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逢甲大學學生報告 ePaper

凹陷濾波器與梳形濾波器的設計與驗證

Design and verification of notch filter and comb filter

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中文摘要

本篇報告主要在於探討凹陷濾波器與梳形濾波器的設計理念以及驗證過程，底下分別使用了四種濾波器來濾除心電訊號的雜訊，這四種濾波器為：單獨的凹陷濾波器、凹陷濾波器加上 Shank 的遞迴濾波器，單獨的梳狀濾波器，梳狀濾波器加上 Shank 的遞迴濾波器。

關鍵字：凹陷濾波器、梳狀濾波器、遞迴濾波器

Abstract

This report mainly discusses the design concept and verification process of notch filter and comb filter. Four filters are used to filter out the noise of the ECG signal. These four filters are Notch Filter, Notch Filter + Shank's Recursive (IIR) Filter, Comb Filter, and Comb Filter + Shank's Recursive (IIR) Filter.

Keyword：comb filter, notch filter, Recursive (IIR) Filter

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一、 60-Hz Notch Filter

fs = 500 Hz

∵ 180° : θ = 250 Hz : 60 Hz

∴ θ = 43.2°

得到

$$H(z) = (1 - e^{j43.2°} \cdot z^{-1})(1 - e^{-j43.2°} \cdot z^{-1}) = 1 - (e^{j43.2°} + e^{-j43.2°}) \cdot z^{-1} + z^{-2}$$

$$= 1 - 2\cos 43.2° \cdot z^{-1} + z^{-2} = 1 - 1.45793725484 \cdot z^{-1} + z^{-2}$$

$$H(1) = 0.5420627452$$

進行正規化

$$H(z) = \frac{1 - 1.45793725484 \cdot z^{-1} + z^{-2}}{0.5420627452}$$

$$= 1.844804884 - 2.689609769 \cdot z^{-1} + 1.844804884 \cdot z^{-2}$$

然而

$$H(z) = \frac{Y(z)}{X(z)} = 1.844804884 - 2.689609769 \cdot z^{-1} + 1.844804884 \cdot z^{-2}$$

經過整理

$$Y(z) = 1.844804884 \cdot X(z) - 2.689609769 \cdot z^{-1} \cdot X(z)$$
$$+1.844804884 \cdot z^{-2} \cdot X(z)$$

進行Z^{-1}得到差分方程式為

$$y[n] = 1.844804884 \cdot x[n] - 2.689609769 \cdot x[n-1] + 1.844804884 \cdot x[n-2]$$

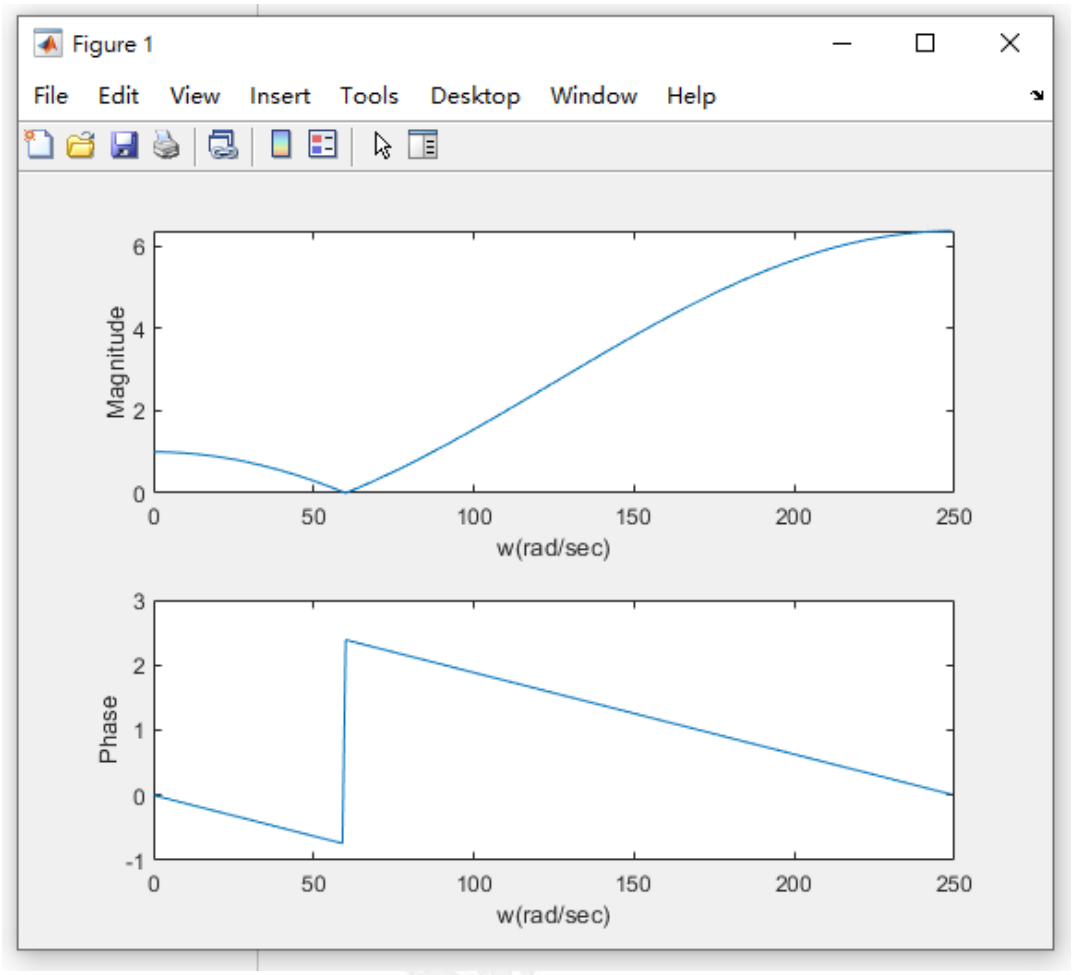


Figure 1
File Edit View Insert Tools Desktop Window Help
Magnitude
6
4
2
0
0 50 100 150 200 250
w(rad/sec)
Phase
3
2
1
0
-1
0 50 100 150 200 250
w(rad/sec)

二、 60-Hz Notch Filter + Shank's Recursive (IIR) Filter

fs = 500 Hz

r = 0.995

∵ 180° : θ = 250 Hz : 60 Hz

∴ θ = 43.2°

得到

$$H(z) = \frac{(1 - e^{j43.2^\circ} \cdot z^{-1})(1 - e^{-j43.2^\circ} \cdot z^{-1})}{(1 - re^{j43.2^\circ} \cdot z^{-1})(1 - re^{-j43.2^\circ} \cdot z^{-1})} = \frac{1 - 1.45793725484 \cdot z^{-1} + z^{-2}}{1 - 2r\cos 43.2^\circ \cdot z^{-1} + r^2 z^{-2}}$$

$$= \frac{1 - 1.45793725484 \cdot z^{-1} + z^{-2}}{1 - 1.450647569 \cdot z^{-1} + 0.990025 \cdot z^{-2}}$$

$$H(1) = 1.004978544$$

進行正規化

$$H(z) = \frac{1 - 1.45793725484 \cdot z^{-1} + z^{-2}}{1.004978544(1 - 1.450647569 \cdot z^{-1} + 0.990025 z^{-2})}$$

$$= \frac{1 - 1.45793725484 \cdot z^{-1} + z^{-2}}{(1.004978544 - 1.457869682 \cdot z^{-1} + 0.994953883 \cdot z^{-2})}$$

然而

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 1.45793725484 \cdot z^{-1} + z^{-2}}{(1.004978544 - 1.457869682 \cdot z^{-1} + 0.994953883 \cdot z^{-2})}$$

經過整理

$$1.004978544 \cdot Y(z) - 1.457869682 \cdot z^{-1} \cdot Y(z) + 0.994953883 \cdot z^{-2} \cdot Y(z)$$
$$= X(z) - 1.45793725484 \cdot z^{-1} \cdot X(z) + z^{-2} \cdot X(z)$$

進行Z^{-1}得到差分方程式為

$$1.004978544 \cdot y[n] - 1.457869682 \cdot y[n-1] + 0.994953883 \cdot y[n-2]$$
$$= x[n] - 1.457869682 \cdot x[n-1] + x[n-2]$$

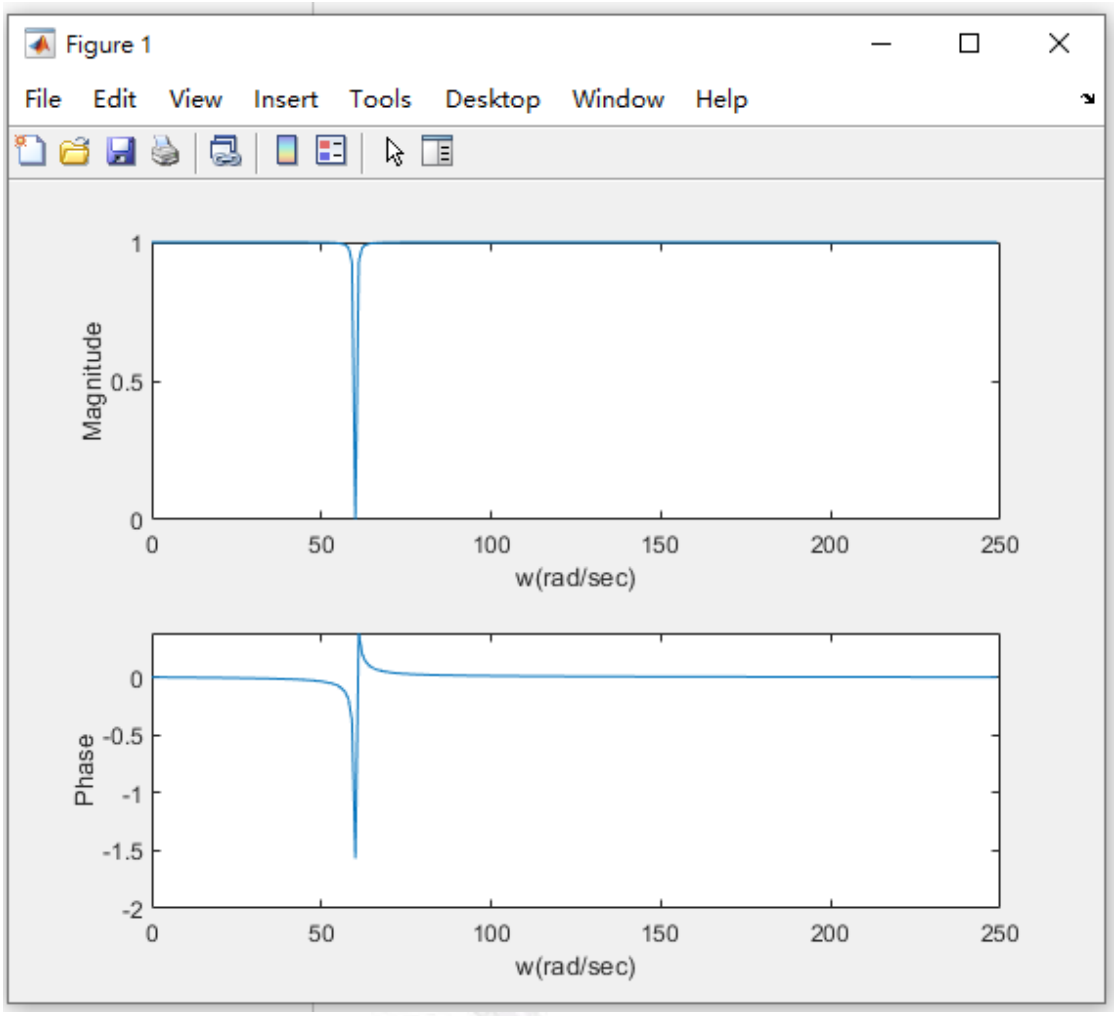


Figure 1
File Edit View Insert Tools Desktop Window Help
Magnitude
1
0.5
0
0 50 100 150 200 250
w(rad/sec)
Phase
0
-0.5
-1
-1.5
-2
0 50 100 150 200 250
w(rad/sec)

