio-chemical and phamarcutical industries. We (Hsieh, 2002 and 2003) estimated the minimum required period of policy convergence towards optimum equilibrium, i.e., the economic growth. The deadline estimate of debt stabilization takes about two years. Second, the optimal stabilization policy is to increase in penalty of debt moratorium in terms of increasing non-zero interest rates. Such debt repayment allows the good firms, farmers and poor to continue to borrow and invest. Third, in the privatization process, the law of insider trading regulation is adopted if the trading volume exceeds, say, 10 percent of the equity capital of each firm or bank. Thus, we reduce the moral hazard through the incentives for good borrowers and good workers; we avoid the adverse selection when we predict the equilibrium earnings; we avoid the free riders through increasing the penalty of increasing debt interest rates on default loans. Fourth, we estimate the optimal fiscal and monetary policy. Our preliminary estimate of the optimal coexistence ratio of policy is as follows:

Table 1. Impacts of Policies Upon Output

Causes	Keynesian Model	Symmetric Model
Increases in government spending	+	+ -
Increases in public education spending	+	+ -
Increases in capital taxation	-	+ -
Increases in money supply	+	+ -

Note:According to the neoclassic economics, the stabilization policies include the zero inflation or positive inflation, and have neutral or negligible positive impacts upon output.

Table 2: Our Preliminary Estimates about Optimal Policy Coordination in Taiwan

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The consumption share in GDP is C*/Y*=60%;

The government share is G**/Y**=16%;

The investment share is I**/Y**=24%.

The share of real money balances are kept stable at M/PY=30% or 50%:

The optimal inflation rate is around 3%;

The optimal real interest rate is 2%;
The optimal nominal interest rate is around 5%.
The share of cumulative public debt in GDP should not exceed 28%.
The share of annual budget deficit in GDP should not exceed 3%.

Note: Hsieh(2003)

In mathematical contribution, suppose that objective function is

Minimize F(x)=xQx for Q a positive matrix

the Lyapunov stability requires a negative eigenvalue solution $-\theta=-x<0$:

$dF(x)/dt=\alpha \exp(-\theta t)$ $\alpha>0$, $\theta>0$ and $t>0$

where it is assumed that the equilibrium is zero, x*=0.

In contrast, we use dynamic quadratic regression to yield the non-zero equilibrium:

$dF(x)/dt==\alpha \exp(-\theta_5 (x(t)-x^*)(u(t)-u^{**})^2$

where $\alpha \neq 0$. Our stability requires that the parameter switches,

$\theta_5<0$ if $x>x^*$ and $\theta_5>0$ if $x<x^*$

The solution (x*, u**) is the extremal; in the switching model, it is the singular fixed point. We assume that the equilibrium x*>0 is not zero. The optimal policy u** may oscillate and switch. Furthermore, for the problem of the unconstrained maximization, we find that the first-order coefficient is positive in the output equation. For the problems of constrained cost-minimization, the first-order coefficient may show a negative value in the price-cost and the unemployment rate equations. In the nonexpansive, closed, convex or concave domain, the relationship between variables are stable and controllable without iterations. We transform the stochastic differential equations into innovative statistical tests for the closed form solution.

In the following, Section 2 presents the model, Section 3 provides theorems, and proofs. Section 4 illustrates the econometric procedure and simulation methods. Section 5 concludes with remarks.

2. The Model

Assumption 1. The state variable x and the control variable u are the observable random variables and are serially correlated and nonstationary.

Assumption 2. v is unobservable random variables and tends to converge to the Brownian motion when observations of the state variable converge to the equilibrium. E(v)=0 with the unit variance $\exp(E(v^2))=1$.

Definition 1: The equilibrium, x(t)=x(t-1)=x*, is the sustainable solution at all horizons, for all time t>0. In equilibrium, the state x and the policy variable u are independent. The expected error v is zero with a unit variance. In disequilibrium, x and u are correlated.

Definition 2: A non-constant entire function is that the equilibrium is not constant and that the function is homothetic or homogeneous.