三、60-Hz Comb Filter

fs = 500 Hz

基頻 = 60 Hz

3 倍頻 = 180 Hz

∵ 180° : θ = 250 Hz : 60 Hz

∵ 180° : θ = 250 Hz : 180 Hz

∴ θ = 43.2° , 129.6°

得到

$$H(z)$$

$$= (1 - e^{j43.2^\circ} \cdot z^{-1})\left(1 - e^{-j43.2^\circ} \cdot z^{-1}\right)(1 - e^{j129.6^\circ} \cdot z^{-1})\,(1 - e^{-j129.6^\circ} \cdot z^{-1})$$

$$= (1 - 2\cos 43.2^\circ \cdot z^{-1} + z^{-2})(1 - 2\cos 129.6^\circ \cdot z^{-1} + z^{-2})$$

$$= (1 - 1.45793725484 \cdot z^{-1} + z^{-2})(1 + 1.2748479794 \cdot z^{-1} + z^{-2})$$

$$= 1 - 0.18308927544 \cdot z^{-1} + 0.14135163765 \cdot z^{-2}$$

$$-0.18308927544 \cdot z^{-3} + z^{-4}$$

$$H(1) = 1.775173087$$

進行正規化

$$H(z)$$

$$= \frac{1 - 0.18308927544 \cdot z^{-1} + 0.14135163765 \cdot z^{-2} - 0.18308927544 \cdot z^{-3} + z^{-4}}{1.775173087}$$

$$= 0.5633253497 - 0.1031388301 \cdot z^{-1} + 0.07962696071 \cdot z^{-2}$$

$$-0.1031388301 \cdot z^{-3} + 0.5633253497 \cdot z^{-4}$$

然而

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= 0.5633253497 - 0.1031388301 \cdot z^{-1} + 0.07962696071 \cdot z^{-2}$$

$$-0.1031388301 \cdot z^{-3} + 0.5633253497 \cdot z^{-4}$$

經過整理

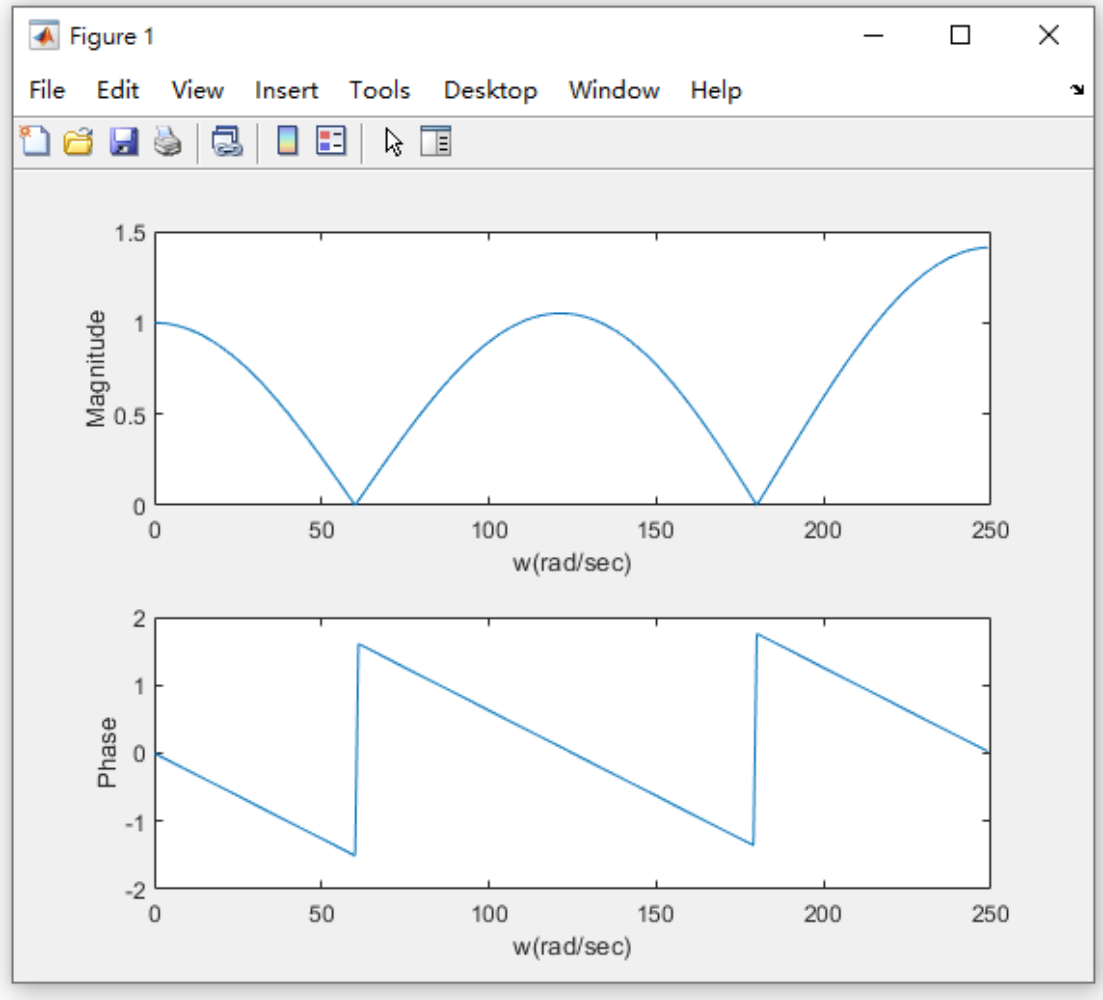
$$Y(z) = 0.5633253497 \cdot X(z) - 0.1031388301 \cdot z^{-1} \cdot X(z)$$

$$+0.07962696071 \cdot z^{-2} \cdot X(z) - 0.1031388301 \cdot z^{-3} \cdot X(z)$$

$$+0.5633253497 \cdot z^{-4} \cdot X(z)$$

進行Z^{-1}得到差分方程式為

$$y[n] = 0.5633253497 \cdot x[n] - 0.1031388301 \cdot x[n-1]$$
$$+0.07962696071 \cdot x[n-2] - 0.1031388301 \cdot x[n-3]$$
$$+0.5633253497 \cdot x[n-4]$$



四、60-Hz Comb Filter + Shank's Recursive (IIR) Filter

fs = 500 Hz

基頻 = 60 Hz

3 倍頻 = 180 Hz

∵ 180° : θ = 250 Hz : 60 Hz

∵ 180° : θ = 250 Hz : 180 Hz

∴ θ = 43.2° , 129.6°

得到

$$H(z) = \frac{(1-e^{j43.2^\circ}\cdot z^{-1})(1-e^{-j43.2^\circ}\cdot z^{-1})(1-e^{j129.6^\circ}\cdot z^{-1})(1-e^{-j129.6^\circ}\cdot z^{-1})}{(1-re^{j43.2^\circ}\cdot z^{-1})(1-re^{-j43.2^\circ}\cdot z^{-1})(1-re^{j129.6^\circ}\cdot z^{-1})(1-re^{-j129.6^\circ}\cdot z^{-1})}$$

$$= \frac{1-0.18308927544\cdot z^{-1}+0.14135163765\cdot z^{-2}-0.18308927544\cdot z^{-3}+z^{-4}}{(1-2r\cos 43.2^\circ\cdot z^{-1}+r^2z^{-2})(1-2r\cos 129.6^\circ\cdot z^{-1}+r^2z^{-2})}$$

$$= \frac{1-0.18308927544\cdot z^{-1}+0.14135163765\cdot z^{-2}-0.18308927544\cdot z^{-3}+z^{-4}}{(1-1.45064756856\cdot z^{-1}+0.990025\cdot z^{-2})(1+1.2684737395\cdot z^{-1}+0.990025\cdot z^{-2})}$$

$$= \frac{1-0.18308927544\cdot z^{-1}+0.14135163765\cdot z^{-2}-0.18308927544\cdot z^{-3}+z^{-4}}{1-0.182173829\cdot z^{-1}+0.13994165402\cdot z^{-2}-0.180356645\cdot z^{-3}+0.98014950062\cdot z^{-4}}$$

$$H(1) = \frac{1.775173087}{1.757560681} = 1.010020938$$

進行正規化

$$H(z) = \frac{\frac{1-0.18308927544\cdot z^{-1}+0.14135163765\cdot z^{-2}-0.18308927544\cdot z^{-3}+z^{-4}}{1.010020938}}{1-0.182173829\cdot z^{-1}+0.13994165402\cdot z^{-2}-0.180356645\cdot z^{-3}+0.98014950062\cdot z^{-4}}$$

然而

$$H(z) = \frac{Y(z)}{X(z)}$$

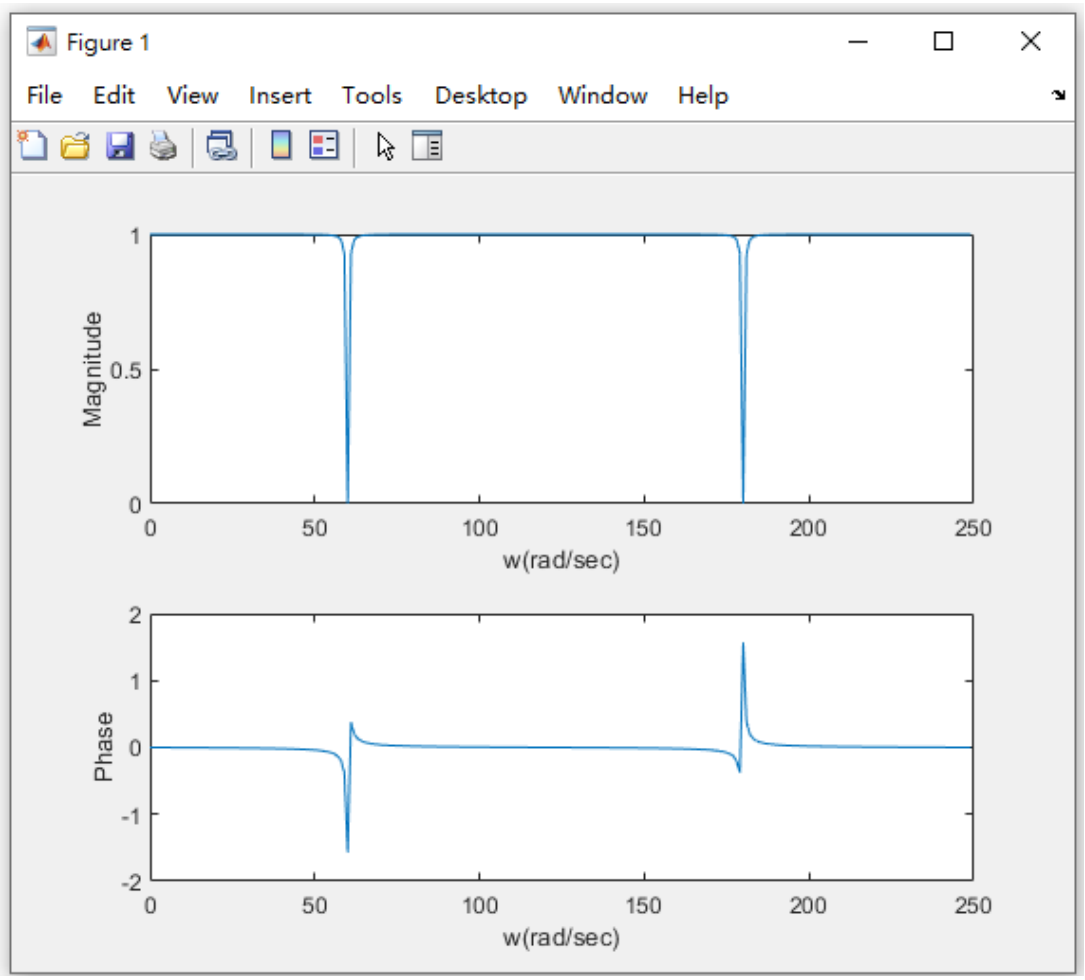
$$= \frac{\frac{1-0.18308927544\cdot z^{-1}+0.14135163765\cdot z^{-2}-0.18308927544\cdot z^{-3}+z^{-4}}{1.010020938}}{1-0.182173829\cdot z^{-1}+0.13994165402\cdot z^{-2}-0.180356645\cdot z^{-3}+0.98014950062\cdot z^{-4}}$$

經過整理

$$1.010020938 \cdot Y(z) - 0.1839993816 \cdot z^{-1} \cdot Y(z) + 0.1413440007 \cdot z^{-2} \cdot Y(z) - 0.1821639878 \cdot z^{-3} \cdot Y(z) + 0.989971518 \cdot z^{-4} \cdot Y(z) = X(z) - 0.18308927544 \cdot z^{-1} \cdot X(z) + 0.14135163765 \cdot z^{-2} \cdot X(z) - 0.18308927544 \cdot z^{-3} \cdot X(z) + z^{-4} \cdot X(z)$$

進行Z^{-1}得到差分方程式為

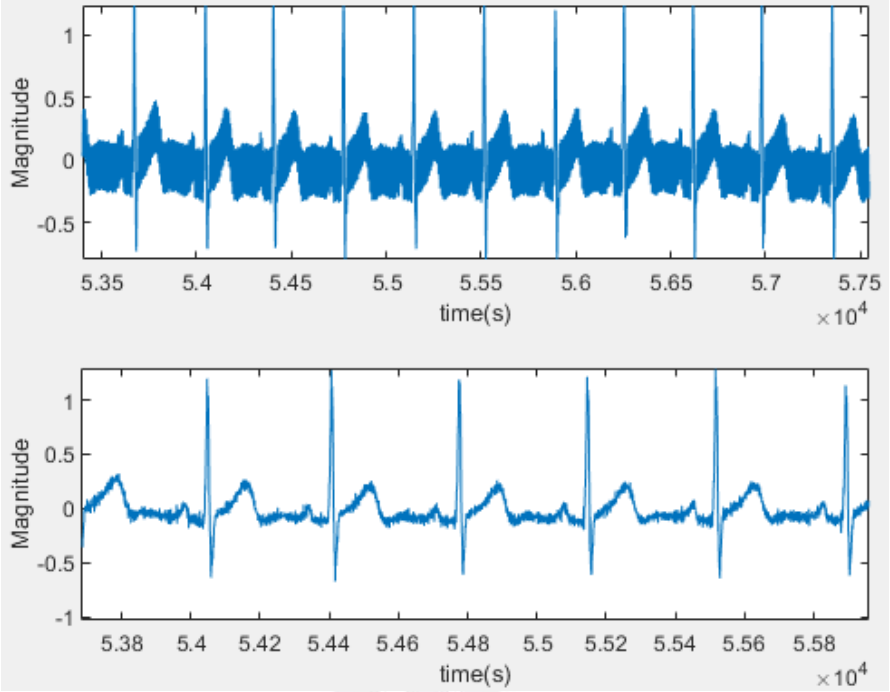
$$1.010020938 \cdot y[n] - 0.1839993816 \cdot y[n-1] + 0.1413440007 \cdot y[n-2] - 0.1821639878 \cdot y[n-3] + 0.989971518 \cdot y[n-4] = x[n] - 0.18308927544 \cdot x[n-1] + 0.14135163765 \cdot x[n-2] - 0.18308927544 \cdot x[n-3] + x[n-4]$$



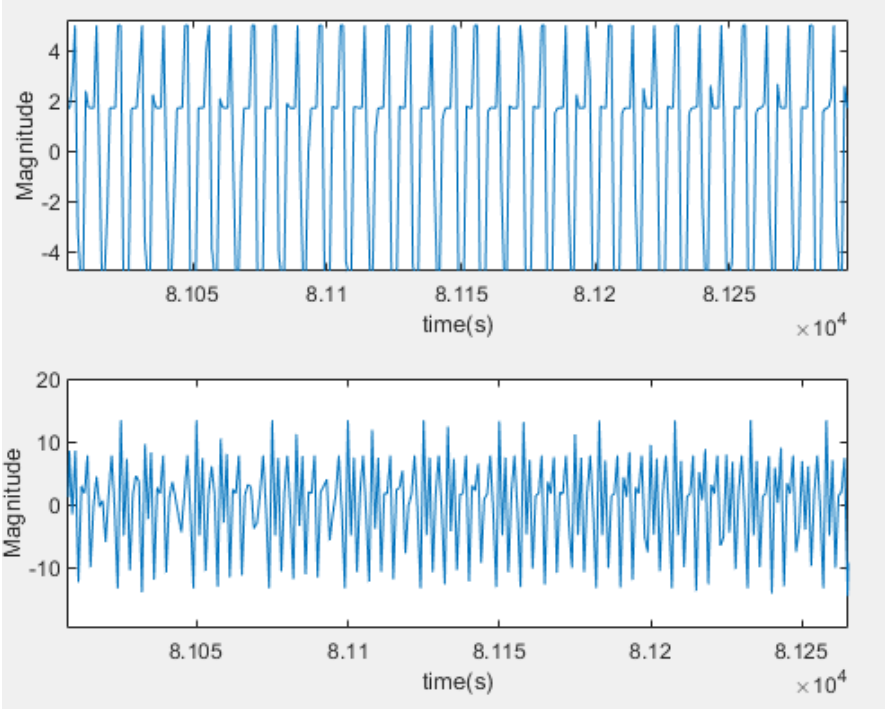
五、利用上述的濾波器過濾心電圖訊號

60-Hz Notch Filter

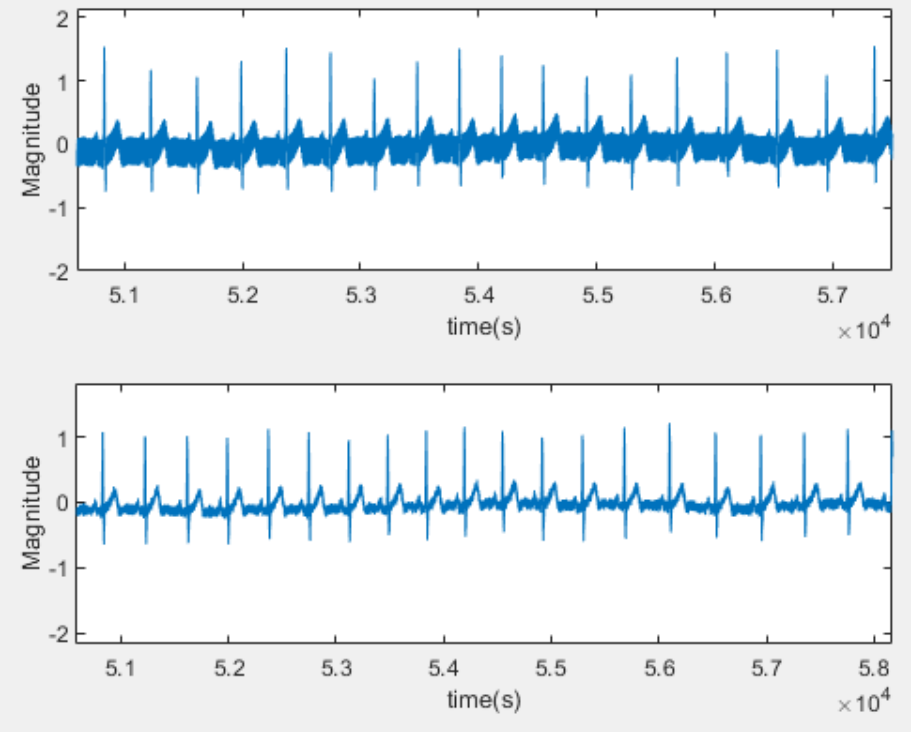
A1a 濾波之後的前後對比



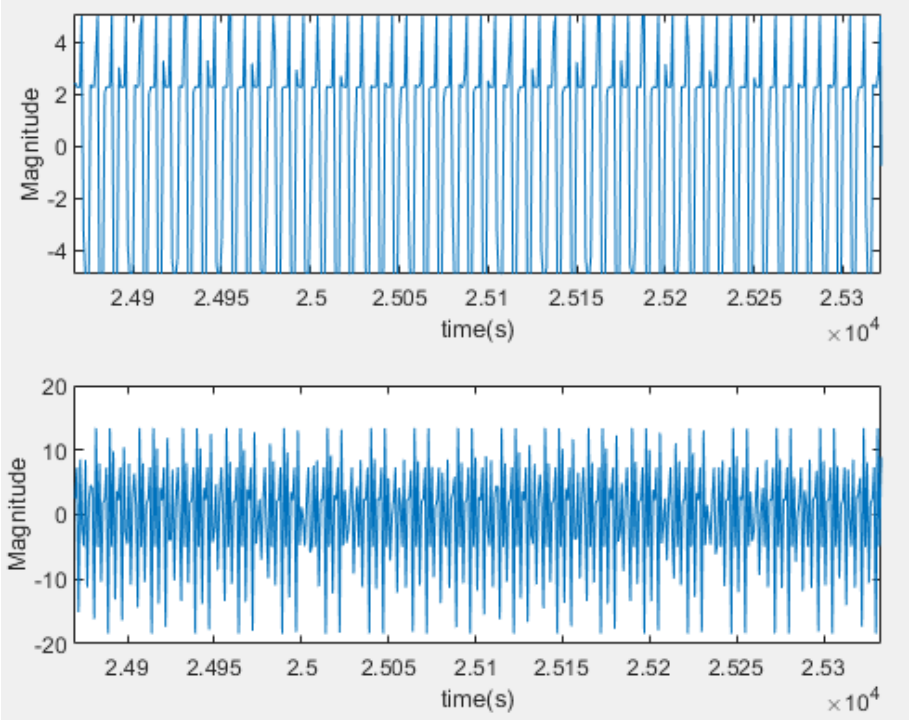
A1b 濾波之後的前後對比



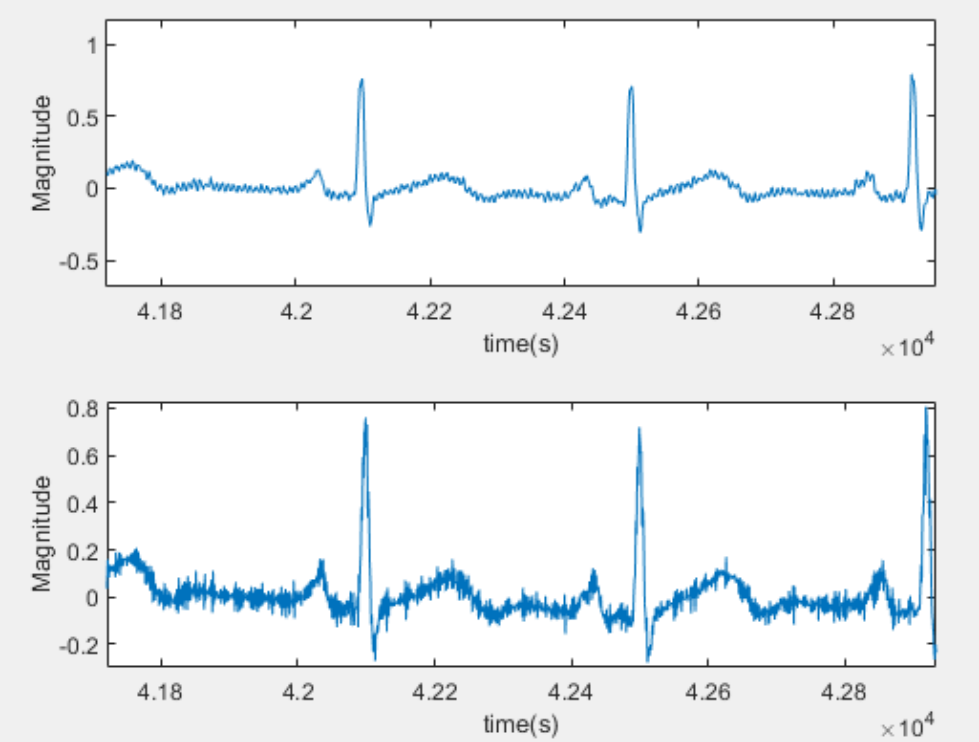
A2a 濾波之後的前後對比



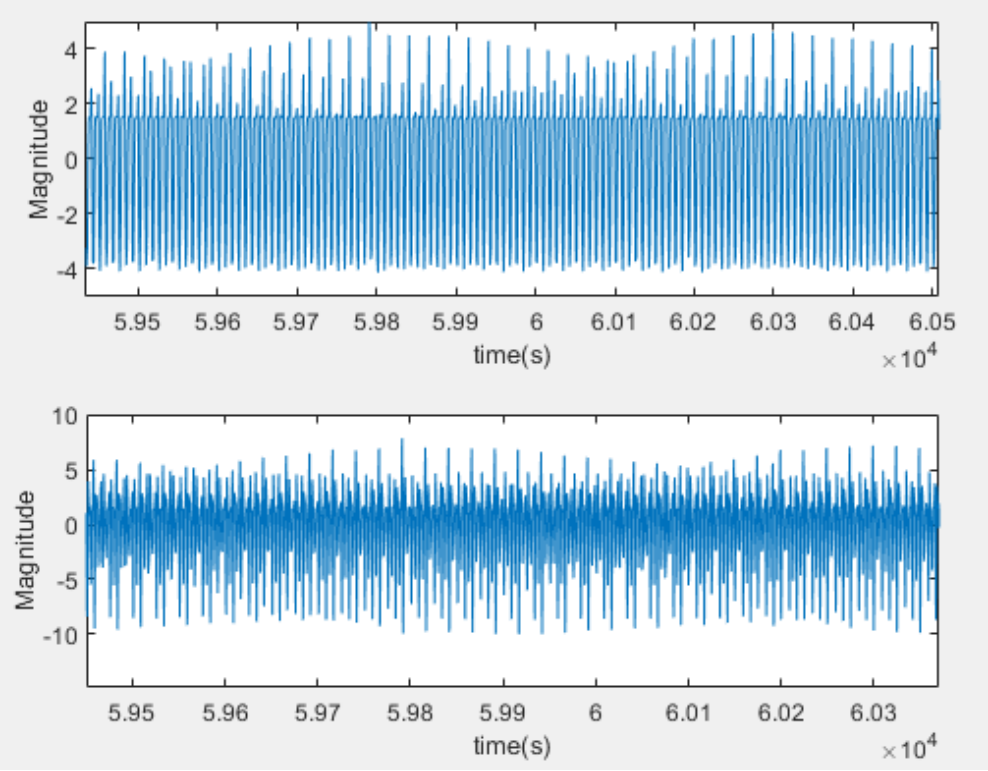
A2b 濾波之後的前後對比



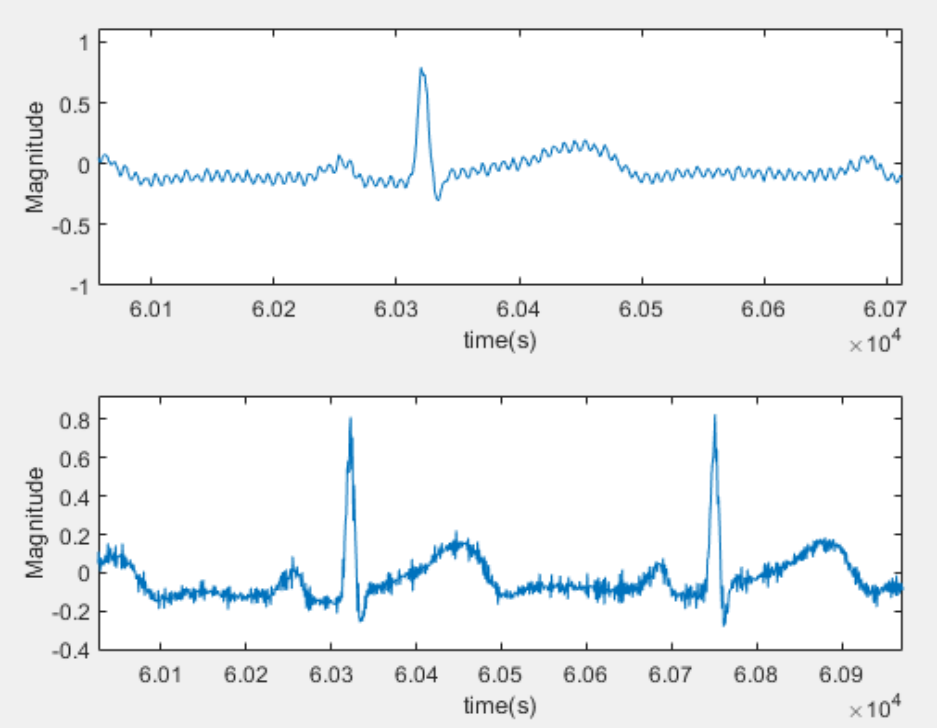