(2.1) $x(i, t+1) = \min_{t,u} \max_{x} F((x(i,t)), \theta u(t))$

$$\text{for } \theta>0,\ u(t)= x(j,t)/\sum_i x(i,t),\ i\neq j$$

where θ is the parameter to be estimated and controllable through controlling the policy. $0<t\leq n$. n is the sample size or period.

Definition 3: Superconvergence probability p(x) implies that state variable x converges to the equilibrium within a finite time $t<\infty$. The law of motion is

(2.2) p(x)=exp(dx/dt)=exp(dF(x,u,t))/dt)=f(x*,u**,t*)+v

where the convergence implies that when the state variable is in equilibrium, the welfare is maximum:

as $u \to u^{**}$, $x \to x^*$, $F(x) \to F(x^*)$. If $x<x^*$, $F(x,u,t)<F(x^*, u^{**},t^*)$ and $f(x^*,u^{**},t^{**})=0$

where t*=t**=t is the minimum required time for convergence $x(t)\to x^*$. The solution (x*, u**, t*) is essentially the same when the model is either dx/dt=f(x,u,t) or exp(dx/dt)=f(x,u,t).

Definition 4: The stationary institutions include the law and organizations where the the nonexpansive compact convex domain Ω exists, and where the relationship coefficient between the state variable x and the control variable u are stable, bounded, controllable, and interpretable. The model is well-posed and the fixed point solution is well-defined and stationary

Suppose B is the public debt and private debt; G is government spending; M is money supply; and P is the price level. r is the interest rate. T is tax revenue; Y is output. F(.) is the welfare function.

Consider the problem of optimization

(2.3) $Y^* = \min_{B,M,G,T} \max_{Y} F(Y, G-T)$

subject to the budget constraint

B(t+1)=(1+r)B(t) +(G(t)-T(t))+(M(t)-M(t-1))/P(t)

or B(t+1)/Y(t+1) –B(t)/Y(t)=(r - Δ Y/Y)(B(t)/Y(t)) +(G(t)-T(t))/Y(t+1)+(M(t)-M(t-1))/P(t)Y(t+1)

where in equilibrium, the debt is stationary and will not accumulate.

Theorem 1: If an increase in public debt can increase income, economic growth, and tax revenues, the public debt tends to decline or is in the steady state:

Proof: The budget constraint implies

(2.4) dB/dt=rB(t) - c(rB(t))

if T(t)-G(t)=c(rB(t)), M(t)=M(t-1) and c=ΔY/Y

$B(t)=B(0)\exp(r-c)t=B(0)\exp(-|r-c|t)\rightarrow 0$

$\exp(p(-|r-c|t) \rightarrow 0$, as $t \rightarrow \infty$ and as $c>r$

Thus, if the marginal productivity of government spending denotes the output growth, and exceeds the interest rate, $\Delta Y/Y>r$, the debt will not accumulate. The optimum deficit satisfies the necessary and the sufficient conditions of optimization:

(2.5) $\partial Y/\partial (G-T)/Y >0$, and $\partial^2 Y/\partial^2 (G-T)/Y \leq 0$. Q.E.D.

Theorem 2: The deadline t* of debt inertia leads to increases in penalties in interest rate, thereby stabilizing the private and public debt.

(2.6) $\partial Y/\partial (G-T)/Y=r>0$, and $\partial^2 Y/\partial^2 (G-T)/Y \leq 0<r$.

Proof: Let t* be the deadline for debt moratorium. After the deadline t*, the penalty interest rate for default debt tends to increase.

(2.7) $B(t)=B(0)\exp(r-\Delta Y/Y)(t-t^*) \leq B(t^*)$ as $t=t^*$

Theorem 3: The deadline of the convergence time t* is the minimum required time when the output growth is maximized.

Proof: Consider simulations on Kuhn-Tucker optimization conditions under constraints. The dynamic quadratic regression detects the concavity or convexity of functions. Suppose the parameters $\theta=(\theta_1,\ldots,\theta_4)$ denote the Lagrangian multipliers. When output Y is growing and non-stationary, below the optimum, deficit tends to increase output growth x, which is stationary, and has a stable, controllable relationship with the deficit u=(G-T)/Y.

Suppose we minimize the predictive percentage errors and maximize the welfare function F(x,u)

(2.8) $\min_{t,u} \max_{x\in\Omega} F(x(t), u)=\log x^*$ almost everywhere for all $t>0$

subject to $x, u \in \Omega$ for $u=(G-T)/Y$ and $x=d\log Y/dt$

the law of motion implies that the marginal revenue equals the marginal cost:

(2.9) $\dfrac{\partial F(x,u)}{\partial x} = \theta \dfrac{\partial F(x,u)}{\partial u}$ $-\infty < \theta < \infty$

where $\theta > 0$ denotes the positive and cooperative relationship between the state variable x and the control variable u and t.

The generalized solution is

(2.10) dL(x,u, θ(t))/dt=dlog x/dt=f(x, u, θ(t))

for x, u $\in \partial \Omega$

where L is the Lagrangian function. θ is the Lagrangian multiplier. x is output growth; the control policy, u $\equiv$(G-T)/Y, is the share of deficits in output. G is the government spending; T is tax revenue; Y is output.

In the econometric procedure, suppose the welfare is a function of output growth and budget deficit over time.

The law of motion (2.10) comprises the supply curve:

(2.11) dx(t)/dt=f(x(t),u(t))

$=\theta_1$ x(t-1)+ θ_3 u(t-1)+v_1(t)

for $\theta_1 > 1$ and $\theta_3 > 0$, x<x*, and u<u**

and the demand curve

(2.12) dx(t)/dt=-f(x(t),-u(t))

$=\theta_2$ x(t-1)- θ_4 u(t-1)+v_2 (t)

for $\theta_2 < 1$ and $-\theta_4 < 0$, x<x*, and u>u**

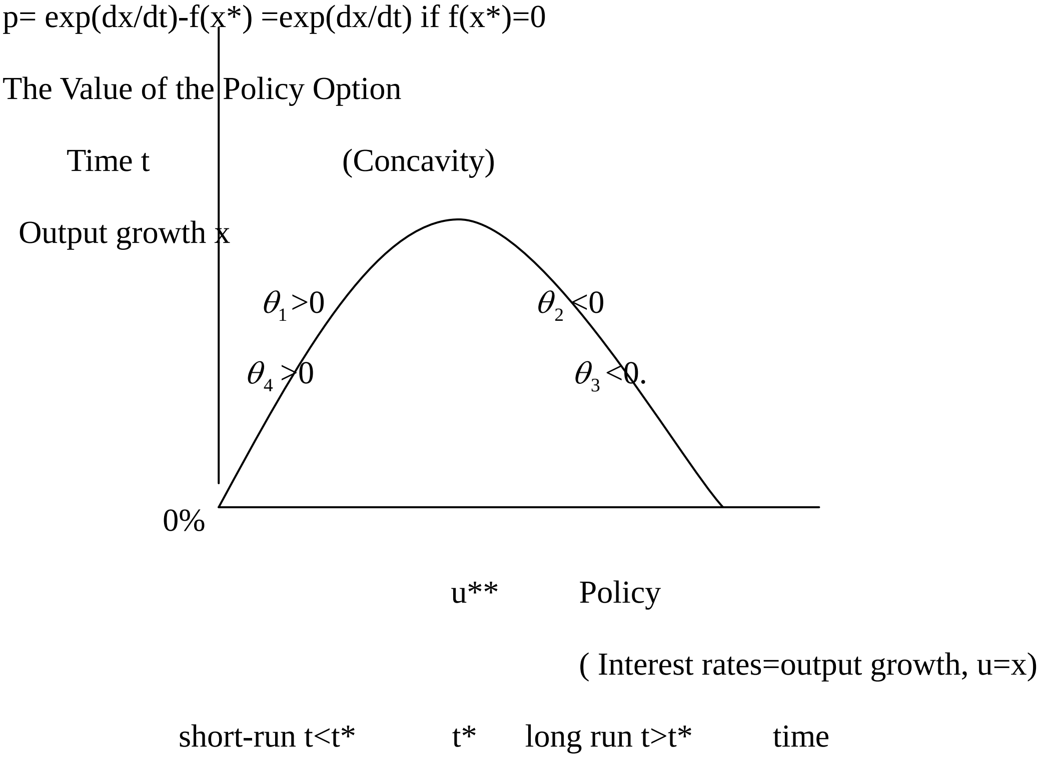
where f(.) is the thrice differentiable function. In (2.11), if the price policy u increases, the output x increases, $\theta_1 > 1$ and $\theta_3 > 0$. In (2.12), if the output supply exceeds the output demand, x>x*, the price policy decreases, $\theta_2 < 1$ and $-\theta_4 < 0$.