D1a 濾波之後的前後對比



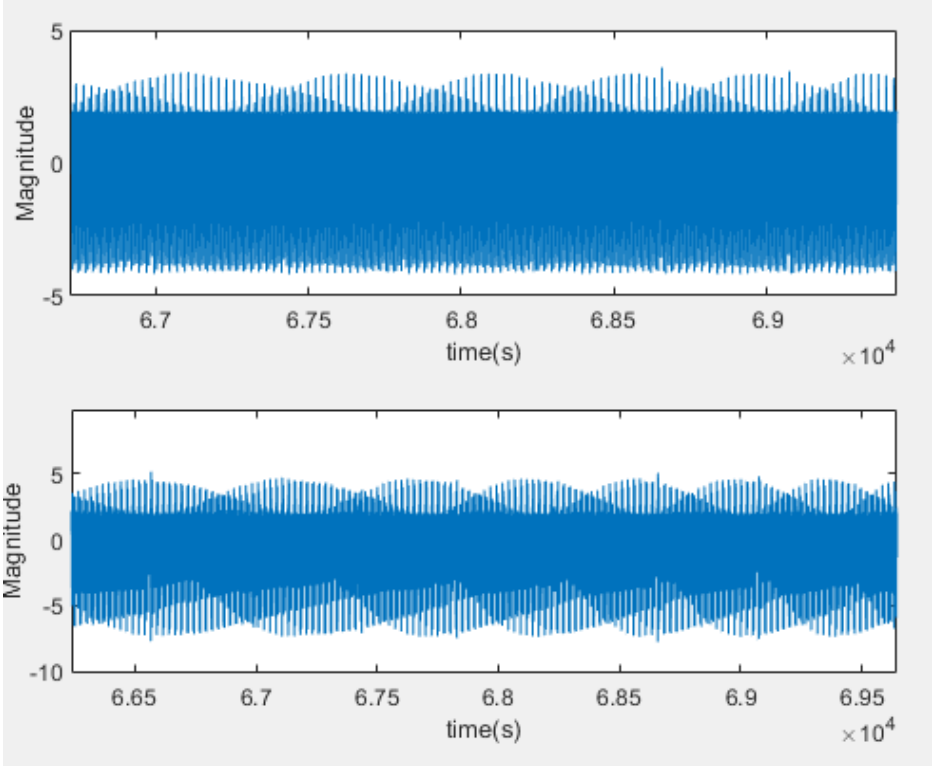
D1b 濾波之後的前後對比



D2a 濾波之後的前後對比

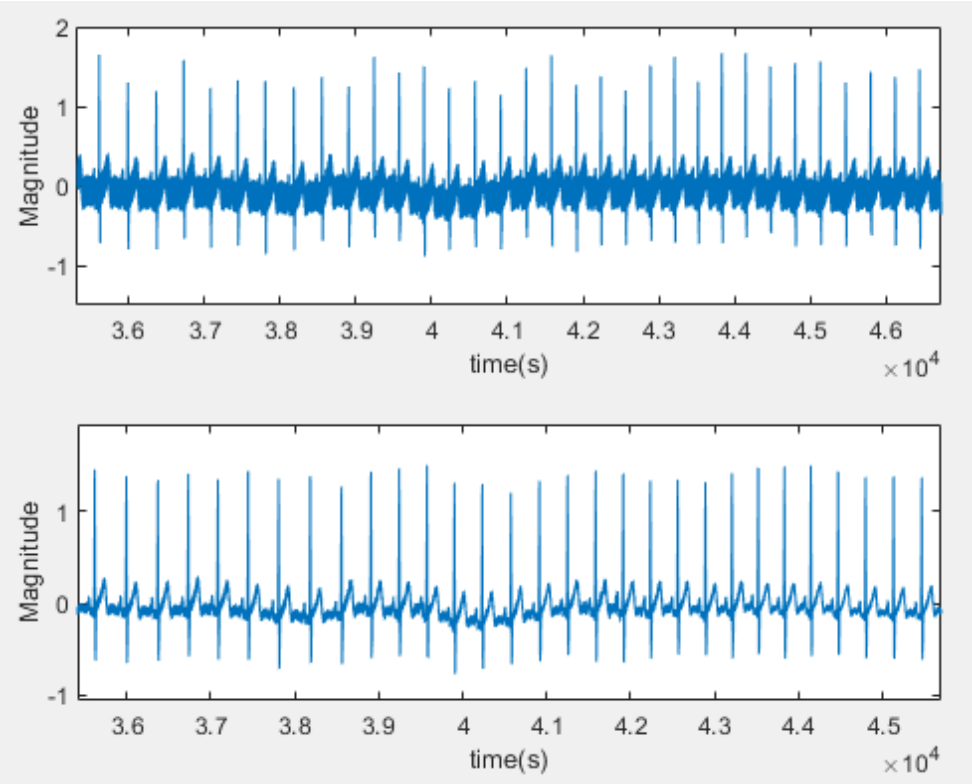


D2b 濾波之後的前後對比

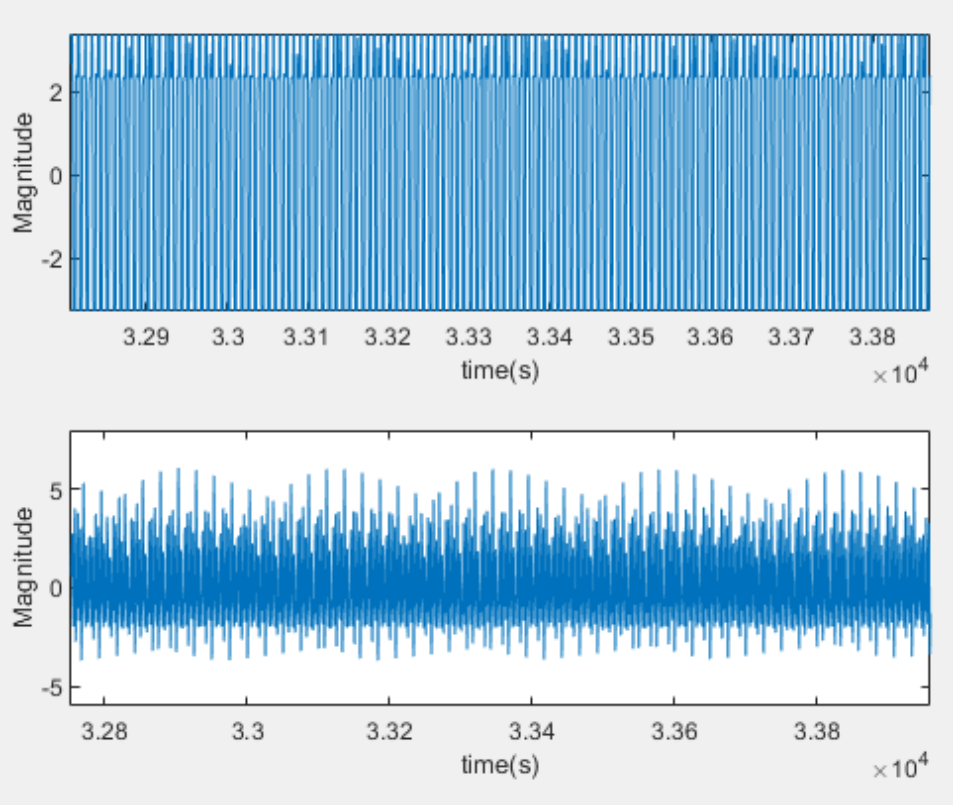


60-Hz Notch Filter+Shank's Recursive (IIR) Filter

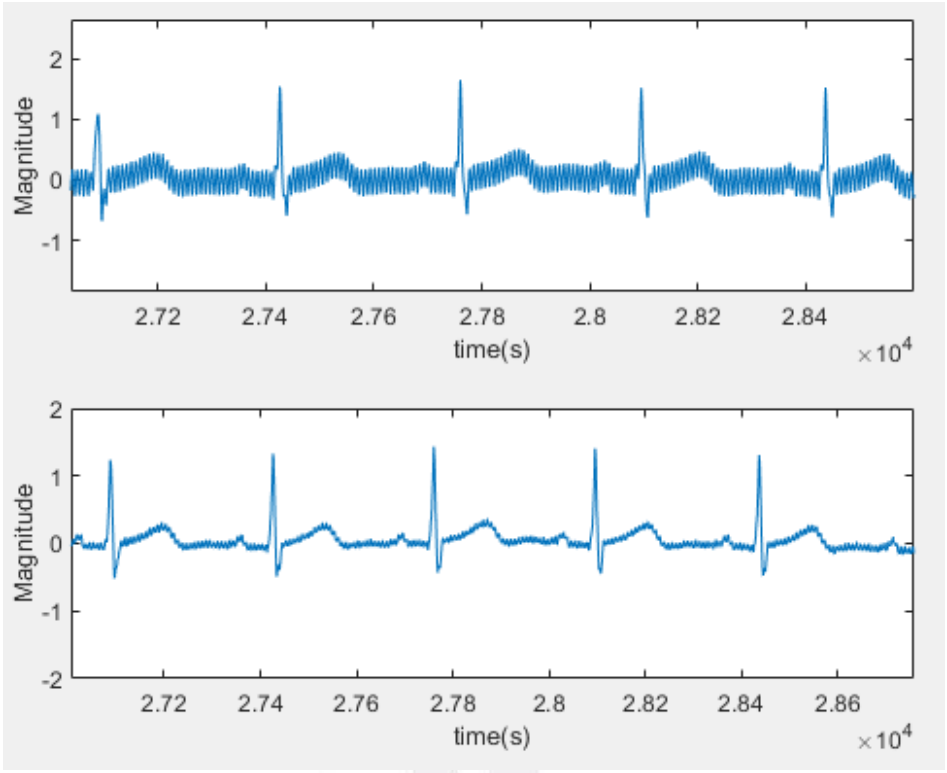
A1a 濾波之後的前後對比



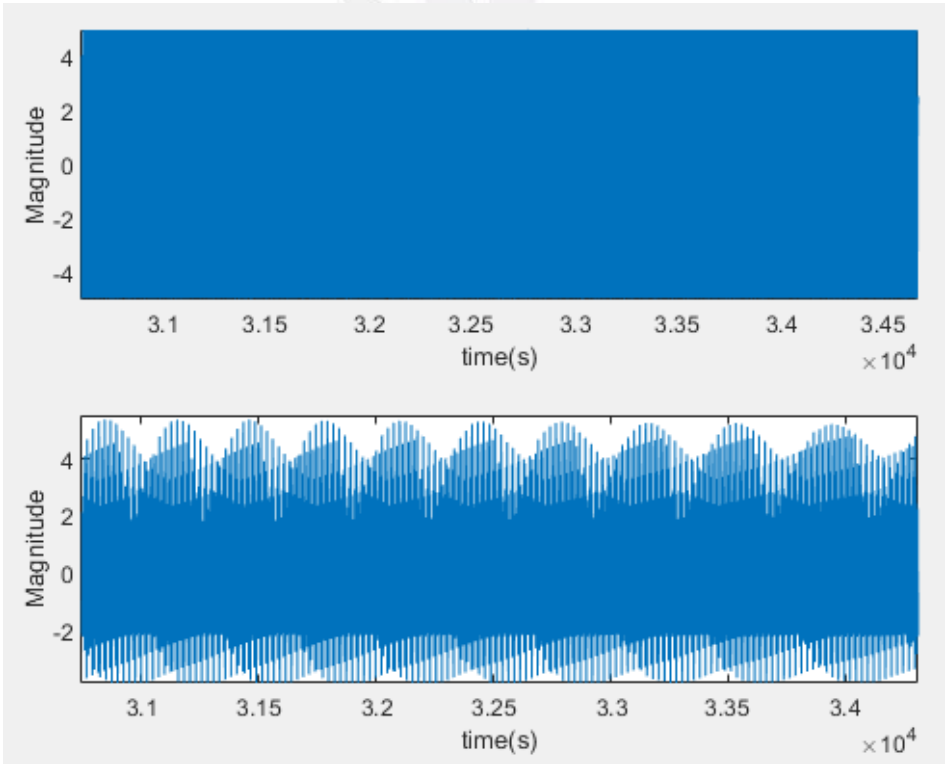