When the equilibrium x* and the optimal policy u** are not zero, (2.11) and (2.12) are rewritten as the non-homogeneous function,

(2.13) dx(t)/dt= θ_1 (x(t-1)-x*)+ θ_3 (u(t-1)- u**)+v_1(t) for $\theta_1 > 0$ and $\theta_3 > 0$ if u<u**

(2.14) dx(t)/dt=θ_2(x(t-1)-x*)-θ_4(u(t-1)-u**)+v_2(t) for θ_2<0 and -θ_4<0 if u>u**,

Figure 1. Maximum output growth x* if θ_1>0 and θ_2<0



Note:The solution is the minimum output growth x* if θ_1<0 and θ_2>0 in the convexity domain.

where x is the state variable; x* is the equilibrium growth. u is a control variable. u** is the optimal limit of deficit. In the supply and demand curves, $\partial F/\partial u=\theta_3>0$; $\partial^2 F/\partial u^2=-\theta_4<0$.

The second-order derivative of the welfare function tends to yield the unique solution. The generalized law of motion is:

(2.15) $dx/dt=\partial^2 F(x, u)/\partial^2 x+\partial^2 F(x, u)/\partial^2 u+\partial^2 F(x)/\partial^2 x\partial u$

According to Hsieh (2003), the econometric process is as follows:

Step 1: Estimate the equilibrium and the optimal policy:

(2.16a) exp(dx/dt)=θ_0 +θ_1x(t-1)+θ_2x^2 (t-1) +θ_3u(t-1)+θ_4u^2 (t-1)+v(t)

where dx/dt$\approx\Delta$x(t)=x(t)-x(t-1)

Step 2: Reduce the heteroschedasticity

(2.16b) p(x)= exp(Δx(t))=$\int_{t=0}^{t^*} f(x(t),u(t))$dt

$$=\theta_2(x(t-1)-x^*)^2+\theta_4(u(t-1)-u^{**})^2+\theta_5(x(t-1)-x^*)(u(t-1)-u^{**})^2$$

$$+\theta_6(\Delta x(t-1))+\theta_7\Delta x(t-2)+v(t)$$

Step 3: Simulation and sensitivity tests through alternative equilibria and optimal policies

(2.16c) $np(x^*)=n\geq\frac{1}{2}\left(\frac{(\theta(x)-\theta(x^*))}{\sigma}\right)^2$ and $2\geq\frac{1}{n}\left(\frac{(\theta(x)-\theta(x^*))}{\sigma}\right)^2$

if p(x*)=1

where even if the function F is nonsmooth and not differentiable, the minmax solution implies that as x(t-1)$\rightarrow$x*, F(x(t-1))$\rightarrow$F(x*), and if x(t)<x*, then F(x(t))<F(x*). σ^2 is the variance of the maximum likelihood estimator θ(x*) which coincides with the uniformlu consistent most powerful unbiased (UCMPU) estimator.