A1b 濾波之後的前後對比



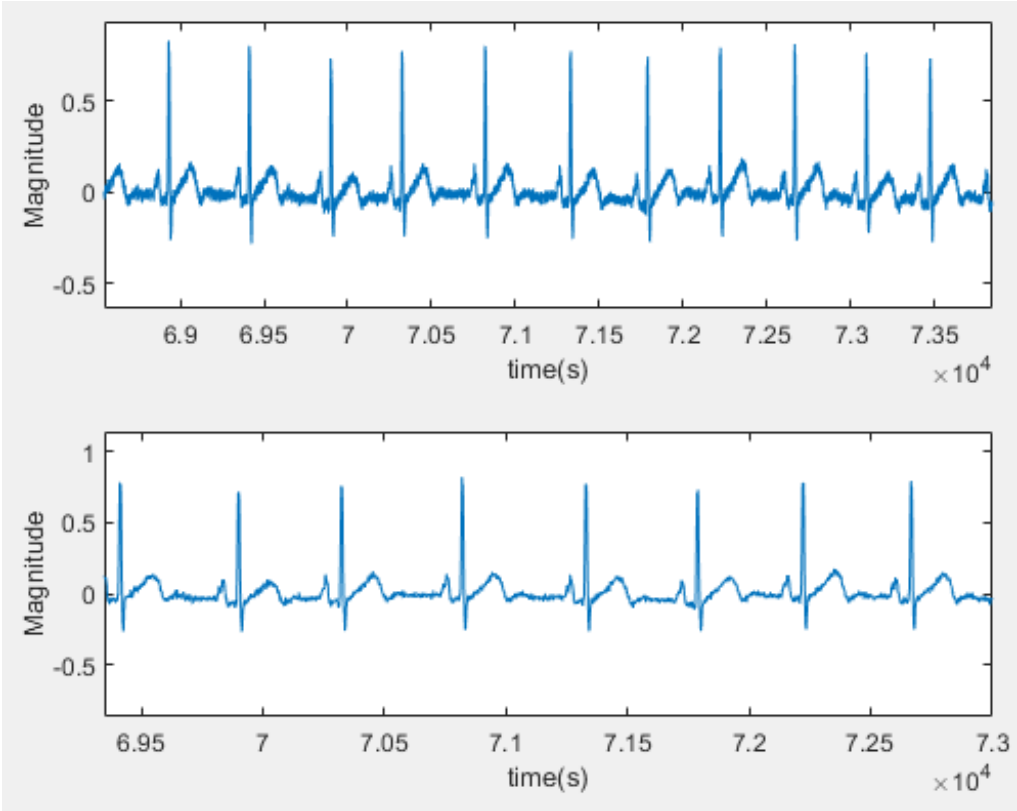
A2a 濾波之後的前後對比



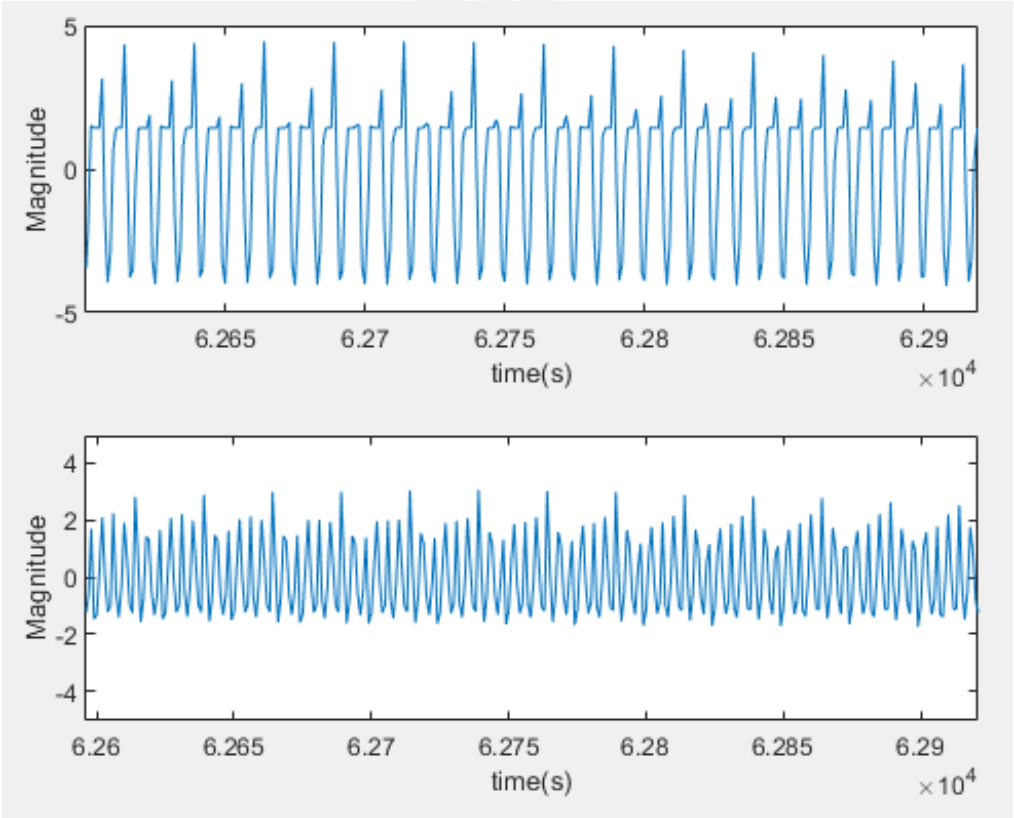
A2b 濾波之後的前後對比



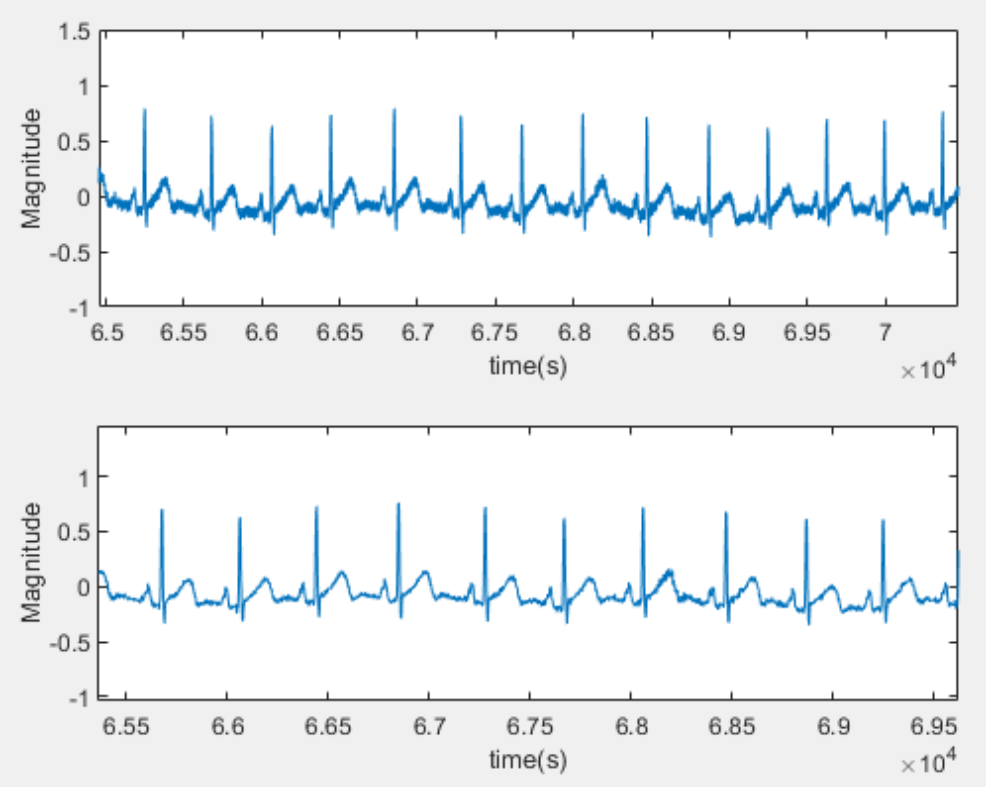
D1a 濾波之後的前後對比



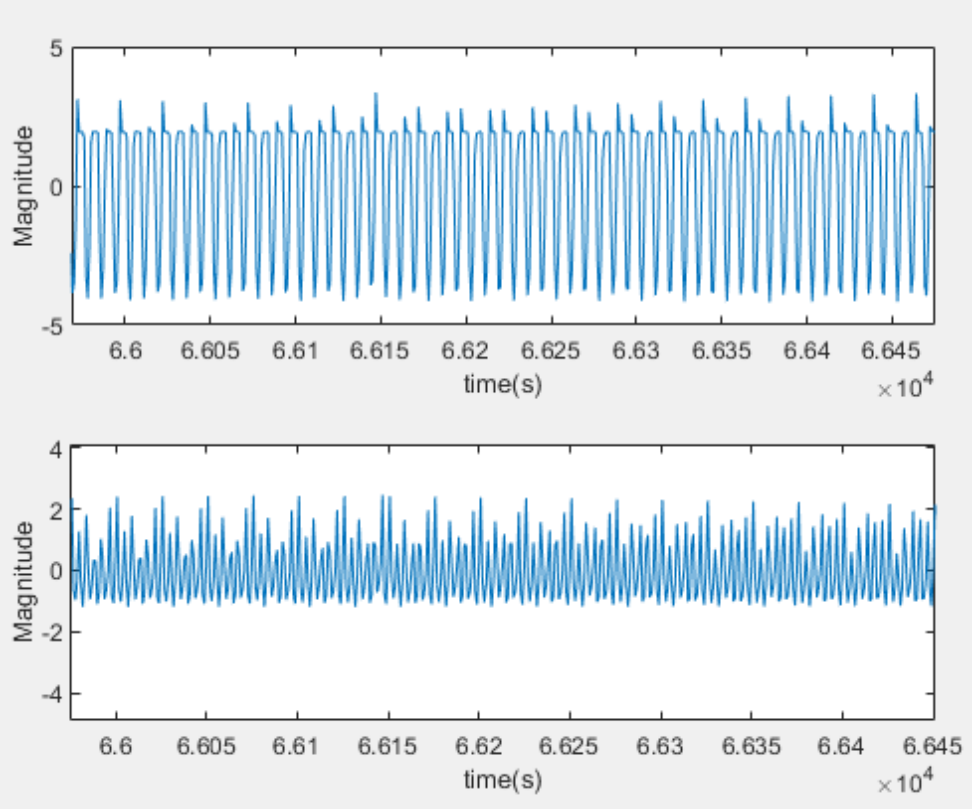
D1b 濾波之後的前後對比



D2a 濾波之後的前後對比

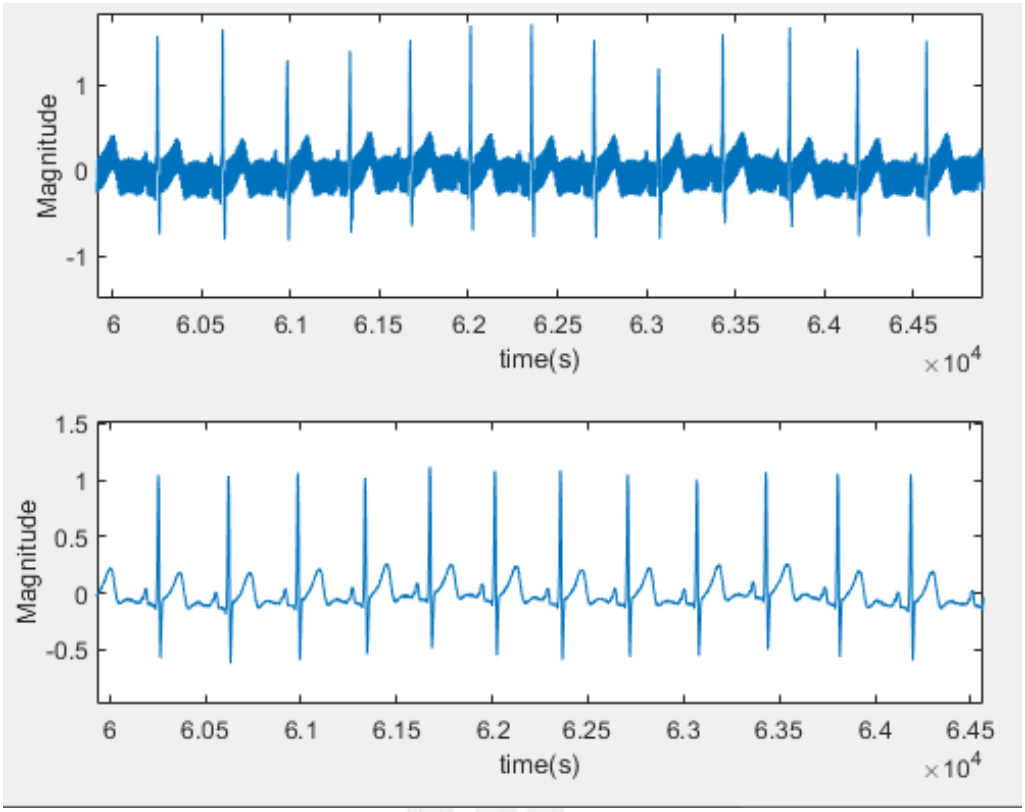


D2b 濾波之後的前後對比