The equilibrium is x*=-$\theta_1/2\theta_2$ and has a upper bound and a lower bound. In the concave domain of an elliptical curve f, if θ_1>0 and θ_2<0, x* is the maximum. In the convex domain of a hyperbolic equation, if θ_1<0 and θ_2>0, x* is the minimum. In the flat sphere of a parabolic equation, if

the first and the second-order coefficient have the same sign, θ_1>0 and $\theta_2\geq 0$, x*=θ_1 is the unique solution and the inflection point. The optimal policy is u*=-$\theta_3/2\theta_4$. $\frac{\partial F(x,u)}{\partial x}=\theta_1<0$ and $\frac{\partial F(x,u)}{\partial u}=\theta_3<0$ imply the crowding- out and the negative impact of policy upon the state variable x. $\frac{\partial^2 F(x,u)}{\partial x^2}=\theta_2>0$ and $\frac{\partial^2 F(x,u)}{\partial u^2}=\theta_4>0$ denote the positive impact of policy.

The control variable u is allowed to oscillate and stabilize the state variable x.

Our accelerated convergence implies that $\Delta x(t)/x(t-1)=\theta_1$. For the half-life speed of adjustment, $p(x)=\alpha^t$ is estimated as

(2.17) $t=\log(1/2)/\alpha \approx \log(1/2)/\log(1+\theta_1)\approx 0.5/(1+\theta_1)<1$,

where after repeated choices of the same policy with the same probability, t<1 implies that less than one year, the solution converges to the equilibrium. Q.E.D.

3. Test Problem on Sensitivity and Robustness

Budget deficit is allowed to oscillate and jump to steer the state variable towards the equilibrium. In the long run, the budget must be balanced over business cycles; in the short run, policy is countercyclical. Public debts provide credible capital in advance. The existence of the optimum budget deficit implies that the government is productive subject to a limited credibility and efficiency of debt repayments. Beyond the optimum, the high debt is not incentive-compatible nor induces saving. High debts tend to raise the interest rate and crowd out private investment. Large loans and debts tend to increase default rates; public debt and investment tend to be financed by money supply. The value of public debt is reduced either when the price inflation or indirect taxation increases, or when the real interest rate increases and discourages loans. Beyond the optimum, government spending starts to reduce investment, output, and consumption. The money-financed deficit tends to raise inflation and exacerbate the productivity of government spending. We estimate the general solution (x*, u**), which satisfies the necessary and sufficient conditions of optimization. We estimate the optimal nominal output (or the gross domestic product) which can be stabilized through policy, including budget deficit, debts, as well as monetary and fiscal policy.

Example 1:Consider simulations on Kuhn-Tucker optimization conditions under resource constraints.

The quarterly data in Taiwan are used to estimate the comparative dynamics over business cycles; the data are available for the period 1983:1 till 2001:4, as published by the government in Taiwan. Let x be output growth; D=u=(G-T)/Y is the share of budget deficit.

The econometric procedure is as follows:

Step 1 : Estimate dynamic quadratic regression. To minimize the percentage forecast error, we stabilize output growth through adjusting the budget deficit:

(4.1) $d\log x(t)/dt \approx (x(t)-x(t-4))/x(t-4)$

$$=-127.17+11.91x(t-1)+630.44\,D(t-1)$$
$$(-4.59)\quad (3.42)\quad (2.36)$$

$$-1441.06D^2(t-1)+0.50d\log x(t-1)-0.32d\log x(t-2)/dt$$
$$(-1.91)\quad (4.44)\quad (-3.51)$$

$\Re^2=0.72$; $\bar{\Re}^2=0.70$; D.W.=1.96; n=76, 1st order autocor. =-0.009

where the values in parentheses are t statistic. x(t-4) denotes the annualized growth rates of quarterly data. $\theta_1>0$ and $\theta_3>0$ implies that as the deficit increases, the Keynesian effective demand increases, and output increases. $\theta_1>0$ and $\theta_4<0$ implies the crowding out effect, where the deficit increases, the private investment and output growth decreases.