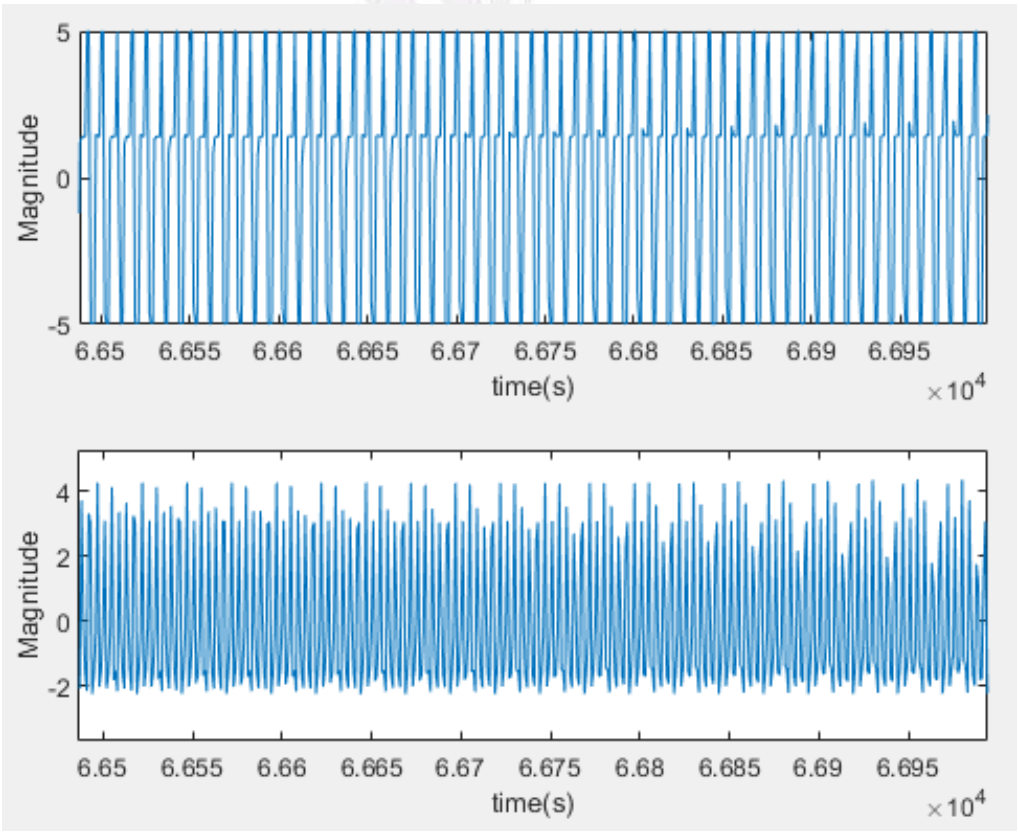


60-Hz Comb Filter

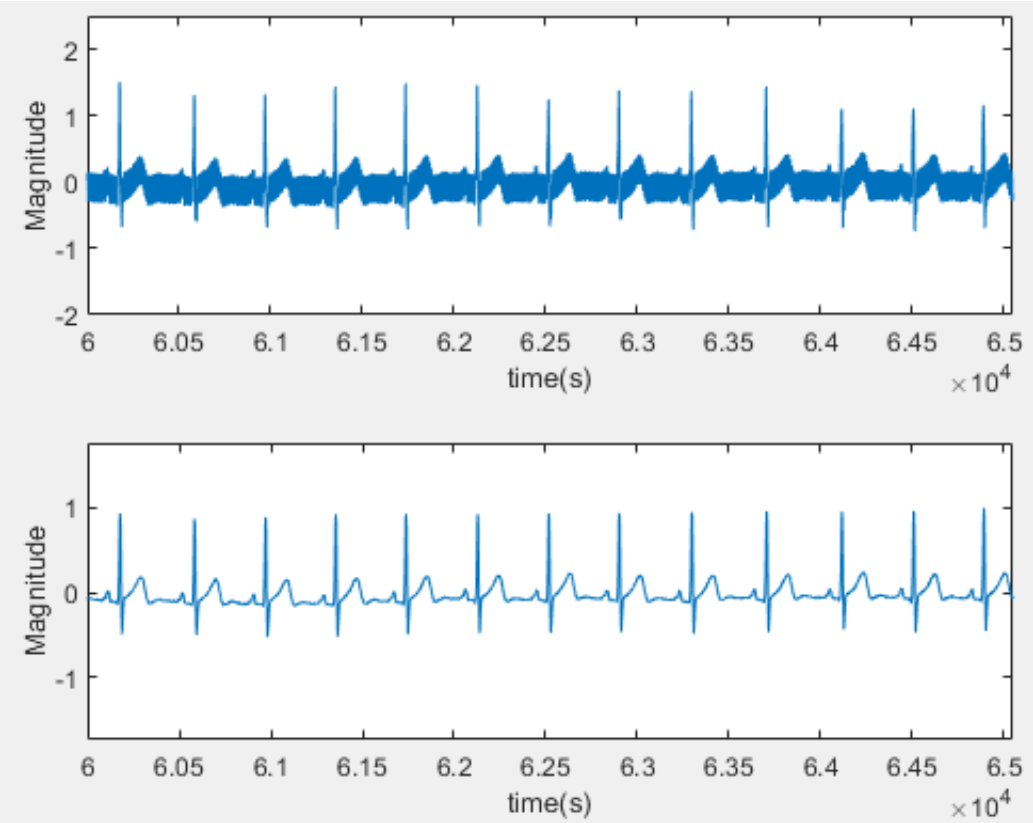
A1a 濾波之後的前後對比



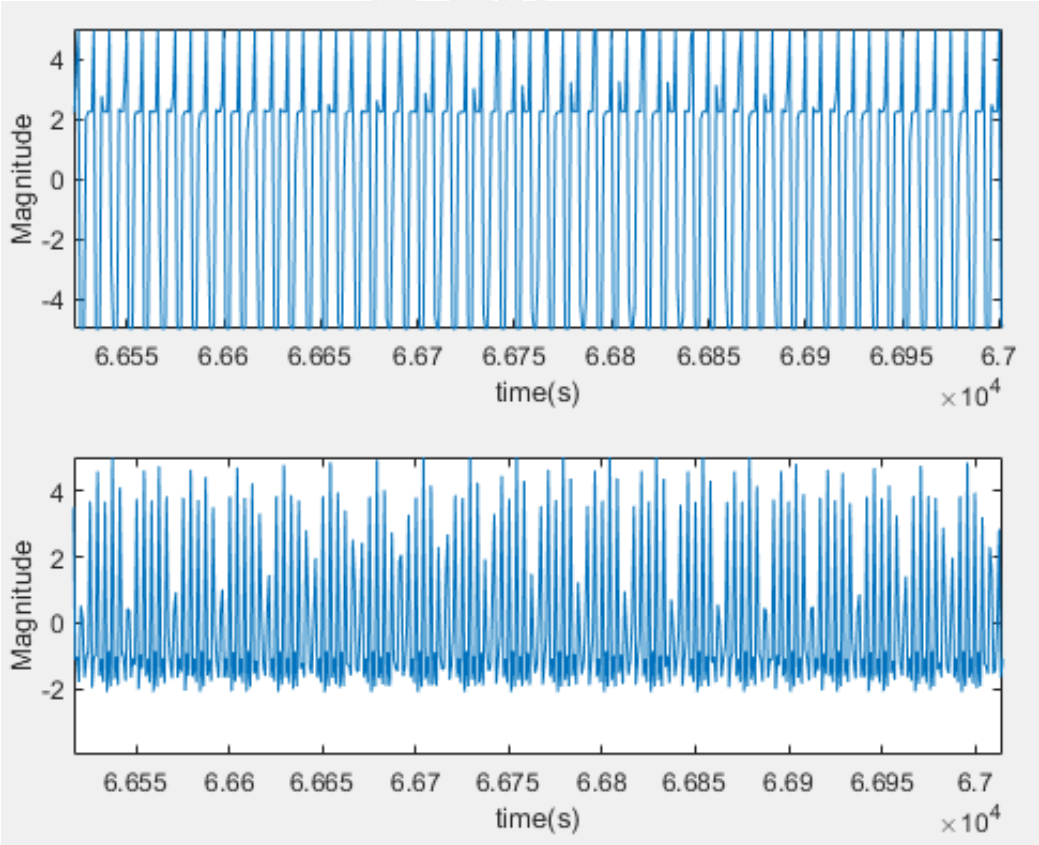
A1b 濾波之後的前後對比



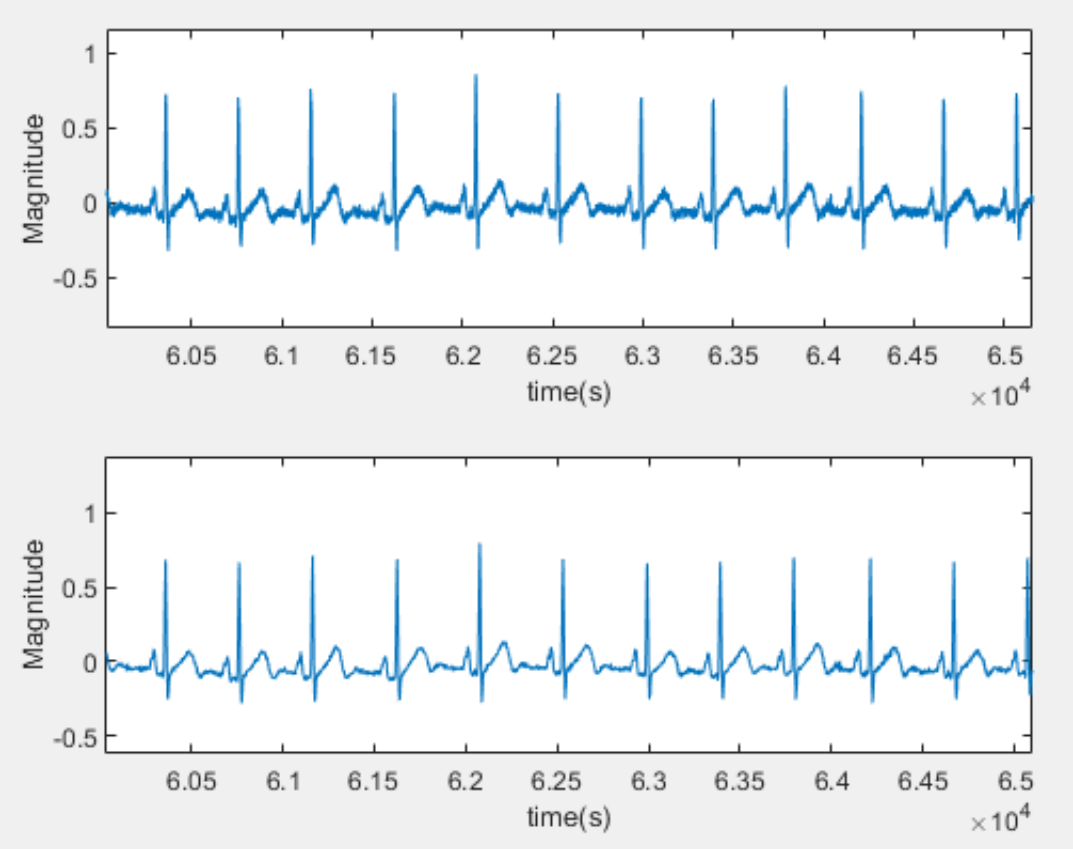
A2a 濾波之後的前後對比



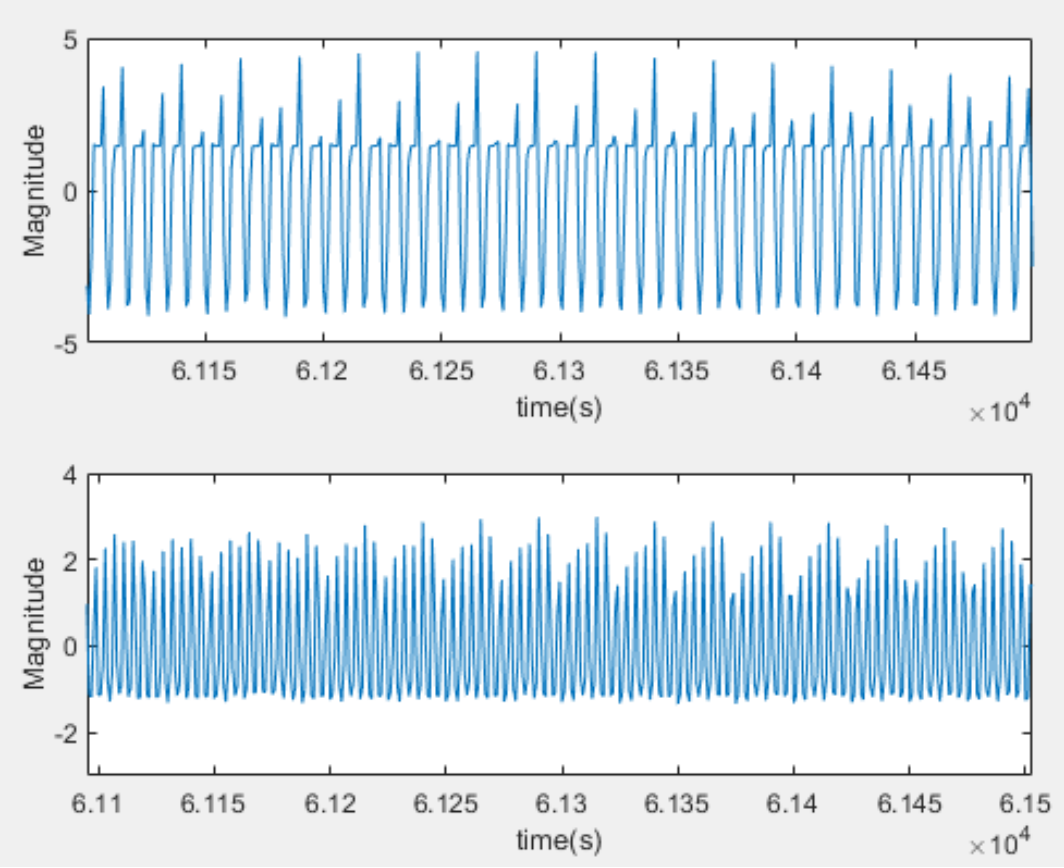
A2b 濾波之後的前後對比



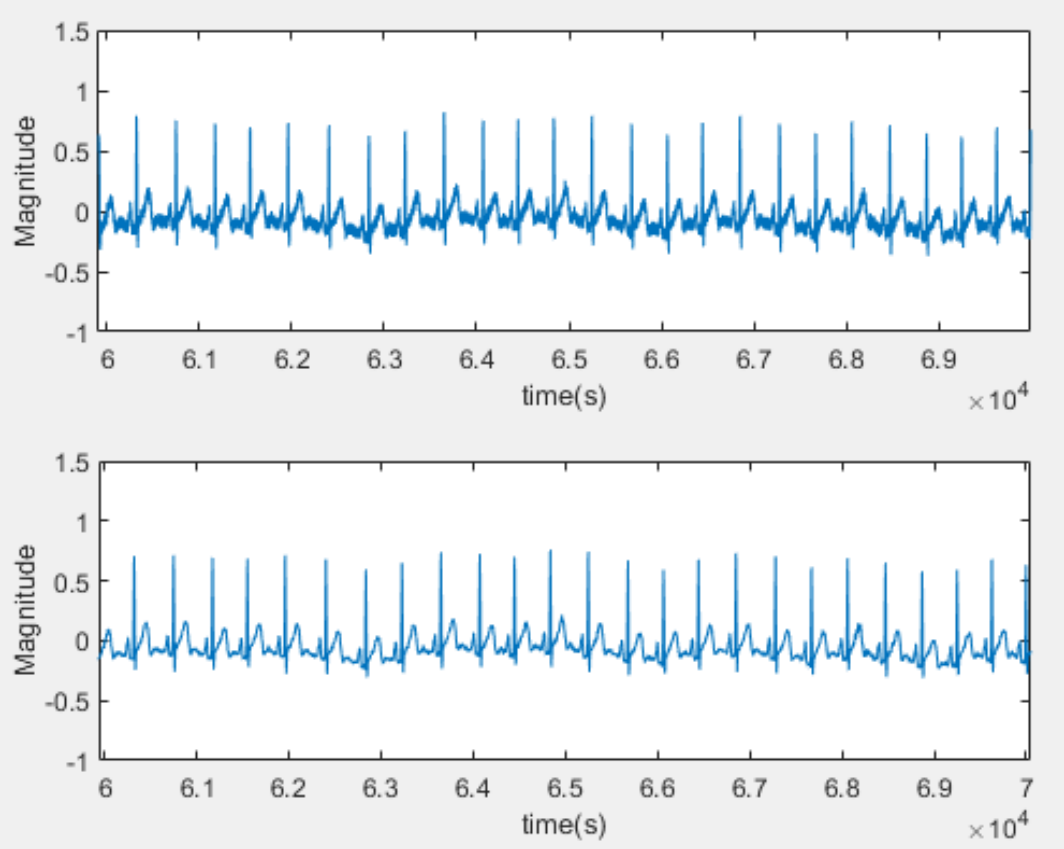
D1a 濾波之後的前後對比