$\theta_1=11.91>0$ denotes that the output growth x is unique. $\theta_2=0$. $\theta_3=630.44>0$ and $\theta_4=-1441.06<0$ imply that the budget deficit is the maximum. The cumulative upper-limit share of budget deficit in GDP is $D\rightarrow D^{**}=26\%\{=-(630.44)/(2)(-1441.06)\}$, and the maximum output growth is a unique inflection point, $x\rightarrow x^*=\theta_1=11\%$ for $\theta_2\geq 0$. Below the upper limit, expansionary countercyclical policies tend to promote output and output growth. $dx/dt\approx x(t)-x(t-1)=\theta_4(D-D^*)^2\leq 0$, implying that an increase in deficits D tends to reduce output growth x(t). When D$\rightarrow$D**, x$\rightarrow$x*. t*=-$\theta_5/2\theta_6$ =-(0.50)/(2)(-0.32)<1, t* is the required period for the time-delay impact and is less than one year, when the deficit stabilizes the output growth or earnings growth.

Remark on super-convergence time: During depression in 2000-2002 in Taiwan, when the central bank reduced the money supply, within one year, the output growth declined to nearly zero. Similarly, in the United States, during the Great Depression in 1929-33, when the money supply decreased, the output growth declined within two years. For structural estimation, the forecast is recursive, Y(t+1)= Y(t)(1+x(t)) with the convergence probability close to one as x(t)$\rightarrow$x(t*)=x*.

Our convergence acceleration implies that Δx(t)/x(t-1)= θ_1=11.91. For the half-life speed of adjustment, t =log(1/2)/ $\alpha \approx$ log(1/2)/log(1+θ_1)$\approx$0.5/(1.1191)<1 year. A contractionary fiscal or monetary policy can lead to recession; and output growth declines within one year.

In a previous version of this paper on the monetary growth model, t =log(1/2)/ $\alpha \approx$ log(1/2)/log(1+θ_1)$\approx$ =$|$ -0.69 $|$/0.63 $\approx$1 year. Thus, when the inflation falls below the optimal inflation, the fiscal and monetary contraction is a significant cause for depression with various delay times within one year. The channel of policy impacts may go from contraction in bank loans and increases in liabilities, and declines in output and employment.

Table 1: The analysis of data in Taiwan

	Mean	Standard Deviation	Minimum	Maximum

Output growth	x	8.26%	5.89%	-14.46%	24%
Budget deficits	(G-T)/Y	11%	5%	2%	37%
Maximum equilibrium growth	x*	11%	3.42%		11%
Limit of budget deficit	D**=(G-T)/Y			3%	26%

Sources: Bureau of auditing and statistics, the government in Taiwan, ROC.

The standard deviation of the equilibrium growth is derived from Equations 4.1 and 4.2.

Remark on the optimal policy: As in a previous version of this paper, the optimal income tax rate is around T/Y=17%; the optimal real interest rate is around 2%; the optimal inflation rate is around 3%; and the optimal money growth is around 8%. The optimal share of government spending in output (gross domestic product) is 10%<(G/Y)**<24%. The optimal real money balance in GDP is around M/PY=30%. G is government spending; Y is output; M is money supply; and P is the price level. In In comparative dynamics, the upper limit of the share budget deficit in output is D=(G-T)/Y=26%. In the short run, using the annual data of the United States, the maximum share of the annual deficit is 3%. Beyond the optimum, large budget deficit tends to expand aggregate spending and causes the price inflation, reducing output growth and the social welfare. To optimize the investment share in GDP around 24 percent, the optimal nominal interest rate is around 5%.

Step 2: Estimate the equilibrium and the cumulative deficits and reduce the heteroschdasticity of errors: Within the finite sample period, $0<t\leq n$, we test the global maximum equilibrium of output growth, $x^*=\theta_1=11.91\%$, and the maximum

budget deficit, D**=((G-T)/Y)**=-$\theta_3/2\theta_4$=26%, during crisis times, and zero deficit D**=0 otherwise. One asterisk denotes the equilibrium; two asterisks denote the optimal policy. Equation (4.1) is refined as

(4.2) dlog x(t)/dt ≈ {x(t)-x(t-4)}/x(t-4)