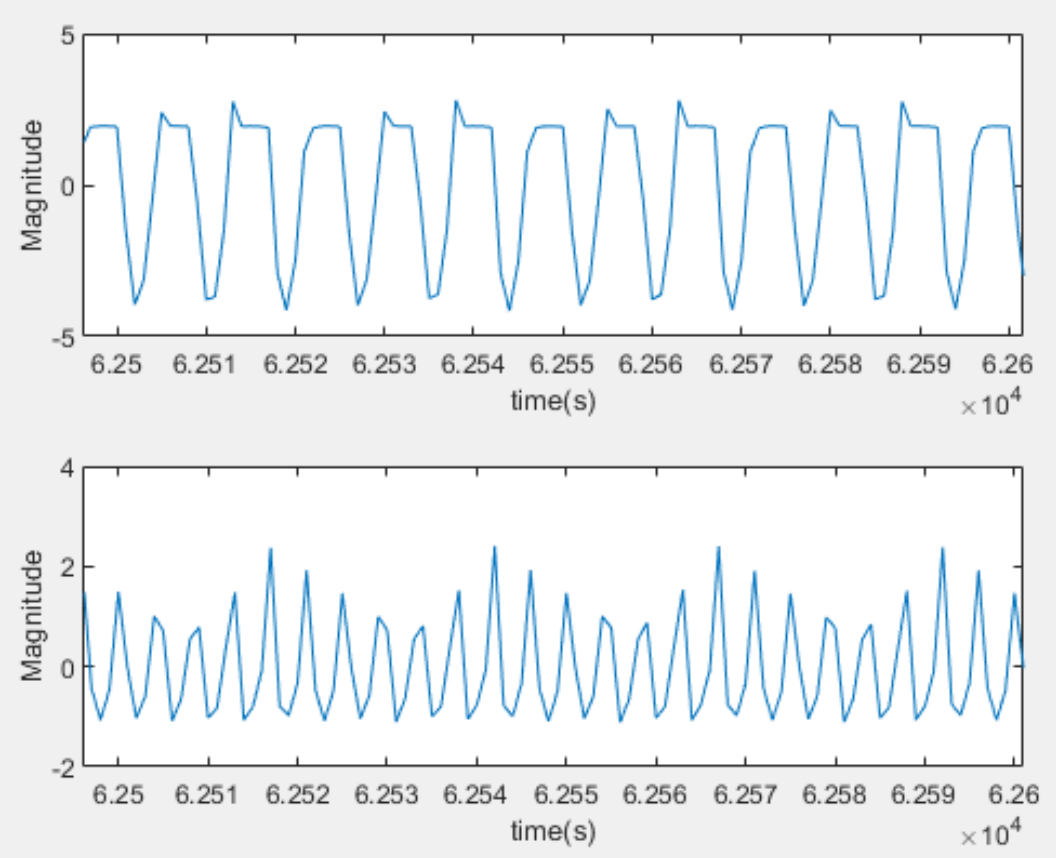
D1b 濾波之後的前後對比



D2a 濾波之後的前後對比

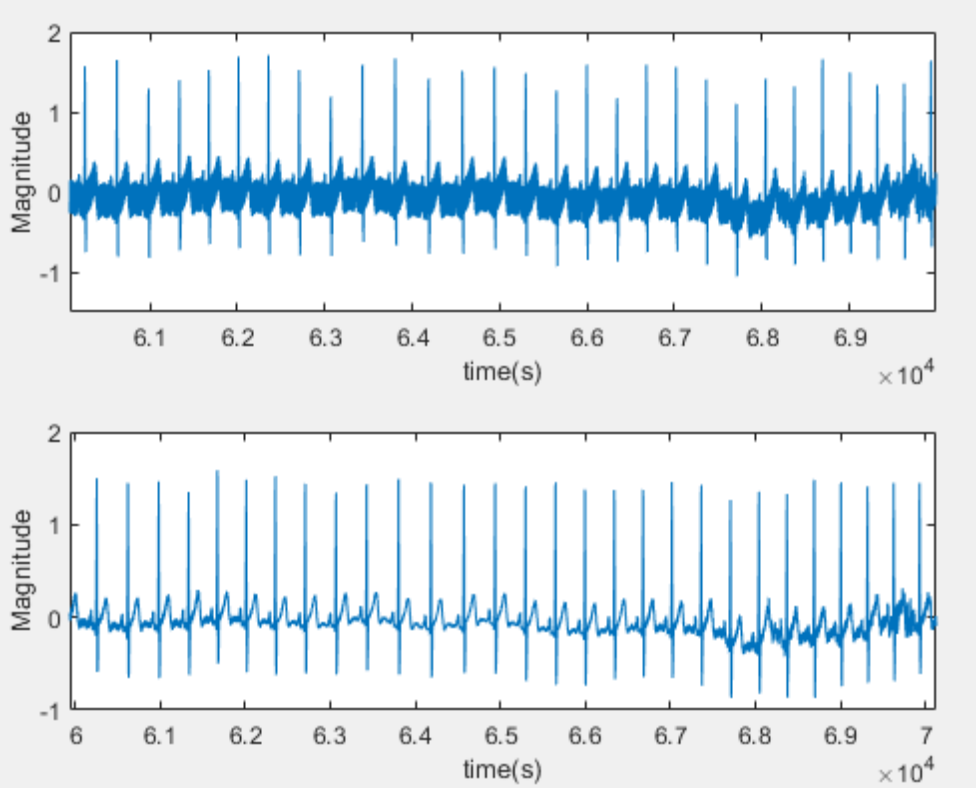


D2b 濾波之後的前後對比

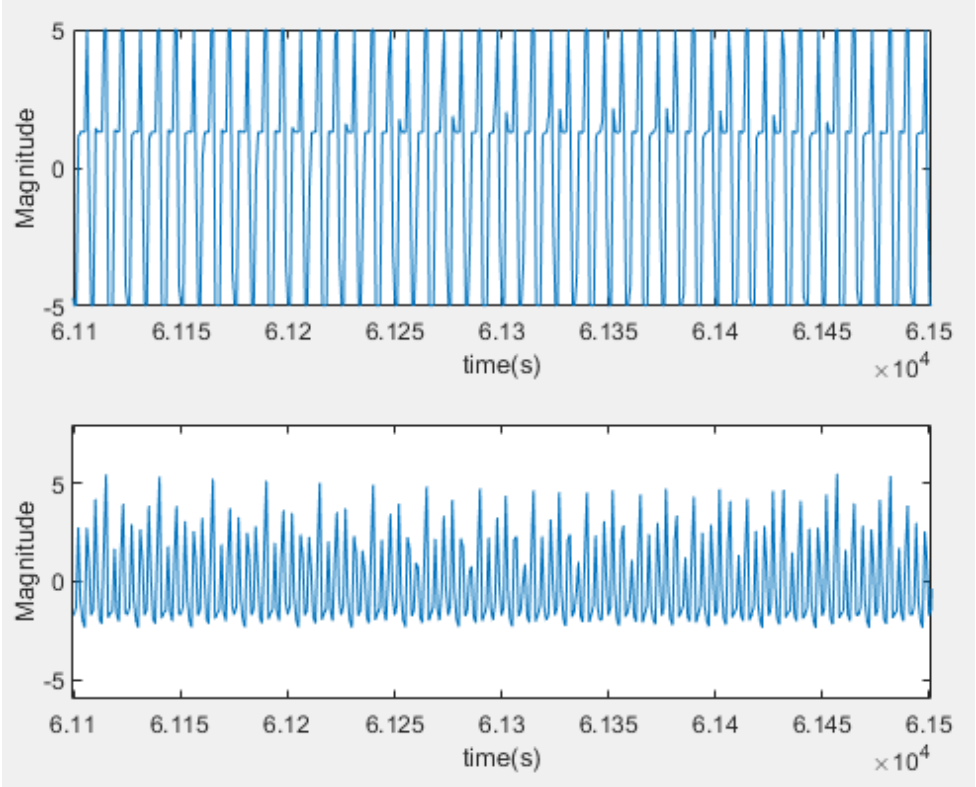


60-Hz Comb Filter+Shank's Recursive (IIR) Filter

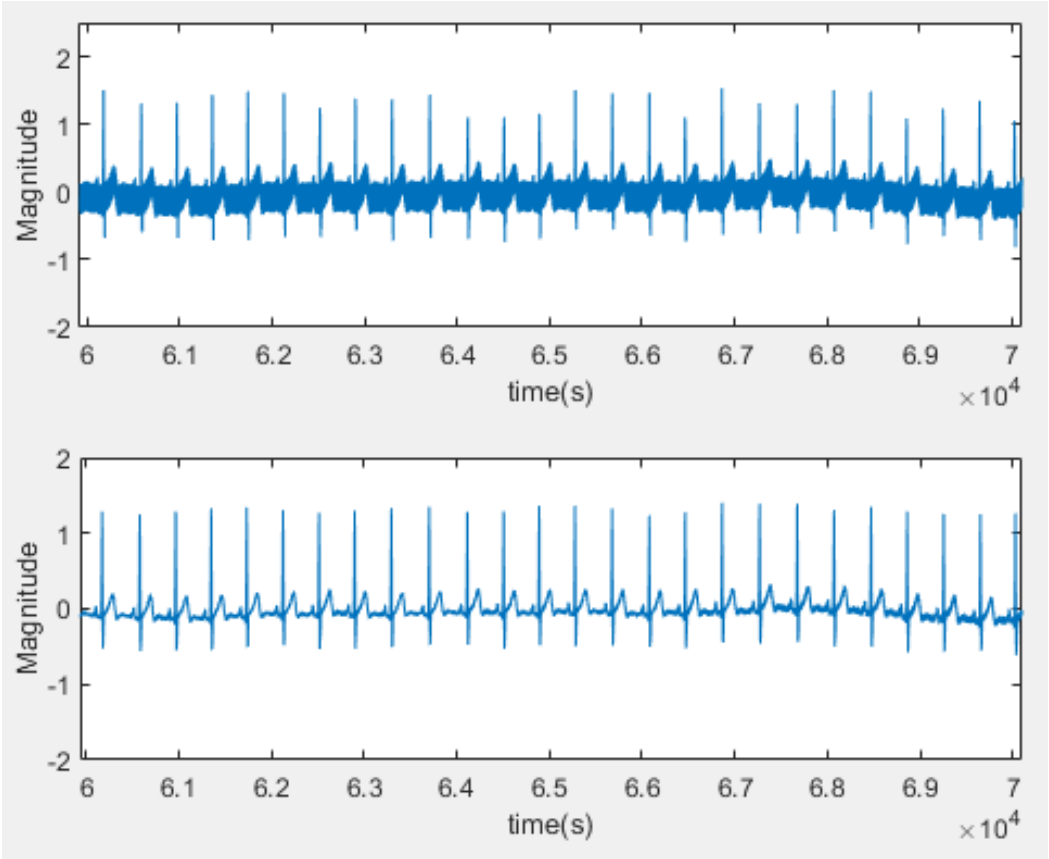
A1a 濾波之後的前後對比



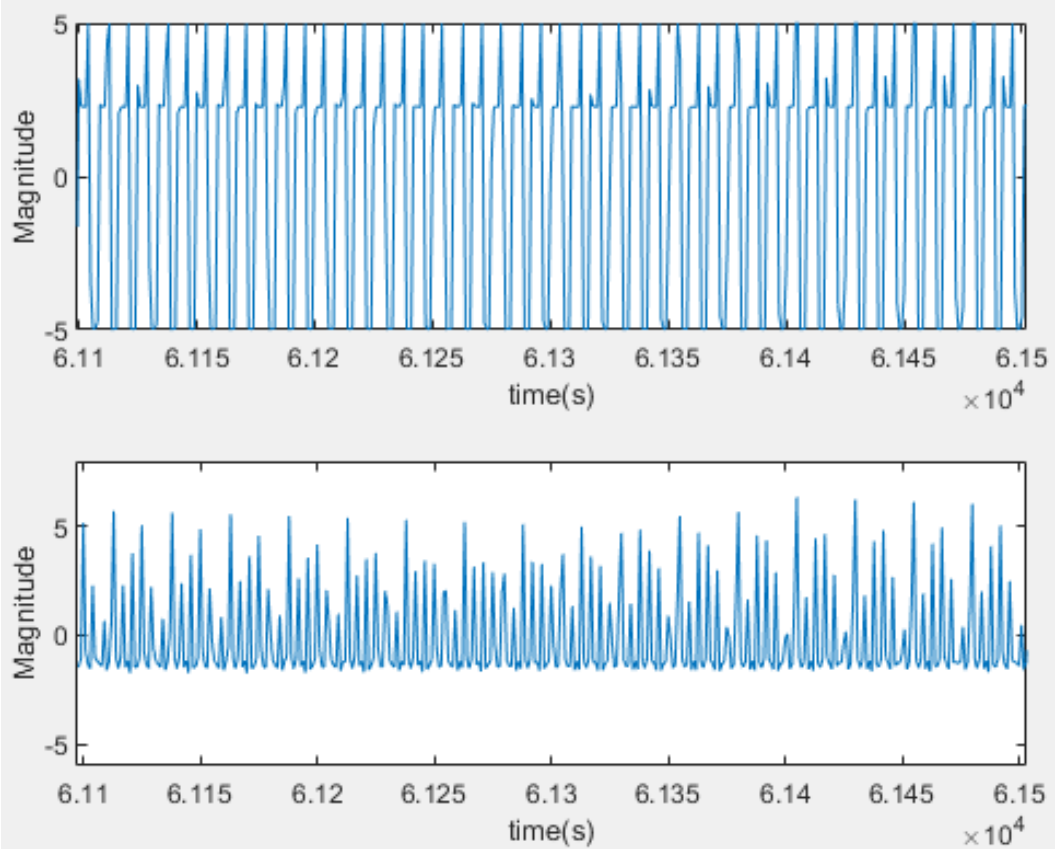
A1b 濾波之後的前後對比