=24.36 –0.39(x(t-1)-11%)^2 +278.04(x(t)-11%)((G(t-1)-T(t-1))/Y(t-1)- 26%)^2

(1.84) (-2.09) (4.10)

+0.68dlogx(t-1)/dt –0.35dlogx(t-2)/dt

(6.37) (-3.63)

$\Re^2$=0.70; $\bar{\Re}^2$=0.68; D.W.=1.90; n=76; the 1st order autocor. =-0.04

where parameter θ_2=-0.39<0 implies that the equilibrium growth x*=11% is maximum, is concave down, and tends to decline. This output growth is the stopping point. In (4.1), θ_4 =-1441.06<0, implying that this budget deficit is the stopping point, tends to be concave down, and has a negative impact upon output. When the budget deficit exceeds 26%, an increase in budget deficits tends to decreases output growth. In (4.2), the expected percentage error is zero.

Step 3 on Simulation and Sensitivity tests on interior solutions:

Example 2: Simulating the alternative equilibrium target and policy. To minimize the forecast error of output growth, we test an alternative hypothesis of the annual deficit constraint D=3% and economic growth x=8%. We find that the balanced budget has a lower convergence probability p=$\Re^2$=0.61 as follows:

(4.3) dlog x(t)/dt ≈ {x(t)-x(t-4)}/x(t-4)

=4.87+1410.29((G(t-1)-T(t-1))/Y(t-1)-3%)^2

(0.66) (1.70)

+583.37(x(t)-8%)((G(t-1)-T(t-1))/Y(t-1)- 3%)^2

(2.40)

+0.87dlogx(t-1)/dt –0.35dlogx(t-2)/dt

(7.72) (-3.16)

$\Re^2$=0.61; $\bar{\Re}^2$=0.58; D.W.=2.14; n=76; the 1st order autocor. =-0.08

where the values in the parentheses are t statistics. The convergence probability in (4.2) is higher than in (4.3), p= $\Re^2$ =0.70>0.61, implying that the limit of cumulative debts is around 26% rather than 3% during crises.

In Equation (4.3), θ_5 =585.37>0 implies that output increases if x(t)<x*=8% when output growth x(t) exceeds the equilibrium. θ_6 =0.87 >0 and θ_7 =-0.35<0 imply the existence of delay cycles of output growth. Consider dx/dt ≈ x(t)-x(t-1)= θ_4 (D-3%)^2. θ_4 =1410.29>0 implies that the volatility of the budget deficit tends to increase output growth, when D*=3% is the minimum budget deficit.

In Equation (4.1), θ_4 =-1441.06<0 implies that the budget deficit has a negative impact in a concave-down growth curve. The maximum cumulative deficit is D=26%. Our laws of motion in (4.1), (4.2) and (4.3) are reduced forms and a closed feedback form, where the fixed point exists. The state variable, x, denotes output growth; and policy is an explanatory or control variable rather than the dependent variable. The equilibrium growth, x*=dlog f(x,t)/dt, is stabilized by the optimal budget policy D**.

Remark:First, counter-business cycles through deficit-financing have positive impacts θ_2 >0 if the deficit is limited to three percent of output. Nevertheless, cumulative policy effects lead to convergence to optimum equilibrium over time. Second, asymmetric conflict theories lead to the optimal solution (x*,u**, t*), which holds in all sciences, including physics and economics. The coexistence equilibrium of the demand and supply shocks is solved by dynamic quadratic regression.

4. Concluding Remarks

The goal of this study is to investigate the convergence probability, the equilibrium, the optimal policy, and the convergence time as well as the prediction.

The predictive value tends to be the optimal non-zero equilibrium. This study proves that the optimal deadline for debt stabilization is the convergence time when the output and output growth are maximized through debt moratorium. The solution is widely applicable. We estimate the maximum feasible default rate, the real interest rate, as well as the insider trading limit if the output growth is maximized.

Note 1: China Daily News, October 24, 2003, "The expanded equity capital may favor the democratic progressive party and sold at the price lower than the market price."

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