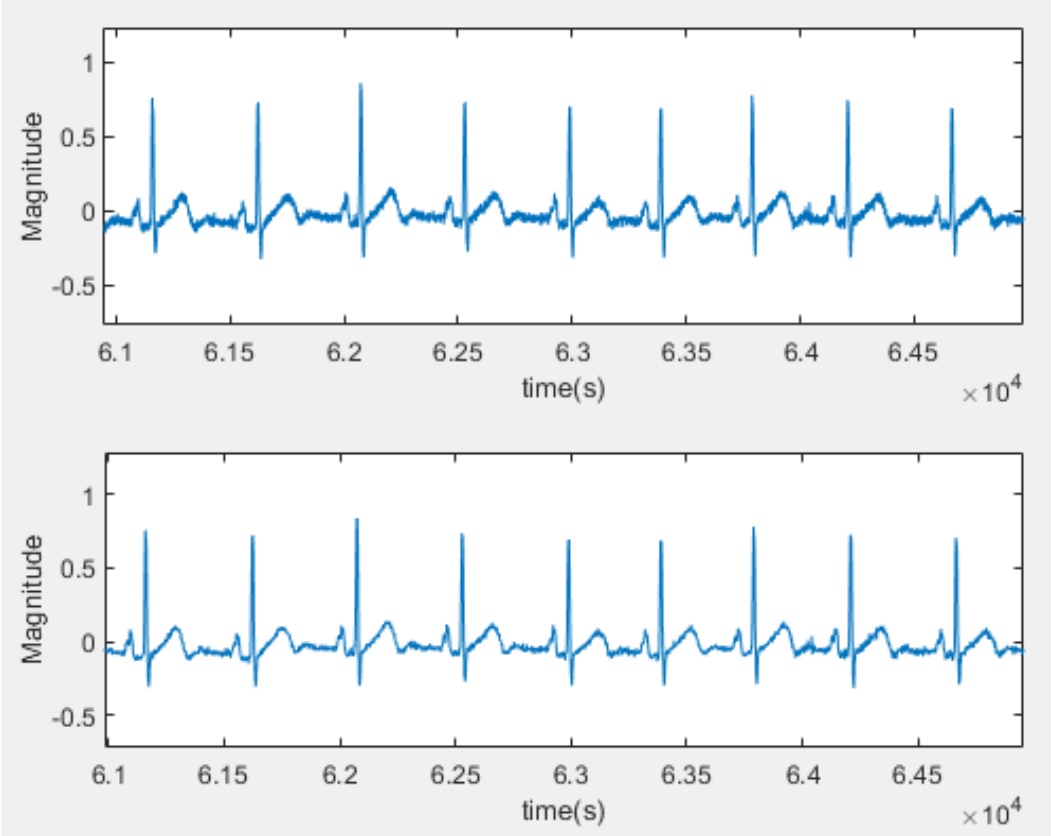
A2a 濾波之後的前後對比



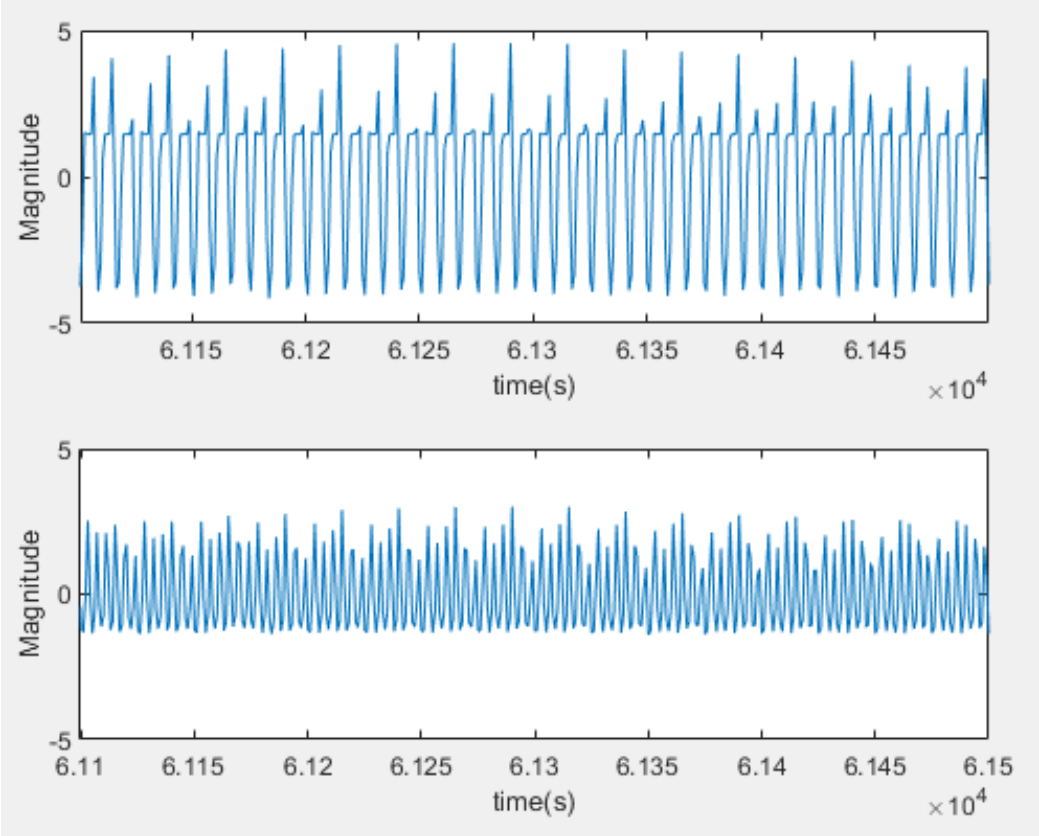
A2b 濾波之後的前後對比



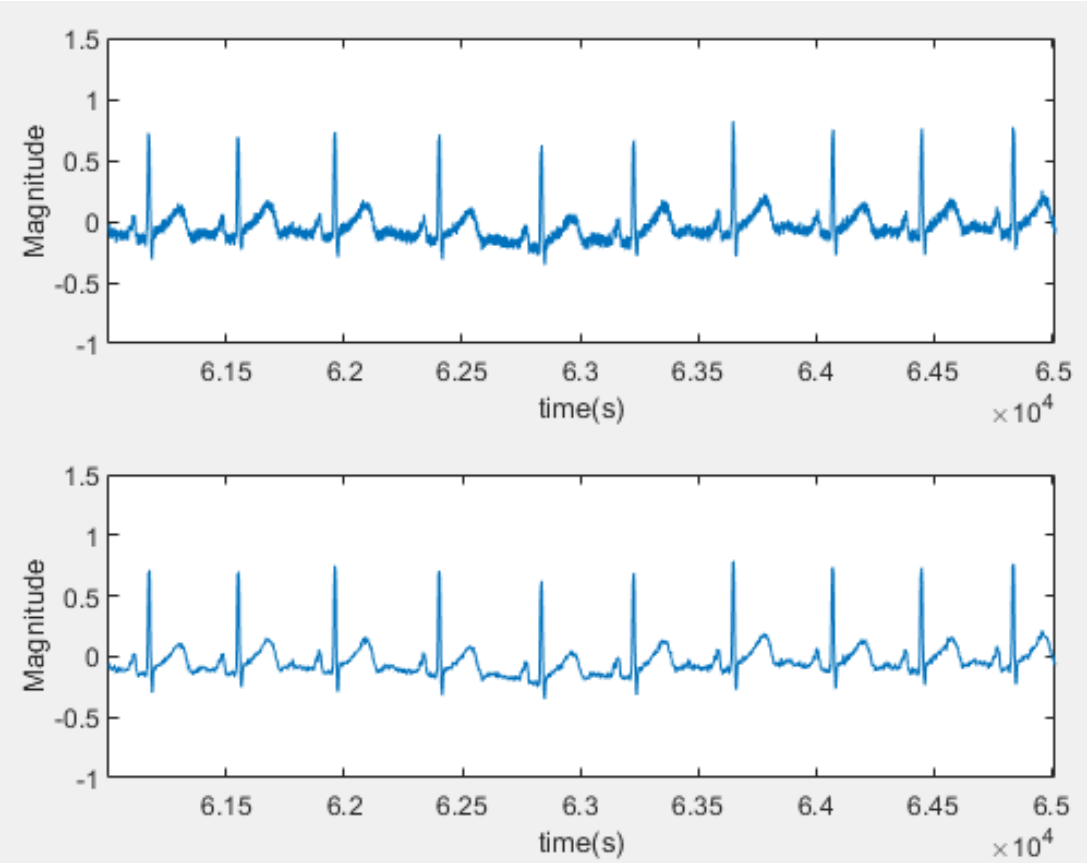
D1a 濾波之後的前後對比



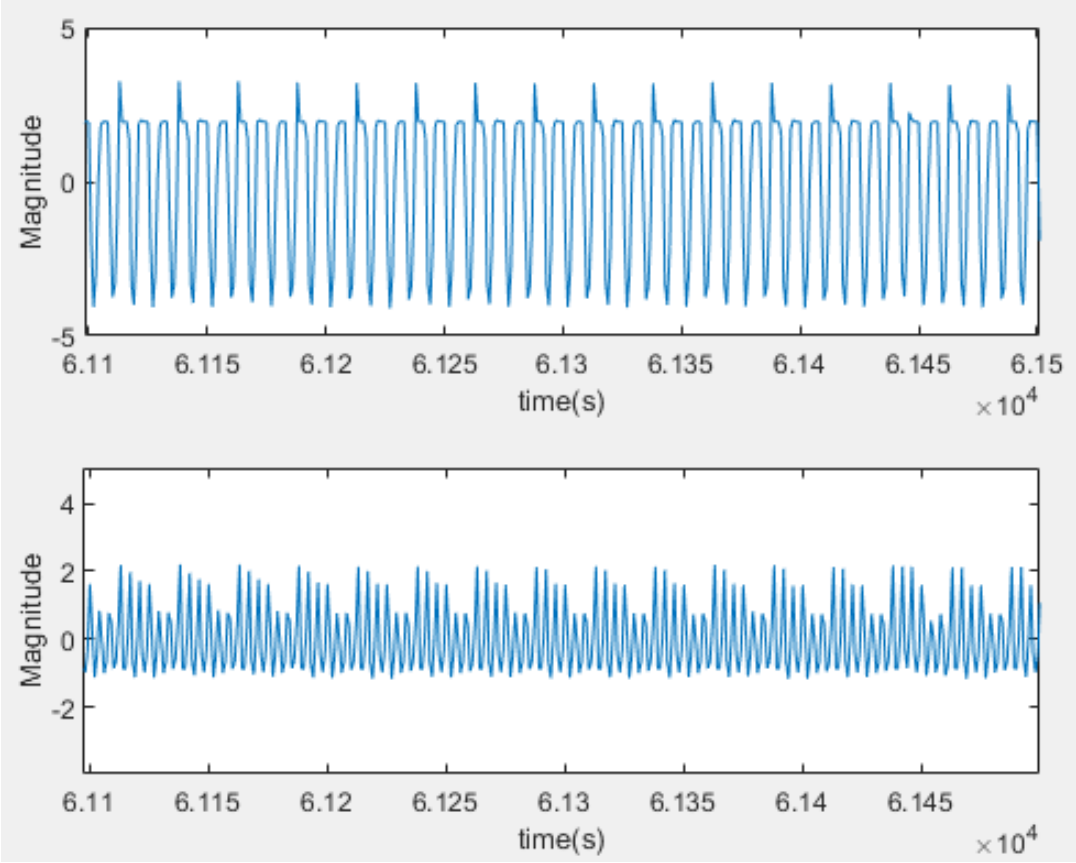
D1b 濾波之後的前後對比



D2a 濾波之後的前後對比



D2b 濾波之後的前後對比



六、參考文獻

R. M. Rangayyan, Biomedical Signal Analysis, 2nd Ed., Wiley, 2015.